Derivative Pricing and Risk Management for Developing emerging Stock Markets: Proposals from Hypothetical Hedging Performance.



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Abstract

Emerging markets are on the edge of introducing derivative products; however, they are unable to calibrate and select from the models that are available in the literature due to the absence of real market derivative prices. The Black and Scholes (BS) model is the sole pricing model that is sufficient for pricing options based on stock index volatility. This is due to the fact that the sophistication of other models necessitates the calibration of additional parameters, which can only converge to true values when calibrated with options data. Moreover, a good part of mispricing under BS model can be addressed when volatility updated with time series information under risk neutral dynamics is used as a proxy to BS's constant volatility calibrated from cross section of stock market information. Delta-hedged profit-loss distribution of hypothetical European derivatives without requiring real market option data under Proxy-BS pricing with several proxies from recent GARCH option pricing literature such as GARCH, NGARCH, GARCH Jump, GARCH Jumps Constant can provide information to emerging markets regarding which Proxy-BS model to use for derivative pricing; at least until real market option data , on developing indices, become available to calibrate and use more sophisticated option pricing model. Such delta-hedged profit-loss distribution can help emerging markets infer the Value-at-Risk and Expected Shortfall for any proxy choice. Then, for the sake of studying the relative riskiness of their chosen proxy-BS model, it can be replicated for number of developed country's indices such as S&P500 markets for which real option data are available. Ultimate goal of this research is to make proposals, with empirical evidence from developed countries derivative markets, regarding which developing country should follow what dynamic proxy for initial pricing of their derivative contracts in the absence of real market derivative contract data on their own indices.

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Certificate

The undersigned certifies that the work, presented here was done by the author (Re-registration No.: 38, Session: 2023-24) himself. The style and contents of the dissertation is suitable for submission for the partial fulfilment of the degree of Doctor of Philosophy in Mathematics under University of Dhaka.



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Candidate's Declaration

I hereby declare that this research work which is presented in this thesis entitled "**Derivative Pricing and Risk Management for Developing emerging Stock Markets: Proposals from Hypothetical Hedging Performance** "submitted in order to the partially fulfil the requirements for the award of the degree of Ph.D. in Mathematics under the faculty of science at the university of Dhaka, Dhaka-1000, Bangladesh is an undeniable record of my work. It has not been submitted elsewhere universities or institutions for the award of any other degree.

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Dedication

To

My late Mother and Father

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Chapter 1

Introduction

1 Introduction

The stock market is among the most risk-sensitive segments of the financial system, with volatility commonly used as a key measure of this risk. Periods of heightened volatility often coincide with major global events such as financial crises, wars, natural disasters, or commodity shocks. A recent and notable example is the sharp market downturn triggered by the COVID-19 pandemic, which highlighted the critical importance of accurately assessing the risks faced by market participants. Given that most individuals are inherently risk-averse, investors seek to anticipate and manage investment risk to protect their capital. As a result, the study of volatility dynamics—particularly within stock markets—has become a central theme in financial research. In recent years, this discussion has extended to include emerging markets. These economies, while increasingly adopting market-oriented frameworks, still represent a relatively small portion of the global financial landscape. Despite having tradable currencies, emerging markets are often perceived as susceptible to sudden regulatory interventions, such as restrictions on exchange rates or capital flows. Additionally, political and economic instability is more common than not in these regions, further distinguishing them from more mature markets and underscoring the need for tailored risk assessment models.

Chapter 2 presents a brief review of the literature, focusing primarily on prior research and empirical studies related to the implementation of Value-at-Risk (VaR) and Expected Shortfall (ES), Conditional Analytic Markov (CAM) and non-conditional analytic Markov models incorporating mean reversion and leverage effects, as well as studies on hypothetical hedging performance.

In chapter 3, we introduce key financial risk measures—Value-at-Risk (VaR) and Expected Shortfall (ES)—along with the fundamental properties of coherent risk measures and the stylized facts commonly observed in financial time series.

In chapter 4, we discuss four user-friendly models from four classes of risk models, each characterized by zero, one, two, or three parameters. The term "user-friendly" is used to

describe these models. The motivation behind selecting sample models from each class is based on the familiarity and user-friendliness of these models in existing research. We think that this familiarity and simplicity of the models, will enhance individual investors' trust in using these models in real-world applications. The organizational choice on model selection should be more stringent and not limited to classes with zero, one, two, or three parametric families. It should also not be influenced solely by user-friendly features, as financial institutions (banks) have sufficient resources to evaluate the efficacy of models.

The historical simulation model is our choice from the class of zero parametric (non-parametric) risk forecasting models; JPMorgan's RiskMetricsTM model, also referred to as the exponentially weighted moving average (EWMA) model, is our choice from the class of risk models with one parameter. The two parametric risk models that we have chosen as our representative selection are the benchmark Black and Scholes dynamics. Ultimately, we have chosen the student-t (with location scale) model among the three parametric risk models in this class.

Currently, Bangladesh is encountering worldwide economic difficulties, including higher costs of goods and a sudden increase in the cost of imports. This has led to an increasing gap in the trade deficit, which in turn puts a major strain on the country's foreign exchange reserves. Furthermore, the escalating number of impaired loans is a significant issue for the banking industry in Bangladesh, as it jeopardizes both its financial health and economic progress. The insufficient evaluation, supervision, and retrieval procedures in Bangladeshi banks are responsible for the buildup of non-performing loans (NPLs) (Rahman & Khan, 2017). (Siddiqui & Hossain, 2018) have identified political meddling and inadequate regulatory control as major contributors to the rise in unacceptable debts in Bangladeshi banks. (Rahman, Hossain, & Alam, 2020) conducted a new investigation to examine the relationship between non-performing loans and Bangladesh's economic health.

The study also suggested strategies to improve the flexibility of the banking sector. Efficient regulatory changes and oversight are essential in managing non-performing loans (NPLs). (Rahman & Ullah, 2022) conducted a recent study that analysed the impact of supervisory modifications on non-performing loans (NPLs) in Bangladesh, emphasizing the need for an effective supervisory strategy. Although there are constraints in the banking industry, Bangladesh is progressing towards removing itself from the United Nations' list of least developed countries (LDC) by 2026. The country is working to achieve its goal by prioritizing multiple efforts. These include expanding shipments in addition to the RMG industry in order to generate job possibilities in an attractive business climate, enhancing the qualified workforce and human resources to develop effective systems, providing substantial assistance to countryside food production, and making outstanding progress in the fishery and fishing businesses. Bangladesh is actively improving its policy framework for attracting private investors, strengthening the banking and finance sectors, and bolstering government agencies, particularly fiscal reforms, to generate greater internal revenue for sustainable progress.

The banking sector in Bangladesh plays an essential role in the country's economic growth by acting as a financial intermediary and facilitating and promoting developments in both the state and commercial sectors. This sector has made significant advancements through a few state digitization attempts, including the implementation of the marketplace infrastructural component to support computerized bidding and conducting business for government financial instruments, as well as the establishment of a computerized financial data centre to enhance risk management procedures in banks.

Furthermore, the finance sector has implemented surveillance of L/C (letter of credit) openings, as well as trade notifications issued by AD (authorized dealer) departments of the corresponding banks and financial institutions. It has also introduced a computerized clearinghouse establishment to facilitate immediate inter-bank fund transfers and has implemented electronic funds transfers, among other initiatives. These enable the

immediate processing of the majority of settlements as a component of the fundamental banking system within a safe online setting for monetary transactions. The banking industry in Bangladesh also provides mobile banking facilities to facilitate faster and more secure financial transactions, extending complete financial services to the country's most remote areas. Furthermore, BASEL-III implementation began in 2015 and followed a gradual approach, with full implementation at the start of 2019 for the Capital Adequacy Ratio (CAR). To ensure thorough and rigorous risk management processes, we have published regulations on stress testing to evaluate the ability of banking and other financial organizations to withstand various unfavourable situations. Private investors are currently not actively engaging in the financial markets as much as they did in the past, despite various efforts. While they do participate, their actions often disrupt the marketplace and cause serious problems for their families and society as a whole. In contrast to other sectors, private investors who invest in banking shares on stock exchanges tend to exhibit foolish behaviour and lack awareness of the financial risks associated with their financial decisions.

Lastly, we have taken into consideration the HS and RiskMetricsTM models in order to choose a selected group of the best-performing, investor-friendly banks in Bangladesh's growing economy, taking into account the efficiency of the ES and VaR projections, subject to our user-friendliness design. This would assist individual investors in selecting a Bangladeshi investment bank based on their exposure to market risk. Our selection of user-friendly representative models of the zero, -one, -two, and -three parametric risk model classes affect this ranking of the top banks for investors. On top of our backtest experiments using a one-year rolling window, we nevertheless cross-checked the validity of the ordering using alternate options of rolling window lengths of two and three years. The joint score function values based on both VaR and ES violations, which correspond to alternate rolling window lengths, are not found, suggesting that a different model would be the most user-friendly choice in Bangladesh's expanding market. Thus, from an

aesthetically pleasing standpoint, presenting the top banks in Bangladesh using the HS model and taking into account the VaR and ES backtest results is a good idea. This exposes a less appealing aspect of the well-known RiskMetricsTM model's finest user-friendly approach in Bangladesh's emerging markets.

In **Chapter 5,** we present analytical formulas for the risk measures Value-at-Risk (VaR) and Expected Shortfall (ES)—a rarity in existing literature. While the standard Black-Scholes (BS) model provides closed-form expressions for these measures, it is often inadequate due to its restrictive assumptions. This chapter proposes an alternative approach that enables analytic risk forecasting conditioned solely on the current asset level, without relying on current volatility, thereby preserving the Markov property. We then conduct a comparative evaluation of the single- and multi-period-ahead forecasting performance of VaR and ES using three models: the analytic BS model, the GARCH-EVT model, and the proposed Conditional Analytic Markov (CAM) model. The GARCH-EVT model, a two-stage hybrid approach, combines Generalized Autoregressive Conditional Heteroscedasticity (GARCH) filtering with Extreme Value Theory (EVT) to capture both time-varying volatility and extreme tail risks. The analysis is carried out across two panels of data: one comprising financial assets (primarily banks), and the other, non-financial assets. The objective is to assess how well the CAM model performs in comparison to both the conditional EVT framework and the traditional BS model in forecasting key risk measures.

We present Markovian and conditional forecasts for Value-at-Risk (VaR) and Expected Shortfall (ES) based on the well-established Cox-Ingersoll-Ross (CIR) dynamics, which effectively incorporate leverage effects and mean reversion. Through backtesting and simulation studies, we evaluate the performance of single-period forecasts, square-root-rule-based multi-period extensions, and fully parametric multi-period characterizations across both financial assets (primarily banking stocks) and non-financial assets. The results

demonstrate that our proposed approach significantly outperforms the conventional GARCH-EVT model in forecasting risk measures. Notably, the method requires no numerical optimization or iterative search procedures, making it both computationally efficient and highly accessible—thus offering strong potential for advancing risk management practices across a wide range of users.

In Chapter 6, we explore the hypothetical delta and delta-gamma hedging performance of several models—NGARCH**,** GARCH**,** GARCH-Jump, and GARCH-Jump Constant—used as proxies for the Black-Scholes (BS) option pricing model. Among these, the GARCH model consistently demonstrates superior performance in delta hedging, although the differences across models are generally modest. We further assess the delta-gamma hedging effectiveness of the same models relative to the BS framework. The findings reveal that the GARCH model outperforms all alternatives across 1-day**,** 3.5-day, and 7-day hedging horizons, and this superiority holds across different moneyness categories, for both delta and delta-gamma hedging strategies.

While this research primarily utilizes Dow Jones data, its core objective is to extend the proposed framework to emerging markets, particularly the Dhaka Stock Exchange (DSE)**,** where options trading has yet to be introduced. The methodology will be applied to DSE data to evaluate its relevance and effectiveness in underdeveloped financial markets**,** offering valuable insights into the feasibility of initiating option trading in such contexts. This extension aims to bridge the gap between theoretical modeling and practical application within developing economies.

In Chapter 7, we provide a comprehensive conclusion and outlines key directions for future research.

Chapter 2

Literature Review

2 Literature Review

This chapter mainly focuses on an overview of previous research and empirical studies on conducting value-at-risk and expected shortfall, Conditional Analytic Markov (CAM) and non-conditional analytic Markov with mean reversion and leverage effect, and hypothetical hedging performance.

2.1 Conducting Value-at-risk and Expected Shortfall

Quantifying risk is a crucial component of risk management in financial organizations, such as banks, insurance companies, and funds for investment. For a significant period of time, risk experts have been actively searching for a reliable risk metric. Based upon an extensive collection of literature, we study the efficiency of forecasting and estimation under several risk measure models, including value-at-risk and expected shortfall. Both developed and emerging financial markets appear to benefit from accurate market loss estimation and forecasting. The proper estimation and forecasting of VaR is the subject of multiple articles, including those by (Alexander & Leigh, 1997), (Christoffersen, Hahn, & Inoue, 2001) and (Manganelli & Engle, 2001) .

Utilizing past financial return data to estimate the quantile metric is a common practice. We refer to this conventional technique as historical simulation (HS). It is straightforward to implement and does not necessitate any explicit return distribution. Nevertheless, any methodology based on historical data has the potential to impact the current patterns of returns. (Hendricks, 1996) found that extending the sample period reduced the variability of value-at-risk (VaR) estimations. Conversely, using shorter sample periods led to less exact VaR estimates relying on the Historical Simulation (HS) method. Another widely used approach to calculating value at risk (VaR) is through Monte Carlo (MC) modelling, which involves generating random future price pathways to estimate the potential fluctuations of financial securities. Value-at-risk (VaR) utilization directly generates the desired quantile from random routes. Nevertheless, certain pricing models are necessary to establish the parameters of the simulation.

A parametric estimate is a frequently used method that requires a precise representation of the return distribution. By estimating volatilities using past returns, also known as past volatility, the variance-covariance approach measures VaR. We use the Exponential Weighted Moving Average (EWMA) approach to calculate VaR, which assigns greater importance to more recent observations to capture the impact of recent volatility on past data. (Bollerslev, 1986) has also endeavoured to produce more precise volatilities using a GARCH model. Parametric estimating, in comparison to historical and MC simulations, necessitates a smaller amount of data and computational resources. This suggests that the approach is not beyond its limitations.

One significant drawback of parametric estimation is that it necessitates a precise return distribution. (Hull & White, 1998) discovered that the distribution of returns for financial securities is seldom Gaussian. GARCH-Normal faces severe criticism due to its inclination to underestimate future risk. This necessitates the use of the GARCH-t model to handle a distribution with fat tails. According to (Lin & Shen, 2006), the student's t-distribution provides a slight improvement in estimating market risk. However, the student's t-distribution lacks the ability to accurately represent the asymmetry present in the distribution of returns.

Following the 1997 Asian Financial Crisis, certain newly emerging economies in Asia encountered weaknesses in their banking sectors as a consequence of insufficient surveillance by regulators and inefficient risk management techniques (Agénor, 1999). Furthermore, banks in developing markets may encounter difficulties in achieving sufficient amounts of financial resources and obtaining finance in comparison to banks in industrialized economies. These issues might impact their capacity to absorb unexpected events and grow their operations. Amidst the global financial meltdown of 2008, numerous banks in developing economies encountered challenges in acquiring funds and obtaining international finance, resulting in a scarcity of solvency. Furthermore, banks in developing economies frequently face vulnerability to price and conversion rate risks as a result of volatility in their own currencies. Such risks can have an impact on the banking industry's

reliability and users' ability to cover loans in foreign currencies. The Turkish banking industry experienced considerable stress during the 2018–2019 Turkish currency and loan crisis, as numerous clients faced difficulties in repaying their obligations structured in foreign currency (Akcay & Güngen, 2019).

It is also reasonable to infer that individual investors are reluctant to utilize their risk models with an extensive time series of returns in the context of our investigation. We typically employ models with one-year rolling windows of return data and generate expected estimates of VaR and ES, as our backtesting investigations indicate that the default parameter lambda in the Risk Metrics model disregards return data older than 100 days. In our study we also select user-friendly models and use returns from more than seven years (2013–2020) to backtest the models.

2.2 Conditional Analytic Markov (CAM) and Non-Conditional Analytic Markov with Mean reversion and Leverage effect

Extreme value theory (EVT) and quantile regression are two recent approaches proposed to address the limits of distribution in estimating value at risk (VaR). The core principle of the EVT is to focus exclusively on modeling the extreme ends of the return distribution. EVT enhances parametric estimation by avoiding the need to model the entire distribution. Although there is substantial evidence supporting the superior accuracy of this method in estimating VaR compared to previous approaches (McNeil, 1997) and (Neftci, 2000), it still faces the challenge of accurately simulating tail distributions. Despite EVT's overall superiority over HS, GARCH-Normal, GARCH-t, and GARCH-skewed t models, it has two primary inherent drawbacks. Initially, in the immediate future, the risk manager may prioritize the assessment of potential losses within the next few days. In such cases, the Extreme Value Theory (EVT) is inadequate for capturing the fluctuating nature of volatility over time. Secondly, it relies on the assumption that the distributions are independently and identically distributed (i.i.d).

A GARCH-type model, which stands for Generalized Autoregressive Conditional Heteroskedasticity, is the most common parametric method for capturing the distribution's time-varying volatility. GARCH-normal, GARCH-t, and GARCH-skewed t are examples of common models. The GARCH-normal model is heavily criticized for its propensity to underestimate future risk. As a result, the GARCH-t model is used to account for fat-tailed distributions. (Lin & Shen, 2006) claim that the student-t distribution somewhat improves the market risk estimate. However, the Student-t distribution is symmetric, it cannot capture the asymmetric nature of the return distribution, which is crucial for accurately modeling financial returns that often exhibit skewness and kurtosis. A model with a symmetric probability density function performs worse than one with a skewed density function, as demonstrated by (Giot & Laurent, 2003) particularly in scenarios where financial returns exhibit significant asymmetry, such as during market downturns.

In recent years, experts have suggested many models and methodology to calculate VaR (Value-at-Risk) and Conditional Value-at-Risk (CVaR) specifically designed to adapt to the changing financial environment and to overcome limitations in current methods. The traditional simulation method calculates Value-at-Risk (VaR) and Conditional Value-at-Risk (VaR) by using past return data. In the latest research, researchers (Alexander C. , 2021) have been primarily concerned with enhancing this method by employing sophisticated methods like as asymmetrical scoring functions, fat-tailed distributions, and integrating non-linear relationships. (Chen & Wang, 2020) introduced an innovative enhancement to past simulations that utilizes high-frequency information in order to enhance precision in turbulent market situations. Although the past modelling approach is straightforward, it is frequently criticized for its limited capacity to accurately represent changing market circumstances and effectively account for tail risk. On the other hand, parametric approaches, such as the Normal, Student's-t, and Generalized Extreme Value (GEV) distributions, have been extensively employed for estimating VaR and Conditional Value-at-Risk (CVaR). Scientists have suggested improvements to these methods in order to more accurately represent various features of asset returns, such as the tendency for

volatility to accumulate and the presence of skewness. (Li, Zhang, & Liu, 2021) proposed the combination of the GARCH-Skewed t-distribution model to specifically tackle the issues of skewness and leptokurtosis, resulting in improved prediction of tail risk. In a recent study by (Jiang & Zhu, 2022)an integrated GARCH-EVT model was proposed to include both volatility clustering and extreme value dependency. Parametric approaches, while their usefulness, are constrained by the presumptions they make about the distribution of data. Consequently, their effectiveness may be compromised during times of substantial market turmoil. Furthermore, EVT has become increasingly popular for evaluating the likelihood of extreme events occurring in financial assets. Event-based models, such as Peaks Over Threshold (POT) and Block Maxima (BM), have been used to estimate VaR and Conditional Value-at-Risk (CVaR). (Huang & Wang, 2022) introduced a hybrid EVT-GARCH model to accurately capture and analyse abnormal market risks. Although EVT has advantages, its dependence on a small number of exceptional observations might result in estimating mistakes, especially for assets that have few extreme events. Moreover, Monte Carlo methods entail the generation of random samples to model future asset returns and measure VaR and Conditional Value-at-Risk (CVaR). Recent research (e.g., (Boudreaul & Gauthier, 2023)) has investigated the use of advanced methods to reduce variance, such as opposite variates and control variates, in order to improve the effectiveness for Monte Carlo simulations. While this method may need a significant amount of computer resources, it offers the advantage of being able to model intricate relationships and to offer a more precise prediction of severe risks. In addition to these models, the copula model allows for the estimate of joint distributions, which is useful for assessing the risk of multi-asset portfolios.

In contrast to previous study viewpoints ((Lu, Yang, Shi, Li, & Abedin, 2022) ; (Medina-Olivares, Calabrese, Y. Z. Dong, & Shi, 2022) this investigation focuses specifically on private investors rather than organizations and firms. Additionally, it examines developing countries such as Bangladesh, where investors' knowledge and expertise are significantly inferior to those in developed regions (Islamaj, Kose, Ohnsorge, & Ye, 2019). This

undoubtedly impacts the hardships experienced by ordinary investors, leading to substantial effects in their community. It is not simply Bangladesh that possesses these characteristics; numerous rising countries exhibit similar traits.

2.3 Hypothetical Hedging Performance of an Emerging Option Market

The literature on option pricing has significantly increased in the past decade. A multitude of models has been introduced to rectify the empirical deficiencies of the traditional Black-Scholes (BS) framework (Black & Scholes, 1973).The objective of this chapter is to empirical investigation of the behaviours of derivative pricing and hedging performance in emerging stock markets. The main concerned in emerging markets on mis-price of option which can provided abnormal profit to some investor and series loss for other investor. Market anomalies, or inefficiencies, are more pronounced in emerging markets compared to mature economies. Prudent traders can exploit these anomalies to yield extraordinary profits, resulting in losses for noise traders. This is a significant consequence not just for the viability of the markets but also for the efficiency of the derivative markets, as they are influenced by their underlying markets (i.e., the stock and capital markets).

Conventional theoretical option pricing models suggest a deterministic option price, so failing to guide the empirical researcher in defining the error term (Ghysels, Patilea, Renault, & Torrès, 1997). The selection of the loss function is crucial since it inherently assumes a specific error framework. Currently, the widespread methodologies for measuring and estimating volatility include historical volatility, derived from static averages and dynamic moving averages; time-varying volatility, assessed using the continuous generalized autoregressive conditional heteroscedasticity (GARCH) model; implied volatility, computed through a six-variable option pricing framework; random volatility, produced via stochastic simulation; and realized volatility, determined from high-frequency data. All these strategies assume that the return on assets is a continuous process, with the jump component either inadequately addressed or entirely overlooked.

Empirical research indicates that leaps frequently occur in China's stock markets and significantly enhance their volatility (Zhou, 2012).

(Chan & Maheu, 2002) constructed a GARCH-jump constant model using an input to the mixture of GARCH-jump model and the GARCH-jump constant model allows the conditional jump intensity to add a time varying approximate autoregressive moving average (ARMA) form and show that proposed model good fit for stock market volatility. GARCH-jump mixing models have been employed in diverse scenarios. Jorion (1988) employs a mixed jump diffusion model to examine the presence of jumps in the sample paths of exchange rates and a stock market index. It was determined that exchange rates exhibit substantial jump components, which are more pronounced than those in the stock market. These discontinuities appear to persist without explicit consideration of potential heteroskedasticity in the standard diffusion process, exhibiting substantially in weekly data but to a smaller extent in monthly data.

(Wing H. Chan, 2002) introduced an autoregressive conditional jump intensity (ARJI) model to clarify jump dynamics in stock market returns to enriches the GARCH-constant jump intensity framework by incorporating a time-varying jump intensity and a dynamic jump size distribution. The intensity of jumps is represented by a concise ARMA framework influenced by a retrospective assessment of jump likelihood.

We observe substantial temporal variation in the conditional jump intensity and the distribution of jump sizes in our analysis of daily stock market returns. This paper demonstrates enhancements in modelling jump dynamics relative to the fundamental GARCH-constant jump intensity model. The time-varying leap intensity yields accurate predictions of stock market volatility. Secondly, including the autoregressive jump intensity utilizes supplementary structure overlooked in a constant intensity model. The ARJI model has markedly elevated ex ante probabilities of jumps on days characterized by stock market deteriorations. Prior research (Bates, 2000) has examined a potential correlation between jump intensity and a stochastic volatility model of volatility. Our

approach establishes an analogous relationship between the parameterization of conditional jump intensity and the GARCH specification.

(Mixon, 2002) contends that the delta must be less than the BS delta to align with the empirical patterns of volatility dynamics. Deltas that deviate from the Black-Scholes delta have been empirically recorded in the works of (Bakshi, Cao, & Chen, 2000). (Bakshi, Cao, & Chen, 2000) demonstrate that the deltas generated by stochastic volatility models are inferior to the Black-Scholes delta for out-of-the-money put options, yet superior for in-the-money put options. (Coleman, Li, & Verma, 2001) report that the deltas derived from a deterministic volatility model are consistently inferior to the Black-Scholes deltas. Errors in option hedging may originate from two distinct sources. The option value is a nonlinear function of the underlying asset, therefore Black-Scholes hedging is instantaneous; but, in reality, only discrete hedging, potentially with infrequent rebalancing due to transaction costs, is feasible. Secondly, inaccuracies in the foundational pricing model can result in suboptimal hedging efficacy.

(STRÖMDAHL, 2023) demonstrates that the profit and loss are about same whether employing delta hedging or delta-gamma hedging. The transaction costs for the delta hedge were considerably lower; however, the delta-gamma hedge with the minimal threshold exhibited the most neutral deltas and gammas. Delta-gamma hedging is advantageous over delta hedging solely when the threshold is low.

(Mozumder, Kabir, & Dempsey, 2018) used the substitute for estimating the most popular GARCH, EGARCH and GJR-GARCH model's volatilities to constant volatility of BS model, for pricing and hedging option portfolios. They analyze the hedging efficacy by contrasting model-dependent hedging variables of delta and delta-gamma with the actual value of the option portfolio derived from comprehensive model valuation across a five-day trading horizon as the risk-management time frame.(Cao & Guo, 2011) show that the mean value of net profit from seven days delta hedging is larger than one day delta hedging and also got mean value of net profit from three days delta hedging is bigger than seven days delta hedging by comparing the numerical results.

Chapter 3

Risk Measures and Related Properties

3. Risk Measures and Related Properties

This section defines key risk measures and their essential properties, which form the foundation for analyses in the subsequent chapters. A solid grasp of these concepts and methodologies will aid the reader in understanding both the implementation and implications of the research presented in this thesis.

3.1 Definition of Value at Risk (VaR)

Value-at-Risk (VaR) is a statistical measure that represents the lower bound of a one-sided confidence interval for potential losses. The value-at-risk (VaR) is determined by specifying a confidence level and a time period. The formula for calculating a value-at-risk for a period of n days, with a confidence level of $\alpha\%$, is as follows:

$$VaR_{\alpha,t}(X) = inf\{x|P(X_t < -x) \leq \alpha\} \tag{3.1}$$

in which $X(t)$ represents the asset's net worth following n days, as defined by $X_t = S_t - S_{t-1}$. This allows us to define the loss as $L_t = -X_t$. In particular, it is obvious that $VaR_{\alpha,t}(X)$ is a decreasing function of α.

The value at risk fulfils the three characteristics of a risk measure, but it does not meet the requirement of subadditivity, rendering it an incoherent risk measure. Diversification can have a negative impact on value-at-risk, even though in reality it is evident that the risk would really reduce. Furthermore, it fails to consider losses that exceed the VaR threshold. For instance, the 5% value-at-risk remains unaffected by losses that are less likely to occur than 5%. This implies the existence of significant yet unlikely losses that have not been taken into consideration. From a theoretical standpoint, the VaR has a significant limitation, although it is nonetheless commonly employed in practical applications. When calculating VaR for longer time periods, it is necessary to consider the interest rate. However, as these approaches are only applicable for VaR calculated for short time periods, they will be ignored.

VaR is a non-negative value, and the negative sign is used to indicate losses. Therefore, VaR represents the likelihood of losses beyond the negative VaR, resulting in a more unfavourable outcome. This concept is demonstrated in Figure 3.1, which displays a graph of a conventional normal profit and loss distribution. The shaded regions represent the 1% and 5% percentiles.

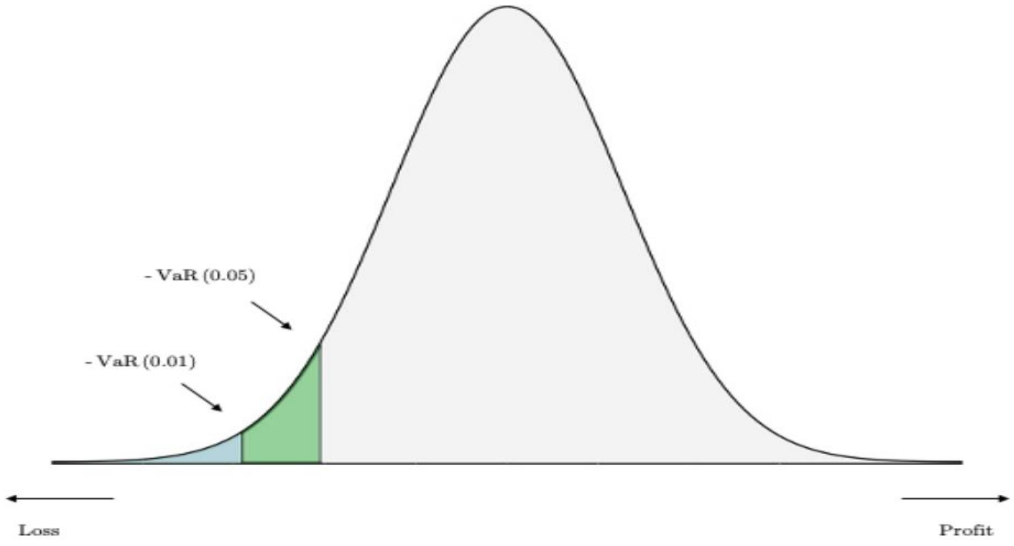


Fig-3.1: Standard normal profit and loss probability density and VaR.

When calculating VaR, three choices need to be made. The selection of the confidence level, the duration of time, and the distribution of profit and loss. The level of confidence is usually established at either 95% or 99% which corresponds to the percentile values $\alpha = 5\%$ and 1%, respectively. Market risk management typically use a time horizon of either one or ten days. This study will utilize a time frame of one day. This pertains to the attempt to predict the daily fluctuations of profit and loss in portfolios that are exposed to market risks. The ultimate determination is to ascertain the probability distribution of profit and loss. Several distributions have been suggested in the academic literature in the past few decades, as evidenced by works such as (Rigby & Stasinopoulos, 2005). Typical options, on the other hand, include the conventional normal distribution or the scaled t-

distribution. The study will utilize the normal distribution, as well as the student-t and skewed student-t distributions, as mentioned before.

Non-parametric approaches for calculating VaR are available, which do not rely on assumptions about the distribution of returns. One drawback of VaR is that it may not consistently meet the subadditivity requirement of a cohesive risk measure. Therefore, it could mistakenly discourage the act of diversifying. Nevertheless, when assuming that returns follow a normal distribution, Value at Risk (VaR) is directly related to volatility, which is a property of being sub additive. Consequently, VaR is considered coherent under this assumption. In their work, (Dan´ıelsson, Jorgensen, Samorodnitsky, Sarma, & Vries, 2013) found that VaR only fails to satisfy the subadditivity requirement when the loss distribution has extremely heavy tails. Specifically, when the degrees of freedom in a student-t distribution exceed two, the property of subadditivity is fulfilled.

Although VaR has certain theoretical flaws, its practical benefits, such its simplicity, ease of implementation, and wide applicability, more than make up for them. Since its inception in 1994, VaR has continued to be the most commonly utilized risk measure. Ever since the Basel-I Accord Amendment of 1996, value-at-risk has been the recognized metric for market risk in relation to regulatory obligations. Because of its poor performance during pressured periods, value-at-risk is being proposed by Basel Committee on Banking Supervision (BCBS) to be replaced as the standard measure for market risk in the Basel-III Accord (Basel III: A global regulatory framework for more resilient banks and banking systems, 2010). This modification is presently being put into effect by banks in accordance with the final regulations of Basel-IV.

3.2 Definition of Expected Shortfall

Expected Shortfall, often referred to as tail value-at-risk, tail conditional expectation (TCE), and conditional Value at Risk (CVaR), is a different method of measuring risk that

specifically tackles the problems associated with VaR. (Philippe Artzner, 1999) formally introduced this concept. Expected shortfall (ES) takes into consideration the data in the extreme end of the distribution that value-at-risk ignores. The expected shortfall is calculated as the average of the $\alpha * 100\%$ worst log returns. ES is defined for a continuous distribution as follows. To be more specific, consider a portfolio whose value at time t is a random variable denoted V_t. At horizon h, the loss denoted by $L_{t,t+h} = -(V_{t+h} - V_t)$. The distribution of $L_{t,t+h}$ is called the loss distribution (Conditional or not).

For a loss $L_{t,t+h}$ with $E(|L_{t,t+h}|) < \infty$ and assumed the conditional distribution of $L_{t,t+h}$ is a continuous and strictly increasing cdf. The ES at level α , is defined as the conditional expectation of the loss given that the loss exceeds the Value-at-risk:

$$ES_{t,h}(\alpha) = E_t[L_{t,t+h} \mid L_{t,t+h} > VaR_{t,h}(\alpha)] \tag{3.2}$$

We have,

$$E_t[L_{t,t+h}\, I_{L_{t,t+h} > VaR_{t,h}(\alpha)}]$$

$$= E_t[L_{t,t+h} \mid L_{t,t+h} > VaR_{t,h}(\alpha)]\, P_t[L_{t,t+h} > VaR_{t,h}(\alpha)]$$

Now $P_t[L_{t,t+h} > VaR_{t,h}(\alpha)] = 1 - P_t[L_{t,t+h} \leq VaR_{t,h}(\alpha)]$

$$= 1 - (1 - \alpha) = \alpha$$

Thus $ES_{t,h}(\alpha) = \frac{1}{\alpha} E_t[L_{t,t+h}\, I_{L_{t,t+h} > VaR_{t,h}(\alpha)}]$

Therefore, mathematically expected shortfall can be defined as follows

$$ES_{t,h}(\alpha) = \frac{1}{\alpha} \int_0^{\alpha} VaR_{t,h}(u)\, du \tag{3.3}$$

3.3 Coherent Risk Measure

Let M denote the aggregate of all risks. A metric for assessing risk ρ is a function that associates a set M with the set of real numbers $\mathbb{R}$. The risk measure for a collection of outcomes X is represented as $\rho(X)$. A risk measure is deemed coherent if it satisfies all of the subsequent properties:

(i) Monotonicity: For all $X.Y \in M$ with $X \leq Y$,

$$\rho(Y) \leq \rho(X)$$

If portfolio X continually surpasses portfolio Y under all conditions, then the risk associated with portfolio X will be elevated.

(ii) Subadditivity: For all $X, Y \in M$,

$$\rho(X + Y) \leq \rho(X) + \rho(Y).$$

The combination of two portfolios results in a total risk that cannot exceed the cumulative risk of the individual portfolios, exemplifying the principle of diversification.

(iii) Positive homogeneity: For all $k \geq 0$and all $X \in M$,

$$\rho(kX) = k\rho(X) .$$

If the portfolio is increased by a factor, the risk will increase proportionally.

(iv) Translation invariance: For all $X, Y \in M$ and all real numbers n,

$$\rho(X + n) = \rho(X) - n.$$

Adding an amount n of cash to the portfolio, this acts as insurance and the risk will be lower.

Although VaR satisfies the three main characteristics of risk assesses - normalization, translation invariance, and monotonicity - it fails to meet the subadditivity state and accurately capture extreme risks. This makes VaR a non-coherent risk measure. Financial organizations are motivated to utilize expected shortfall due to this reality. According to (Rockafeller & Uryasev, 2000), ES is a risk measure that maintains the property of subadditivity when assessing market risk. Therefore, the BASEL committee on Banking Supervision has substituted the Value-at-Risk (VaR) metric with the Expected Shortfall (ES) measure in order to assess the market risk component of a financial institution. Unfortunately, it is not possible to achieve Expected Shortfall (ES) as the only solution that minimizes the anticipated loss function, as stated by (Gneiting T. , 2011). However, (Tobias Fissler, 2016) demonstrate that both Value-at-Risk (VaR) and ES can be elicited

together, which allows for a comprehensive evaluation of ES and VaR performance within a unified framework, as proposed by (Tobias Fissler, 2016).

3.4 Stylized Fact About Financial Time Series

3.4.1 Skewness

Skewness of a distribution is defined as the asymmetrical characteristic of the distribution of a random variable X and is known as the third standardized central moment. In negatively skewed distributions, the majority of the probability is concentrated to the left of the mode, whereas in positively skewed distributions, it is the opposite. Examples of positively skewed distributions include the log-normal distribution, whereas normal distributions and other symmetric distributions exhibit a skewness of zero. This is true despite the distributions having identical mean and standard deviation. Empirically, asset log-returns have a positive skew. Prior to the 2008 financial crisis, returns exhibited positive skewness during extended durations of favourable returns; however, in the succeeding periods, returns displayed significant negative skewness. In most instances of financial time series, the return distribution is seldom fully normal.

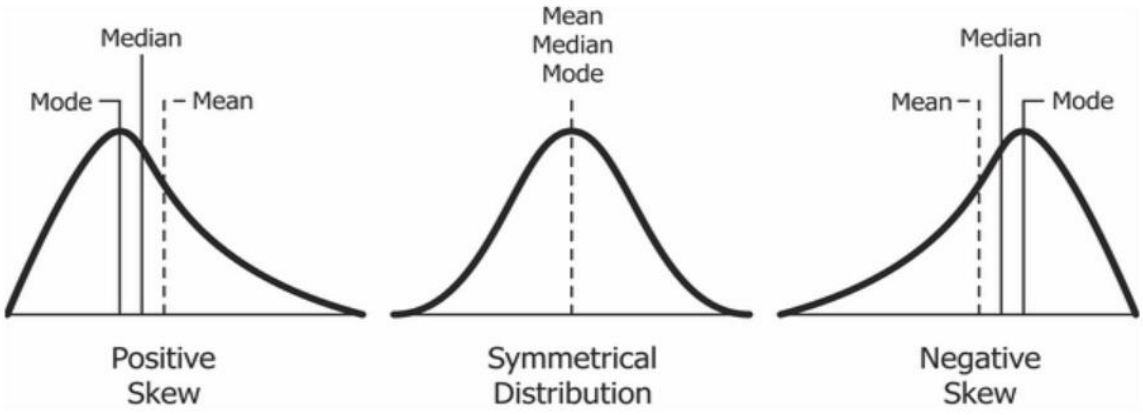


Figure-3.2: **The three types of skews[1].**

[1] Source: Skewness - Clojure for Data Science [Book]

3.4.2 Kurtosis

Kurtosis quantifies the tails of a distribution and is known as the fourth standardized central moment of that distribution. The kurtosis of a normal distribution is three; hence, excess kurtosis is calculated by subtracting three to facilitate comparison. Leptokurtic kurtosis refers to a distribution that is more peaked around its mean and exhibits thicker tails compared to a normal distribution. The platykurtic distribution is significantly "lower" and has thinner tails, as illustrated in Figure 3.3 below.

Empirical financial time series returns exhibit leptokurtic kurtosis, indicating the presence of fat tails. Fat tails indicate that while most returns cluster around the mean, the likelihood of extreme outliers is greater than that found in a normal distribution. In a financial context, these outliers will correspond to significant gains or losses.

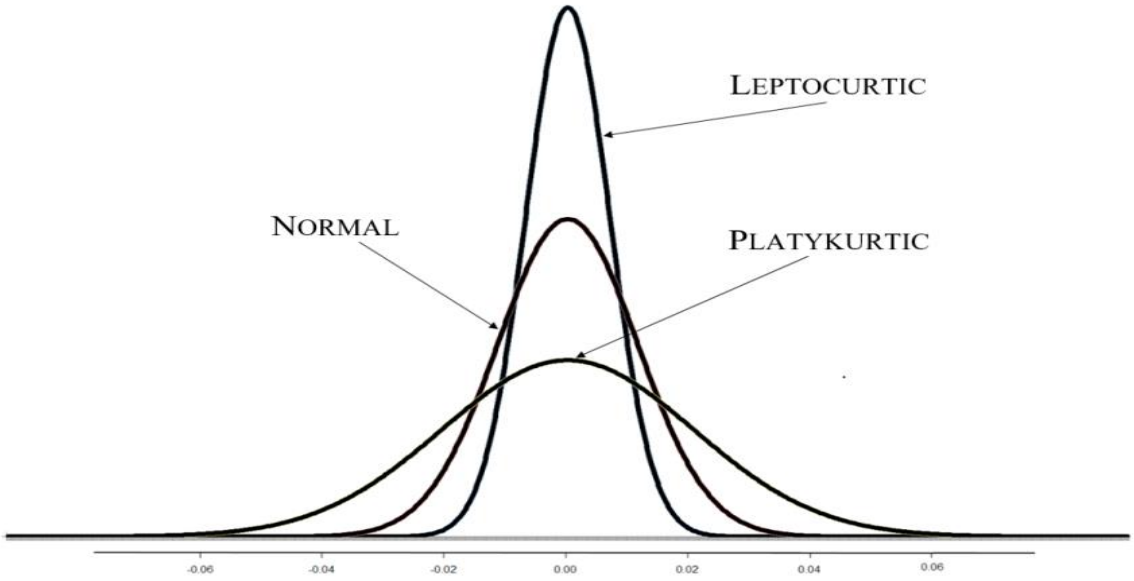


Figure-3.3: Different types of kurtoses.

3.4.3 Volatility Clustering

Volatility clustering is a recognized phenomenon in asset returns, characterized by the positive autocorrelation of volatility over extended durations. It is frequently noted in the statistics that volatile periods are typically succeeded by other volatile periods, while calm periods are followed by further calm periods. Consequently, the probability density function of the return ought to conform to the Gaussian distribution. Nonetheless, the combination of significant changes in financial price series is shown to coincide with the heavy-tailed characteristic of the return distribution (Ding, Granger, & Engle, 1993) (Engle, 1982). A significant fluctuation is likely to succeed a prior significant fluctuation, while a minor fluctuation is likely to succeed a prior minor fluctuation, hence refuting the iid assumption. This behavior is referred to as volatility clustering. The financial market is marked by unforeseen disruptions. In this context, volatility clustering has significant consequences for financial risk management, particularly in the computation of value-at-risk or expected shortfall for the portfolio. Upon the occurrence of an unforeseen shock, the volatility of a financial market significantly escalates. Moreover, the presence of volatility clustering indicates the sustained occurrence of severe volatility over a period. Since the risk measures are derived on previous return data, investors must modify the estimates to effectively manage and safeguard the institution's capacity against the increased risk. Therefore, it is essential to examine the presence and causes of volatility clustering in financial time series.

From the econometrics perspective, the conventional method to identify the reasons of volatility clustering is the Autoregressive Conditionally Heteroscedastic (ARCH) model, which has been extended to the Generalized ARCH (GARCH) model. GARCH models were among the first models to take into account the volatility clustering phenomenon. These methods are robust and descriptive techniques for analyzing volatility clustering; nevertheless, neither model explains the underlying reasons for the appearance of such distributions.

3.5 Leverage Effect

The leverage effect is an asymmetric phenomenon indicating that negative returns exert a more significant impact on future volatility than positive returns of same magnitude. The asymmetry signifies the inverse relationship between an asset's return and its fluctuations in volatility. Generally, increasing asset values correlate with decreasing volatility, and conversely. The term leverage denotes an economic explanation of this phenomenon, as articulated in (Black & Scholes, 1973) and (Christie, 1982) : when asset prices decrease, corporations become inherently more leveraged as the relative value of their debt increases in comparison to their equity.

Subsequently, it has been validated in several empirical research as per (ENGLE & NG, 1993). Rising asset prices empirically correlate with reduced volatility, whereas declining prices result in heightened insecurity and increased volatility. The concept of "leverage" is explained by the relationship between debt and equity in organizations; as the value of debt escalates in comparison to equity during asset price declines, stock volatility intensifies. This remains a hypothesis, but the name has been embraced by scholars to elucidate the statistical regularity in question, as noted by (Ait-Sahalia, Fan, & Li, 2013).

(Yu, 2005) calculated many discrete-time models incorporating a leverage effect. The observed effect appears excessively substantial to be exclusively attributed to an increase in financial leverage. (Figlewski & Wang, 2000) observed, among other findings, that volatility remains unaffected by changes in leverage due to alterations in debt or number of shares, but only in response to fluctuations in stock prices which is the consequence in association with financial leverage.

3.6 Estimation and Rolling Window

As VaR violations are infrequent events, evaluating the precision of VaR predictions through subsequent model performance necessitates an extended timeframe. Consequently, back-testing assesses VaR predictions by examining a model's performance throughout a

historical timeframe. To comprehend the notion of back-testing, it is crucial to differentiate between the estimating window and the testing window.

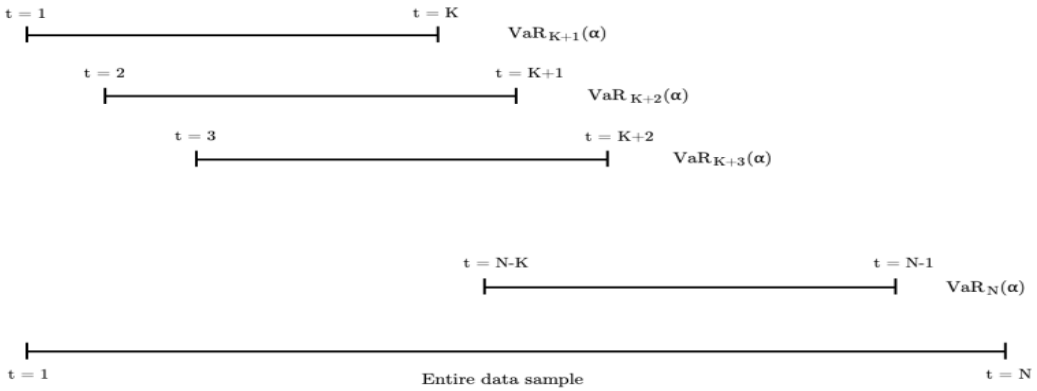


Figure-3.4: Rolling window for forecasting Value-at-Risk (VaR).[2]

Value-at-risk (VaR) is a predominant instrument for forecasting future returns based on current data. Implementing this tool necessitates the delineation of the estimating window and the testing window. (Danı´elsson, 2011) states, "the estimation window is the number of observations used to forecast risk". If the number of observations in the estimate window varies between models, the model necessitating the greatest number of observations should be regarded as the length of the estimating window. The testing window refers to the data sample utilized for the application and assessment of the VaR forecast.

If N represents the total sample size, then N equals the sum of the estimation window and the testing window. The data from the estimation period of the initial K days is utilized to

[2] Estimating and forecasting volatility of stock market returns using GARCH models – with an application to value-at-risk and expected shortfall by Catharina Bratterud Lilleengen. Master's thesis in Industrial Mathematics. Department of Mathematical Sciences. Faculty of Information Technology and Electrical Engineering, Norwegian University of Science and Technology.

generate a VaR prediction for day $K + 1$. The estimation window is subsequently advanced by one day to derive a risk forecast for day $K + 2$. This process persists until a Value-at-Risk (VaR) forecast for day N is achieved, resulting in a cumulative total of T risk forecasts. Figure 3.4 demonstrates the functionality of the rolling window approach.

In Value-at-risk and expected shortfall forecasting, it is essential to recognize the significance of the window size, defined as the quantity of observations incorporated into the empirical distribution of returns. Altering the window size may affect the forecasts. A reduced window size results in more fluctuations in the VaR forecast, as extreme observations are anticipated to vary more significantly from day to day compared to those nearer to the mean of the return distribution. A bigger window size is less susceptible to outliers; but it requires more time for the VaR forecast to adapt to structural shifts in risk. Nonetheless, as the value that meets or surpasses VaR is anticipated to fluctuate in only one out of every $1/\alpha$ observations, the VaR predictions are projected to remain stable for the most of the time. The generally accepted minimum sample size for HS is $^{3}/_{\alpha}$ usually ranging from 250 to 1000 (Danı´elsson, 2011). Upon acquiring the VaR projections, they may be compared with the pre-existing daily returns. If the actual return on a given day above the value-at-risk prediction for that day, a VaR violation is deemed to have happened (Danı´elsson, 2011).

Chapter-4

Conducting value-at-risk and expected shortfall backtest for emerging market using user-friendly models.

4.1 Introduction

In the past twenty years, there has been an increasing interest in predicting value at risk (VaR), as it is a significant risk metric for financial and insurance companies. Moreover, financial and insurance companies frequently employ VaR projections to evaluate various models and accurately capture the changing patterns of financial returns over time. There are a wide range of approaches available for building and assessing VaR forecasts. This study provides an overview of the newly proposed benchmarks discussed in recent literature.

Although there have been significant theoretical advancements in the area of derivative pricing, many regularly used derivatives cannot be accurately priced using existing formulas. Thus, practitioners are compelled to utilize Monte Carlo simulations and confront the limitations imposed by their computing expenses. The more accurately model can comprehend all aspects of the financial system, the greater likelihood that the time required for an exact simulation would become expensive. Consequently, several simplifications of the models are being implemented to reduce execution time without compromising the application's required simulation accuracy. Therefore, when calculating risk measures for large portfolios, it is almost necessary to optimize simulations.

We evaluate the effectiveness of four commonly used models for assessing market risks in Bangladesh's growing market. These models utilize Value-at-risk and Expected Shortfall risk measures. Subsequently, we conduct a backtesting analysis on the risk estimates for twenty public and private banks that operating in Bangladesh. Back testing is the process of evaluating the accuracy of a risk assessment method where the forecasting of a risk measure is compared to the actual financial losses observed over some time. The relevance of backtesting for financial regulation in banking and insurance is tremendous while VaR and ES are adopted as regulatory risk measures.

Based on our findings, it is logical to conclude that independent traders are unlikely to utilize risk models that depend on comprehensive past data on returns. The RiskMetricsTM model, as a default setting, considers return data that is more than 100 days old as not relevant for risk estimates. For our backtest research, we usually analyse one-year rolling windows of return data. We then employ these dynamic forecasts of risk estimations, such as VaR and ES. Additionally, our study emphasizes the use of accessible models. Although the RiskMetricsTM model has been widely recognized for its accurate predictions of VaR, its ability to anticipate Expected Shortfall has decreased, making it less accessible for risk projections. HS regularly outperformed both the dual-parameter and trio-parameter available typical models in this research as well. We conducted additional tests to evaluate the effect of our decision on accessible window length on our outcomes. We also conducted multiple iterations of dynamic backtesting for both Value at Risk and Expected Shortfall using a rolling window prediction that covered a period of two years. In order to confirm the strength and reliability of our study, we further performed tests by utilizing a distinct initial date for the backtest and used these tests over a continuous period of three years. Curiously, as the span of the rolling window rises, HS loses some of its comparative supremacy over the RiskMetricsTM model. Nevertheless, it outperforms the RiskMetricsTM model and consistently ranks as the top choice among the four accessible models in all situations. Forecasts and backtests using rolling windows of two or three years are unable to identify the most efficient risk models among the representative models in the two and three-parametric categories.

The unfortunate incidents of unaware retail investors, who eagerly risked their family's estate and other assets in the stock market, particularly in the supposedly strong banking sector (enticed by government initiatives proclaiming its strength), highlight the need for these investors to employ user-friendly models to assess their daily investment risks. Investors must understand the prospective benefits of their investments in order to foster

the development of stable stock markets. This will generate real momentum and prevent society from making unwise decisions.

We have selected the historical simulation model as our preferred option among the zero parametric risk forecasting models. Additionally, we have chosen JPMorgan's RiskMetricsTM model, usually acknowledged as the exponentially weighted moving average (EWMA) model, as our preferred option among the risk approaches with one parameter. The two parametric risk frameworks we have selected as our representative choices are the standard Black and Scholes dynamics. In the final stages, we have selected the student-t (with location scale) model out of the three parametric risk models presented in this class. The tests suggested by (Acerbi & Szekely, General properties of backtestable statistics, 2017) and (Acerbi & Szekely, Backtesting expected shortfall, 2014), which are devoid of distributional assumptions, should be used to backtest the risk measure expected shortfall, which is advised by Basel-III under no set distributional hypothesis, given the context and goal of this work. In our case, other expected shortfall backtests that depend on distributional assumptions will be contradictory. However, we take into consideration two hypotheses: the unconditional-normal and unconditional-T hypotheses, for distributional assumption-free expected shortfall backtests. We also take into consideration backtesting for VaR using eight distinct tests, but this is not in line with Basel-III. These include the traffic light test, conditional coverage test. conditional coverage independence test, proportion of failure test, time till first failure test, binomial test, time between failures test, and time between failures independence tests.

It is also reasonable to presume, given the parameters of our study, that private investors are not inclined to use risk models using long-term return series. Due to the RiskMetricsTM model's default assumption that return data more mature than a hundred days is irrelevant for risk projections, we usually use models that consider one-year rolling windows of return data. In our backtest studies, we incorporate these steadily predicted risk estimates of value-

at-risk and expected shortfall. We stress the choice of user-friendly models in our study as well.

We use returns spanning more than seven years (2013-2020) to backtest the models. Applying the joint scoring function for estimation, (Tobias Fissler, 2016) to all twenty banks in our chosen developing economy, we prioritize user-friendliness and derive an order of choice for the four models. The combined score function takes into account both the frequency and severity of breaches and is based on corresponding return series, VaR, and ES forecasts. This demonstrates that the HS model, which has a one-year rolling window and excellent user-friendly features, is a much better option for individual investors to think about in Bangladesh's emerging markets. More importantly, while the RiskMetricsTM model has become very accepted as a VaR forecast model, its effectiveness as an expected shortfall forecast model has made it less appealing than it was originally as the friendliest model for risk predictions. In this investigation as well, HS regularly performs better than the user-friendly representative two- and three-parametric models. We reiterated the similar dynamic backtests for both value-at-risk and expected shortfall consuming two years of testing window risk forecasts to confirm whether this has anything to do with our accessible window duration choice. For a robustness study, we adopted an alternative starting date with testing window length seven hundred fifty days for the backtest forecasts.

Overall, in the four user-friendly models after increasing the testing window, historical simulations model shows comparatively advance position whereas the RiskMetricsTM model decreases a little bit, even though it still performs better than the RiskMetricsTM model. Considering rolling windows of two- or three-years length none of the user-friendly models of the two and three-parametric classes of risk models completed the best for forecast and backtest.

4.2 Data and Research Methodology

4.2.1 Data

Risk measurements are a means of assessing the level of risk taken on by determining the quantity of capital needed to mitigate potential unanticipated losses. In this study, we have gathered approximately 2,500 observations of the daily closing share prices of profitable banks that are registered on the Dhaka Stock Exchange (DSE) since 2010 and those observations are used to compute the day-to-day logarithmic returns of each bank. The determination of this analysis is to estimate the value-at-risk and expected shortfall measures for each bank. Furthermore, we conduct back-testing on these measures to assess the effectiveness of the accessible models. We have identified twenty profitable banks that are taken into consideration for this study.

In order to determine an appropriate user-friendly model, we analyzed the day-to-day log return of each commercial bank from Jan, 2010 to Aug, 2020, taking into interpretation of backtest indication and joint score values. The estimation of value-at-risk and expected shortfall for the twenty profitable banks requires the implementation of several easily understandable methods.

The following banks are included in the data series for the period from 1st January 2010 to 31st August 2020: Al-ArafaIslami Bank Ltd (ABL), Bank Asia Ltd (BA), BRAC Bank Ltd (BRAC), Eastern Bank Ltd (EBL), Export-Import Bank Ltd (EXIM), First Security Islami Bank Ltd (FSIBL), Islami Bank Bangladesh Ltd (IBBL), IFIC Bank Ltd (IFIC), Jamuna Bank Ltd (JBL), Mutual Trust Bank (MTBL) Ltd, Mercantile Bank Ltd (MBL), National Credit and Commerce (NCC) Bank Ltd, ONE Bank Ltd (ONE), Prime Bank Ltd (PBL), Premier Bank Ltd (PRBL), Rupali Bank Ltd (RBL), Standard Bank Ltd (SBL), and Trust Bank Ltd (TBL).

4.2.2 Five Years Financial Highlight of 20 Local banks in Bangladesh

Banking in emerging nations frequently functions within regulatory frameworks that are less developed, resulting in regulations that may be more permissive or less rigorously imposed relative to established economies. This can result in heightened risks and weaknesses inside the monetary system. Despite its gross domestic product of over USD 460 billion and ranking as the world's 35th largest economy, Bangladesh is classified as a developing nation due to its impressive history of steady growth and development. This development has been driven by a significant demographic advantage, a thriving ready-made garments exportation business, reliable remittance influxes, and stable macroeconomic circumstances, resulting in fast expansion over the last two decades.

We have chosen Bangladeshi banks for our analysis primarily to uncover the risk understanding and risk-taking behaviour of unintentional temporary participants in the field of banking, which represents one of the key growing industries that is fuelling the sustained economic development of this nation. The following table 4.1 to 4.20 shows the growth of our selected commercial bank.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	16256.61	18604.42	16556.64	13072.79	16109.65
Total Asset (BDT in million)	338465.30	381051.81	418298.65	462687.05	534756.11
Return on Assets (ROA) %	0.73	0.64	0.61	0.46	0.39
Return on Equity (ROE) %	10.46	10.05	10.10	8.21	8.13
Return on Investment (ROI) %	9.53	9.99	8.53	6.96	6.20
Export (BDT in million)	114481.90	108967.10	104939.40	135786.90	181070.20
Import (BDT in million)	168573.80	171611.70	169378.90	261566.10	300430.90
Remittance (BDT in million)	36917.20	37713.40	43807.00	52662.10	88531.70
Total Operating Income (BDT in million)	237208.95	149725.01	119173.30	21835806.00	27441152.00
Loan and Advances (BDT in million)	12690.72	14100.41	11769.60	16566.84	22437.37
Net Asset Value per Share (BDT)	10.14	10.23	10.32	22.56	22.58
Non-Performing Loan (NPL) Ratio %	4.79	4.82	3.80	4.81	5.41

Table-4.1: Five years highlight of **Al-Arafa Islami Bank Ltd (ABL)** from 2018 to 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	222472	253710	303028	317782	337695
Total Asset (BDT in million)	307291	353800	408961	438293	468401
Return on Assets (ROA)%	0.76	0.59	0.53	0.64	0.67
Return on Equity (ROE)%	10.21	8.16	7.81	9.89	10.88
Return on Investment (ROI)%	6.42	5.99	6.88	8.89	6.10
Export (BDT in million)	165203	158115	141289	225789	210588
Import (BDT in million)	136733	132465	108549	141496	171275
Remittance (BDT in million)	60113	79750	98911	128441	132243
Total Operating Income (BDT in million)	8060.40	9322.46	6070.84	9192.53	10694.80
Loan and Advances (BDT in million)	214618.15	227298.96	244642.03	262266.70	277870.36
Net Asset Value per Share (BDT)	20.02	21.22	23.48	23.46	24.56
Non-Performing Loan (NPL) Ratio %	4.10	4.61	3.24	5.14	4.87

Table-4.2: Five years highlight of **Bank Asia Ltd (BA)** from 2018 to 2022.Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	233509	268309	289054	314591	384467
Total Asset (BDT in million)	315417	369404	396982	449084	563239
Return on Assets (ROA) %	1.87	1.64	1.19	1.31	1.14
Return on Equity (ROE) %	19.25	15.60	10.69	11.00	10.22
Return on Investment (ROI) %	9.00	8.04	5.79	6.04	4.69
Export (BDT in million)	60800	71360	68863	93786	148284
Import (BDT in million)	110575	90661	81820	134102	211111
Remittance (BDT in million)	89127	78260	75673	103522	107035
Total Operating Income (BDT in million)	8822	9948	8602	10944	11129
Loan and Advances (BDT in million)	238008	264091	273063	321212	410676
Net Asset Value per Share (Taka)	25.65	29.35	33.38	36.36	39.02
Non-Performing Loan (NPL) Ratio %	3.10	3.99	2.93	3.90	3.72

Table-4.3: Five years highlight of **BRAC Bank Ltd (BRAC)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	205170	246704	254781	282063.63	331890.38
Total Asset (BDT in million)	324780	354689	382926	416902	506847
Return on Assets (ROA) %	0.70	0.70	1.10	1.20	1.00
Return on Equity (ROE) %	8.20	9.90	14.80	15.80	14.10
Return on Investment (ROI) %	7.51	6.46	8.29	5.28	6.36
Export (BDT in million)	117520	130241	119987	186029	272920
Import (BDT in million)	172937	197590	180817	326480	467213
Remittance (BDT in million)	48525	57148	51579	69910	129702
Total Operating Income (BDT in million)	15902	18285	16737	21403	24588
Loan and Advances (BDT in million)	231391	246944	268202	286379.68	354773.69
Net Asset Value per Share (BDT)	25.2	25	28.4	29.3	27.4
Non-Performing Loan (NPL) Ratio %	5.30	5.80	4.00	4.90	3.90

Table-4.4: Five years highlight of **The City Bank Limited (CBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	262467.70	302159.20	362611.00	401500.30	438131.30
Total Asset (BDT in million)	346468.80	390362.00	472355.40	514399.80	555473.60
Return on Assets (ROA) %	1.30	1.20	1.30	1.10	1.10
Return on Equity (ROE) %	19.70	17.20	18.40	16.10	14.40
Return on Investment (ROI) %	6.90	7.80	8.50	6.40	6.70
Export (BDT in million)	151484.40	141688.50	109085.50	156652.40	179471.90
Import (BDT in million)	167371.30	143652.80	127924.60	178831.70	183429.40
Remittance (BDT in million)	12803.24	40453.83	60844.66	216435.53	200547.86
Total Operating Income (BDT in million)	8693.00	11260.00	10501.30	11607.20	12362.80
Loan and Advances (BDT in million)	231553.90	256239.70	273382.90	319448.10	364000.80
Net Asset Value per Share (BDT)	115.4	54.9	58.6	58.4	59.85
Non-Performing Loan (NPL) Ratio %	4.1	4.4	2.2	3.7	4.3

Table-4.5: Five years highlight of **Dutch Bangla Bank Ltd (DBBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	199629	240164	242358	268019	317097
Total Asset (BDT in million)	282451	335163	336936	388815	455989
Return on Assets (ROA) %	1.15	1.30	1.22	1.28	1.21
Return on Equity (ROE) %	13.83	16.52	15.04	15.51	15.46
Return on Investment (ROI) %	1.93	2.37	2.25	12.95	6.93
Export (BDT in million)	135070	147306	130898	164629	227359
Import (BDT in million)	179884	174027	160413	262593	267297
Remittance (BDT in million)	42371	48551	31902	21793	28726
Total Operating Income (BDT in million)	13139	14597	14134	17158	17789
Loan and Advances (BDT in million)	209306	232051	228944	269394	308916
Net Asset Value per Share (BDT)	31.12	31.49	35.69	32.52	32.66
Non-Performing Loan (NPL) Ratio %	2.35	3.35	2.72	3.70	2.78

Table-4.6: Five years highlight of **Eastern Bank Ltd (EBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	300378.01	355689.16	394462.90	420673.77	426015.24
Total Asset (BDT in million)	317510.07	433017.99	484850.87	541665.25	540408.10
Return on Assets (ROA) %	0.66	0.59	0.61	0.42	0.69
Return on Equity (ROE) %	8.35	8.30	9.42	6.95	11.76
Return on Investment (ROI) %	8.31	8.26	7.29	5.91	6.10
Export (BDT in million)	171731.30	170652.60	161290.20	211974.50	276498.20
Import (BDT in million)	189530.40	200801.05	177825.26	280093.00	250137.77
Remittance (BDT in million)	5271.70	6456.60	5313.56	5811.80	6270.82
Total Operating Income (BDT in million)	12217.89	13016.53	13116.56	12337.16	14229.03
Loan and Advances (BDT in million)	-	-	-	-	-
Net Asset Value per Share (BDT)	19.98	20.67	21.66	21.72	22.02
Non-Performing Loan (NPL) Ratio %	5.11	4.33	3.82	3.98	3.86

Table-4.7: Five years highlight of **Export-Import Bank Ltd (EXIM)** from 2018 to2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	320011.19	376609.7	428687.22	469035.38	473025.03
Total Asset (BDT in million)	372183.32	437646.63	497250.79	544795.19	616453.58
Return on Assets (ROA) %	0.45	0.51	0.60	0.64	0.51
Return on Equity (ROE) %	11.97	13.43	15.41	16.22	13.06
Return on Investment (ROI) %	2.51	4.58	2.57	1.51	2.87
Export (BDT in million)	10337.7	12280.7	12412.1	23965.6	37150.4
Import (BDT in million)	92431.3	107385.7	43021.1	53845.7	63218.8
Remittance (BDT in million)	12995	14913.6	15003.1	15110	17137.4
Total Operating Income (BDT in million)	11207.96	12380.79	13460.49	15145.71	16915.29
Loan and Advances (BDT in million)	-	-	-	-	-
Net Asset Value per Share (BDT)	15.49	15.98	18.01	19.45	21.32
Non-Performing Loan (NPL) Ratio %	3.34	4.94	-	-	-

Table-4.8: Five years highlight of **First Security Islami Bank Limited (FSIBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	822573.12	946291.53	1176476.48	1381979.53	1410445.43
Total Asset (BDT in million)	1170422.26	1316169.23	1624916.66	1900838.42	2106095.67
Return on Assets (ROA) %	0.67	0.50	0.35	0.30	0.34
Return on Equity (ROE) %	11.07	9.39	7.38	7.19	8.93
Return on Investment (ROI) %	8.60	8.59	7.40	6.39	6.32
Export (BDT in million)	251592.00	234443.00	224970.00	301782.00	369637.00
Import (BDT in million)	399816.00	399884.00	419046.00	645295.00	754045.00
Remittance (BDT in million)	275212.00	305514.00	486274.00	505175.00	446969.00
Total Operating Income (BDT in million)	39597	43190	30706	43327	62529
Loan and Advances (BDT in million)	-	-	-	-	-
Net Asset Value per Share (BDT)	34.10	36.43	38.27	40.10	42.34
Non-Performing Loan (NPL) Ratio	4.12	3.82	3.41	3.31	3.70

Table-4.9: Five years highlight of **Islami Bank Bangladesh Limited (IBBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	226364	258368	296369	333142	375548
Total Asset (BDT in million)	283073	316950	356794	405878	465273
Return on Assets (ROA) %	0.59	0.81	0.17	0.52	0.68
Return on Equity (ROE) %	7.40	10.46	2.26	7.55	10.38
Return on Investment (ROI) %	7.53	5.77	6.55	7.54	6.18
Export (BDT in million)	98257	94645	74061	87674	120981
Import (BDT in million)	101642	86203	77078	110718	106372
Remittance (BDT in million)	15617.10	17450	5611.25	9467.36	11471.05
Total Operating Income (BDT in million)	3895	5120	2902	7168	6602
Loan and Advances (BDT in million)	206930	228589	260650	305061	354454
Net Asset Value per Share (BDT)	15.02	15.18	14.81	15.19	16.85
Non-Performing Loan (NPL) Ratio %	6.16	5.37	3.97	6.09	5.61

Table-4.10: Five years highlight of **IFIC Bank Ltd (IFIC)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	188073.50	202509.52	191103.99	212052.50	225070.78
Total Asset (BDT in million)	225018.22	242928.46	241533.71	264321.51	282636.72
Return on Assets (ROA) %	1.10	1.11	1.10	0.98	0.58
Return on Equity (ROE) %	13.83	14.80	12.93	10.88	7.69
Return on Investment (ROI) %	6.36	6.56	10.06	9.52	8.22
Export (BDT in million)	106070.70	103951.30	92321.21	114381.70	143542.70
Import (BDT in million)	130242.30	125264.80	111097.00	165227.90	175200.80
Remittance (BDT in million)	15000.00	18175.60	18120.00	24730.00	49292.76
Total Operating Income (BDT in million)	9817.78	11434.53	10860.69	10971.05	12822.34
Loan and Advances (BDT in million)	165402.85	177278.78	162658.43	174824.78	180490.79
Net Asset Value per Share (BDT)	24.12	22.90	32.23	28.52	26.36
Non-Performing Loan (NPL) Ratio %	3.77	3.70	2.95	2.97	5.32

Table-4.11: Five years highlight of **Jamuna Bank Limited (JBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	243643.42	262961.05	252817.40	274905.61	294060.00
Total Asset (BDT in million)	291385.64	316363.47	330785.56	359411.36	382328.47
Return on Assets (ROA) %	1.09	0.78	0.73	0.99	0.59
Return on Equity (ROE) %	16.55	10.99	10.05	14.70	8.87
Return on Investment (ROI) %	9.13	7.70	8.45	7.68	6.77
Export (BDT in million)	158307.70	163152.30	135418.10	164212.50	209222.50
Import (BDT in million)	196389.70	184650.00	172773.50	278533.10	254945.00
Remittance (BDT in million)	35943.00	35239.00	30893.00	40848.60	68272.90
Total Operating Income (BDT in million)	28117.54	31292.16	27275.79	26894.62	29555.47
Loan and Advances (BDT in million)	224230.61	236890.45	248994.39	266766.60	280870.00
Net Asset Value per Share (BDT)	22.93	22.31	22.46	23.62	23.30
Non-Performing Loan (NPL) Ratio %	4.82	4.86	4.72	4.54	7.09

Table-4.12: Five years highlight of **Mercantile Bank Limited (MBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	166160	190948	188734	211782	232657
Total Asset (BDT in million)	22464	256913	269269	306501	353644
Return on Assets (ROA) %	0.82	0.56	0.37	1.03	0.72
Return on Equity (ROE) %	13.83	9.03	5.83	16.22	11.56
Return on Investment (ROI) %	8.92	8.23	9.77	8.73	7.32
Export	61930.11	66213.18	58434.09	85754.91	127459.86
Import	118566.33	119455.02	102280.86	175079.40	228477.48
Remittance	47981.13	49341.96	37486.02	45724.62	87693.21
Total Operating Income (BDT in million)	9892.12	11207.18	9987.00	12804.11	16102.18
Loan and Advances (BDT in million)	166145	189730	203744	226012	253836
Net Asset Value per Share (BDT)	23.24	23.16	23.03	24.19	23.87
Non-Performing Loan (NPL) Ratio %	5.39	5.39	4.61	5.80	5.77

Table-4.13: Five years highlight of **Mutual Trust Bank Limited (MTBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	191344	200018	197021	203244	215850
Total Asset (BDT in million)	316707	340926	337372	377689	410402
Return on Assets (ROA) %	0.82	0.86	0.87	0.93	0.92
Return on Equity (ROE) %	10.45	11.51	11.10	11.63	11.68
Return on Investment (ROI) %	3.37	4.19	3.31	3.38	2.46
Export (BDT in million)	48577	68293	58355	71237	87600
Import (BDT in million)	101944	107718	96148	132891	144541
Remittance (BDT in million)	29493	42850	44616	46018	55127
Total Operating Income (BDT in million)	10418	11544	10123	11647	12675
Loan and Advances (BDT in million)	173867	179037	178159	189490	209738
Net Asset Value per Share (BDT)	17.38	19.05	20.48	20.99	21.78
Non-Performing Loan (NPL) Ratio %	5.80	4.81	4.98	4.67	6.85

Table-4.14: Five years highlight of **National Credit and Commerce Bank (NCC)** limited from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	202546	228091	229485	230659	226059
Total Asset (BDT in million)	266050	297874	303517	310366	314841
Return on Assets (ROA) %	0.56	0.59	0.44	0.25	0.45
Return on Equity (ROE) %	9.68	10.34	7.54	4.33	7.57
Return on Investment (ROI) %	6.88	6.00	8.51	6.01	7.01
Export (BDT in million)	94797	99496	81118	97273	107669
Import (BDT in million)	137904	130548	103742	137954	123559
Remittance (BDT in million)	6445	7013	4462	4204	2399.95
Total Operating Income (BDT in million)	10214	9331	7999	8388	10236
Loan and Advances (BDT in million)	198909	215228	220342	222694	225709
Net Asset Value per Share (BDT)	18.78	19.04	19.14	18.71	18.74
Non-Performing Loan (NPL) Ratio %	7.05	9.24	8.60	11.55	13.97

Table-4.15: Five years highlight of **One Bank Limited (ONE)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	169439.78	209346.02	241630.24	268803.05	296439.86
Total Asset (BDT in million)	216133.21	261900.49	315723.19	356574.98	384771.82
Return on Assets (ROA) %	2.06	1.40	0.71	0.97	1.09
Return on Equity (ROE) %	29.15	19.70	10.62	15.41	17.17
Return on Investment (ROI) %	6.61	5.12	10.69	6.43	5.18
Export (BDT in million)	89601.38	109235.04	109637.17	172080.71	257627.99
Import (BDT in million)	105322.94	140628.46	155414.83	270335.56	292227.64
Remittance (BDT in million)	37069.99	52755.30	40126.10	60388.68	92724.06
Total Operating Income (BDT in million)	11059.58	12951.78	12145.29	14573.73	18308.25
Loan and Advances (BDT in million)	156960.04	188945.47	213204.68	249829.58	264549.64
Net Asset Value per Share (BDT)	13.32	16.21	17.64	19.34	21.59
Non-Performing Loan (NPL) Ratio %	3.99	6.70	2.58	2.73	3.04

Table-4.16: Five years highlight of **The Premier Bank Ltd (PRBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	197518	216444	233028	243070	265841
Total Asset (BDT in million)	295613	323788	347502	389878	433410
Return on Assets (ROA) %	0.76	0.54	0.54	0.84	0.98
Return on Equity (ROE) %	8.31	5.93	6.31	10.61	12.93
Return on Investment (ROI) %	8.64	7.85	9.00	7.22	6.09
Export (BDT in million)	134616	137787	118091	154746	206693
Import (BDT in million)	182263	185735	147811	223419	199737
Remittance (BDT in million)	45755	50353	49267	38710	46786
Total Operating Income (BDT in million)	12899	14131	12653	15049	16502
Loan and Advances (BDT in million)	205810	213955	232400	263015	296481
Net Asset Value per Share (BDT)	24.63	24.86	25.4	26.4	28.53
Non-Performing Loan (NPL) Ratio %	6.16	4.66	3.46	4.83	3.42

Table-4.17: Five years highlight of **Prime Bank Limited (PBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	389549.50	414624.30	532299.90	576430.60	588675.80
Total Asset (BDT in million)	463289.80	497249.30	630547.70	665248.00	686727.20
Return on Assets (ROA) %	0.08	0.11	0.03	0.05	0.03
Return on Equity (ROE) %	0.62	0.80	0.92	2.05	1.26
Return on Investment (ROI) %	7.81	8.42	9.11	8.40	8.94
Export (BDT in million)	26002.00	26892.70	22834.50	33225.90	40678.30
Import (BDT in million)	114021.50	154018.30	112076.00	273002.20	176627.00
Remittance (BDT in million)	17173.20	22569.20	65802.00	48918.50	56325.30
Total Operating Income (BDT in million)	3095	1932.30	1596.70	1186.60	1067.40
Loan and Advances (BDT in million)	247490.60	306724.00	336835.20	380833.70	435400.80
Net Asset Value per Share (BDT)	41.23	40.75	39.62	36.37	35.18
Non-Performing Loan (NPL) Ratio %	17.90	15.04	11.79	17.50	21.19

Table-4.18: Five years highlight of **Rupali Bank Limited (RBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	154961	175468	169639	166419	170803
Total Asset (BDT in million)	197148	221890	219272	225085	233925
Return on Assets (ROA) %	0.67	0.72	0.49	0.35	0.49
Return on Equity (ROE) %	8.47	9.26	6.45	4.51	6.34
Return on Investment (ROI) %	8.22	6.51	22.35	1.96	4.74
Export (BDT in million)	51,884	56,326	43,631	44,593	49,160
Import (BDT in million)	74,464	75,399	49,195	70,522	74,581
Remittance (BDT in million)	9,579	10,364	8,407	12,527	12,565
Total Operating Income (BDT in million)	7144	7754	7801	6257	6728
Loan and Advances (BDT in million)	-	-	-	-	-
Net Asset Value per Share (BDT)	17.01	16.96	16.67	16.73	16.67
Non-Performing Loan (NPL) Ratio %	8.44	5.29	4.92	6.06	7.77

Table-4.19: Five years highlight of **Standard Bank Limited (SBL)** from 2018 to 2022. Source: Annual report 2022.

Parameter	2018	2019	2020	2021	2022
Total Deposit (BDT in million)	212680.61	242024.76	288403.90	284880.71	325930.65
Total Asset (BDT in million)	259638.37	294892.13	354778.33	365507.54	417902.46
Return on Assets (ROA) %	0.73	0.73	0.50	0.75	0.72
Return on Equity (ROE) %	15.14	14.60	11.18	15.55	15.70
Return on Investment (ROI) %	7.67	8.34	6.41	8.18	8.43
Export (BDT in million)	36552.45	37732.85	34680.82	46547.89	67790.26
Import (BDT in million)	119622.46	66079.04	72201.38	134751.59	146107.00
Remittance (BDT in million)	50112.60	48864.86	10291.57	16505.75	66888.00
Total Operating Income (BDT in million)	10085.71	11133.40	11728.76	16053.17	16763.58
Loan and Advances (BDT in million)	197128.06	210766.74	221063.69	257295.38	298376.45
Net Asset Value per Share (BDT)	23.14	24.38	26.49	25.80	25.50
Non-Performing Loan (NPL) Ratio %	7.90	5.49	4.51	3.65	4.76

Table-4.20: Five years highlight of **Trust Bank Ltd (TBL)** from 2018 to 2022. Source: Annual report 2022.

4.2.3 Research Methodology

Our study is based on the of several banks' daily closing prices, which are traded at the Dhaka Stock Exchange. Log-returns of these closing price are defined as

$$\boldsymbol{r_t = log\left(\frac{S_t}{S_{t-1}}\right)} \tag{4.1}$$

The RiskMetricsTM analyze was created by JP Morgan in 1994 and its framework was published in 1996 ((Longerstaey & Spencer, 1996), (Christoffersen P. , 2012); (Su & Knowles, 2006)). In 1998, the BASEL committee for banking supervision endorsed this method and advised all member banks to maintain capital reserves based on the estimated VaR. (Jordan & Mackay, 1995) and (Linsmeier & Pearson, 1996) initially established the definition of VaR.

4.2.4 Different User-friendly models

This thesis employs many easy-to-use mathematical models. We have selected 20 local commercial bank's daily closing price for both the generation of the data to be investigated and the fitting of models to our selected data. The following section provides an overview of the models employed in this thesis.

4.2.5 Gaussian Normal Distribution Approach

One of the most important ideas in modern financial theory is the Black-Scholes model, often known as the Black-Scholes-Merton (BSM) model. In this research work, we use BSM dynamics to evaluate the performance of the risk measures value-at-risk and expected shortfall based on backtests, along with our own manageable design. The Black-Scholes stochastic differential equation (SDE), which is a type of geometric Brownian motion SDE, is the fundamental basis for this model. It is used to derive the Black-Scholes formula, which is employed for pricing options. The formula was first introduced by Black and

Scholes in 1973, and later expanded upon by Merton in 1974. According to this model, an asset's development looks like this:

$$dS_t = \mu S_t dt + \sigma S_t dB_t$$
$$S_{t+dt} = S_t + S_t \mu dt + S_t \sigma \sqrt{dt}\; z \sim N\left(S_t + S_t \mu dt, S_t \sigma \sqrt{dt}\right) \quad (4.2)$$

Both the drift value μ and the diffusion parameter σ are constants, with σ being greater than zero. Furthermore, in this context, B_t represents a standard Brownian motion, where dB_t is defined as $z\sqrt{dt}$ with z denoting a standard normal random number. When estimating μ and σ using daily log returns and using $dt = \frac{1}{252}$ instead of $dt = 1$, the VaR and ES formulas can be modified to replace μ and σ with μdt and $\sigma\sqrt{dt}$, respectively. This model can only be used to derive analytical estimations of VaR and ES in situations when the drift and diffusion coefficients are not stochastic, as mentioned by (McNeil, Frey, & Embrechts, Quantitative Risk Management Concepts, Techniques and Tools, 2005) .

$$VaR(\alpha) = -(\mu - \sigma \Phi^{-1}(1-\alpha)) \quad (4.3)$$

and

$$ES(\alpha) = \frac{-(\mu - \sigma\varphi(\Phi^{-1}(1-\alpha)))}{\alpha} \quad (4.4)$$

where μ is the average return on daily share price, σ is the standard deviation of that return, φ is the standard normal density function and Φ refers to the standard normal distribution function. $\alpha = 2.5\%$, in our investigation, in compliance to Basel-III.

4.2.6 Historical Simulation model

According to (Farid, 2014), the Historical Simulation (HS) method is widely recognized as the most popular and effective approach among scholars. The HS approach is characterized by its simplicity in that it does not presuppose any type of distribution, and the outcome is contingent upon the caliber and accessibility of the data. (Gajadharsingh, 2013) defines the

empirical quantile method, also known as Historical Simulation, as a straightforward approach for assessing risk metrics. It relies on the observed distribution of past data regarding the financial portfolio's performance. Formally, VaR is computed by directly observing the empirical quantiles of historical returns. According to (Wiener, 1999), historical simulation is classified as a nonparametric approach for determining VaR. Nonparametric techniques have the characteristic of utilizing the empirical distribution, derived from the observed data, instead than relying on assumptions about the theoretical distributions of returns as in the parametric approach. The primary characteristic of historical simulation is its simplicity of execution.

An approach for VaR forecasts known as historical simulation uses historical data to estimate the quantile without making any explicit assumptions about the return distribution's form (Taylor, 2003) It is criticized for characterizing the current return and forecasting future risks based on historical data. It may become required to investigate any shift in the underlying market conditions in order to determine whether or not the data has to be weighted (Huang, 2009). Furthermore, (Hendricks, 1996)found that while a shorter sample period makes the VaR measure imprecise, a bigger sample reduces the variability of VaR estimates in historical simulations. The formula for calculating the value at risk for a future time period (t+1) is as follows:

For a sequence of historical observations $\{r_{t-j}\}_{j=0}^{n-1}$; where r_{t-j} is the non-overlapping $t-day$ returns This only indicates that

$$VaR_{t+1}(\alpha) = -Percentile\left\{\{r_t, r_{t-1}, r_{t-2}, \dots\dots, r_{t-(n-1)}\}, 100\alpha\right\} \tag{4.5}$$

and the ES

$$ES_{t+1}(\alpha) = -\frac{1}{\alpha n}\sum_{j=0}^{n-1} r_{t-j}\ I_{\{r_{t-j}<-VaR_{t+1}(\alpha)\}}.$$

We consider, $\alpha = 2.5\%$ and n is the sample size.

4.2.7 RiskMetricsTM Method

In 1994, RiskMetrics was first released. It includes datasets and methods for figuring out a market indexes or a portfolio of stocks' value at risk (VaR). Morgan and Reuters collaborated in 1996 to develop the method and provide professionals, executives, and investigators with open access to the statistics. By advising customers on managing risks, the goal is to enhance and encourage market uncertainty honesty and which will in turn establish a standard for measuring risk. Morgan computes the value at risk (VaR) by taking the conditional variance, which is a weighted average moved by one period, and then squaring the logarithms at period t-1, as described by (Sobreira & Louro, 2020). Unlike preceding models, this one does not give past returns identical weight. Instead, it allocates unequal weights in an exponentially decreasing manner, as demonstrated below, giving the most recent returns more weights than previous returns on assets because they include crucial information about the state of the market that will influence future movements:

$$\widehat{\boldsymbol{\sigma_t^2}} = \frac{\mathbf{1}}{\boldsymbol{k}} \sum_{\boldsymbol{j=1}}^{\boldsymbol{w_e}} \boldsymbol{\lambda^{j-1} r_{t-j}^2} \tag{4.6}$$

where $\sigma_t{}^2$ is a conditional variance.

The normalizing constant k is followed by the notation: $k = \sum_{j=1}^{w_e} \lambda^{j-1} = \frac{1-\lambda^{w_e}}{1-\lambda}$. And λ, as suggested by JP Morgan, the default value for the decay factor in practice is 0.94, which indicates the relative importance of historical data. The recursive volatility updating algorithm in this model can be obtained through simplification (Christoffersen P. , 2012).

$$\widehat{\boldsymbol{\sigma_{t+1}^2}} = \lambda\, \widehat{\boldsymbol{\sigma_t^2}} + (\mathbf{1} - \lambda)\boldsymbol{r_t^2} \tag{4.7}$$

from which getting the estimates of risk measure VaR turns out to be methodical ($\boldsymbol{VaR_{t+1}(\alpha) = -\widehat{\sigma_{t+1}}\Phi_\alpha^{-1}}$) (Longerstaey and Spencer, 1996; McMillian et al., 2009).

Once VaR is obtained ES can be obtained as an average of VaR's on the tail ((Engvall, 2016), (Emmer, Kratz, & Tasche, 2015) and (Dowd, Measuring Market Risk, 2005)).

4.2.8 Student-t Distribution Method

In order to measure market risk, we have implemented the use of a student t-distribution with location and scale parameters which allows us to determine both the average and the standard deviation of the log returns distribution while also defining the value at risk based on the degrees of freedom (ν), confidence level (cl), and holding period (h) as:

$$\boldsymbol{VaR_{h,cl}(\alpha) = -hp\mu + t_{cl,\nu}{}^{-1}(\alpha)\sqrt{h}\sqrt{\frac{\nu-2}{\nu}}\sigma} \tag{4.8}$$

Here, $t_{cl,\nu}(.)$ denote the cumulative distribution function of the student-t distribution and ν denote the degrees of freedom.

Using the four accessible models, it is also possible to estimate the expected shortfall after the VaR estimates for the twenty banks at the 97.5% level within the BASEL-III accord. Eq. (3.2) above describes ES as an integral of VaR, and we can roughly express this integral as the sum of various value-at-risks that correspond to various VaR levels.

$$\boldsymbol{ES_{\propto}(X) = \frac{1}{\propto}\int_0^{\propto} VaR_u(X)du \approx \frac{1}{N}\sum_{k=1}^{N} VaR_{\frac{k\propto}{N}}(X)} \tag{4.9}$$

The estimation that Emmer and Tasche (2015) proposed

$$\boldsymbol{ES_{\propto} = \frac{1}{4}[VaR_{\alpha}(X) + VaR_{0.75\alpha}(X) + VaR_{0.50\alpha}(X) + VaR_{0.25\alpha}(X)]} \tag{4.10}$$

Nevertheless, (Engvall, 2016) discovered that using the following approach for integration produced VaR estimates with historical simulation that was more accurate.

$$\boldsymbol{ES}_{\propto} = \frac{\mathbf{1}}{\mathbf{5}}[\mathbf{VaR}_{\boldsymbol{\alpha}}(\mathbf{X}) + \mathbf{VaR}_{\mathbf{0.80}\boldsymbol{\alpha}}(\mathbf{X}) + \mathbf{VaR}_{\mathbf{0.60}\boldsymbol{\alpha}}(\mathbf{X}) + \mathbf{VaR}_{\mathbf{0.40}\boldsymbol{\alpha}}(\mathbf{X}) + \mathbf{VaR}_{\mathbf{0.20}\boldsymbol{\alpha}}(\mathbf{X})] \quad (4.11)$$

Here we present the primary feature of our understandable structure that has been adopted by private investors at Dhaka Stock Exchange, as well as asset management firms in Bangladesh. This design aims to make expected shortfall a more accessible and less complex risk measure as compared to value at risk. The equation (4.10) demonstrates that ES is calculated as the average of four value-at-risk values at four different levels. Indeed, the ES is computed as the average of five VaR estimates at five separate VaR levels (as shown in Eq. (4.9). Additionally, it is also calculated as the average of an infinite number of VaR estimates at an infinite number of VaR levels as shown in Eq. (4.9). These ES estimates are expected to be marginally superior to the ones obtained using our user-friendly approach. Nevertheless, in our study, we adhere to the convention favoured by non-technical individual investors in Bangladesh. Therefore, we calculate the expected shortfall (ES) as the average of four value at risk measures in our empirical investigation.

Having calculated the Value at Risk (VaR) and Expected Shortfall (ES) for each of the 20 commercial banks in Bangladesh using four user-friendly models, as previously mentioned, we are now ready to conduct a series of tests. Specifically, we will perform eight tests for VaR and two tests for ES, in order to examine and validate the widely accepted hypotheses related to backtesting VaR and ES. The primary objective is to determine the most suitable user-friendly model based on each individual market participant's perspective within our user-friendliness design. Each hypothesis has a specific emphasis and examines if observed actual losses align with the predictions made by the model.

4.3 Backtesting Value-at-Risk (VaR)

Backtesting procedures are crucial for risk managers and regulators who are interested in evaluating the market risk exposure of a financial institution. These approaches are statistical tests specifically developed to detect excessive risk-taking by financial organizations. They assess the frequency of instances where the VaR model being examined is exceeded. It is widely recognized in the field of forecasting that econometric methods, which accurately describe the behavior of extreme quantiles within a given dataset, may not necessarily be the most effective for predicting their future behavior. Financial institutions have the option to select risk models for predicting conditional VaR. These models may be poorly described, but they nonetheless manage to meet the requirements for unconditional and independence backtesting.

Backtesting is a technique that utilizes past data to assess the precision and efficiency. Backtesting VaR is a method used to compare the projected losses with the actual losses that occur during a specific time period. The data obtained from backtesting give insights into specific time periods in which the Value at Risk (VaR) is overestimated or when the losses exceed the initially predicted VaR value. Researchers and institutions can recalculate these VaR values if the backtesting results are inaccurate, thereby enabling them to mitigate their vulnerability to unforeseen losses.

4.3.1 Binomial (Bin) Test

A frequently used method to evaluate the accuracy of a VaR model is by tallying the occurrences of VaR exceptions, which are defined as the days when the losses in a portfolio surpass the VaR estimates. If the number of exceptions is lower than the indicated confidence level, the algorithm overestimates the level of danger. In contrast, an excessive

number of exceptions indicates a failure to accurately assess the level of risk. It is uncommon for us to witness the precise number of exceptions indicated by the confidence level. Thus, the determination of whether the number of exceptions is reasonable or not, and whether the model will be accepted or rejected, ultimately relies on statistical analysis.

The simplest test to perform is to compare the observed (say, x) number of exceptions (violations) to the expected number of exceptions. Given the properties of a binomial distribution, we may create a confidence interval for the expected number of exceptions. The test utilizes either precise probabilities given by the binomial distribution or an approximation based on the normal distribution The probability of mistakenly rejecting a good model when x exceptions occur can be determined by calculating the $p - value$ for the observed number of exceptions, x, which represents the likelihood of detecting x exceptions. In this case, the VaR model should fail as an accepted-or-rejected outcome for a given test confidence level if x is beyond the test confidence range for the expected number of exceptions. The Z-score for this test statistic is typically calculated as follows:

$$\boldsymbol{Z_{bin} = \frac{x - Np}{\sqrt{Np(1-p)}}} \tag{4.12}$$

Where x is the number of exceptions, N is the number of the observation used in backtest and $p = 1 - VaRLevel$. The $z - score$ follows a normal distribution rather closely. When N is 250 or more, and p is between 1 and 10%, this rough estimate produces results that are in line with other tests for standard VaR backtesting investigations. On the other hand, it fails miserably when $N\ or\ p$ is tiny. If the $z - score$ is greater than the absolute value, then the bin test's tail probability is that the distribution is not normally distributed.
Probability of Tails = $1 - F(\,|Z_{scoreBin}|\,)$ where F represents the normal cumulative standard distribution. $p - Value_{Bin}$ represents the estimated chance of observing either a large number of failures or fewer when there are too few failures observed in comparison

to the expected failures. This is (roughly) the likelihood of observing that many failures or more in the case of an excessive number of failures.

Two times the tail probability is the definition of the bin test p-value. This is as a result of the two-sided nature of the binomial test

$$\text{Tail Probability} = 1 - F(\,|Z_{scoreBin}|\,)$$

where *F* represents the cumulative distribution function of the standard normal distribution. The $p - Value_{Bin}$ represents the likelihood of witnessing the number of failures many or fewer, where the number of observed failures is significantly lower than the expected number of failures.

The $p - value$ of the binomial test is calculated as twice the likelihood of observing a result as extreme as, or more extreme than, the one obtained. The reason for this is that the binomial test is a test that considers both sides of the hypothesis. If we take alpha to be 1 minus the test confidence level, then the test will reject if the tail probability is less than one half of alpha, or something similar if

$$P_{ValueBin} = 2 * TailProbability < alpha\ .$$

By employing the binomial distribution, we may assess the precision of the VaR model. During a statistical backtest that involves evaluating a null hypothesis to see if a model is 'excellent', there is a negotiation between two sorts of errors. $type\ I$ error pertains to the likelihood of erroneously rejecting a valid model, while $type\ II$ error pertains to the likelihood of failing to reject an invalid model. A test with high statistical power would effectively reduce each of these two probabilities (Jorion, 2001).

4.3.2 Proportion of Failures (POF) test

(Kupiec, Techniques for verifying the accuracy of risk measurement models, 1995) proposed the most generally recognized test that is based on failure rates. Kupiec's test,

commonly referred to as the POF-test (proportion of failures), assesses the consistency between the number of exceptions and the confidence level. Assuming that the model is valid under null hypothesis the number of exceptions is expected to follow the binomial distribution. Therefore, the only necessary data for conducting a POF-test is the total number of observations (T), the number of exceptions (x), and the desired confidence level (c) (Dowd, Measuring Market Risk, 2005).

In a POF-test, the null hypothesis is

$$H_0 : p = \hat{p} = \frac{x}{T}$$

And the goal is to determine whether the theoretical failure rate, p and the empirical failure rate, $\hat{p}$ differ significantly.

The Kupiec test has the following test statistic:

$$\boldsymbol{LR_{uc} = -2 \ln \frac{(1-p)^{T-x} p^{x}}{\left[1-\left(\frac{x}{T}\right)\right]^{T-x}\left(\frac{x}{T}\right)^{x}}} \tag{4.13}$$

Assuming that the model is true, the likelihood ratio test statistic $\boldsymbol{LR_{uc}}$ follows a chi-squared distribution with one degree of freedom in the long run. If the $\boldsymbol{LR_{uc}}$ statistic above the critical value of the χ^2 distribution, the null hypothesis will be rejected, indicating that the model is considered erroneous.

Kupiec's POF-test suffers by two deficiencies. Initially, it is important to note that the test lacks statistical strength due to the sample sizes being in line with the current regulatory framework, which is limited to one year. Kupiec himself has already acknowledged this power deficiency. Furthermore, the POF-test solely takes into account the frequency of losses and does not evaluate the temporal aspect of when they occur. Consequently, it

might not be able to reject a model that generates aggregated exceptions. Therefore, it is important that model backtesting does not only depend on tests of unconditional coverage (Campbell, 2005).

4.3.3 Christoffersen's Interval forecast test (CCI)

(Christoffersen P. F., Evaluating Interval Forecasts, 1998) created the most renowned assessment of conditional coverage. The user applies the same log-likelihood testing framework as Kupiec, but enhances the test by including an extra statistic to measure the independence of exceptions. Moreover, his model evaluates whether the probability of a violation happening on a specific day is affected by the outcome of the previous day. The test is conducted by initially establishing an indicator variable that takes a value of one if there is a VaR violation and a value of zero in the absence of VaR violation.

$$I_t = \begin{cases} 1 \ , if \ there \ is \ a \ violation \\ 0, \ \ in \ the \ absence \ of \ \ violation \end{cases}$$

Then describe T_{ij} as the number of days when situation j happened presuming that scenario i happened on the day before. One can show the result in a $2\ x\ 2$ crosstab to show illustration.

	$I_{t-1} = 0$	$I_{t-1} = 1$	
$I_t = 0$	T_{00}	T_{10}	$T_{00} + T_{10}$
$I_t = 1$	T_{01}	T_{11}	$T_{01} + T_{11}$
	$T_{00} + T_{01}$	$T_{10} + T_{11}$	T

Furthermore, let π_i denote the likelihood of witnessing a violation given that state i occurred on the earlier day:

$$\pi_0 = \frac{T_{01}}{T_{00}+ T_{01}} \quad \pi_1 = \frac{T_{11}}{T_{10}+T_{11}} \text{ and } \quad \pi = \frac{T_{01} + T_{11}}{T_{00}+ T_{01}+T_{10} + T_{11}}$$

If the model is valid, then the occurrence of a violation today should not be influenced by whether or not a violation occurred on the prior day. Simply put, the null hypothesis states that the probabilities π_0 and π_1 should be identical. The appropriate test statistic for determining the independence of violation is a likelihood-ratio:

$$\boldsymbol{LR_{ind}} = -\mathbf{2}\ln\frac{(\mathbf{1}-\boldsymbol{\pi})^{\boldsymbol{Too}+T_{10}}\,\pi^{T_{01}+T_{11}}}{[\mathbf{1}-\boldsymbol{\pi_{01}}]^{\boldsymbol{Too}}\quad \boldsymbol{\pi_{01}^{To1}}(\mathbf{1}-\boldsymbol{\pi_{11}})^{\boldsymbol{T1o}}\boldsymbol{\pi_{11}^{T11}}} \tag{4.14}$$

The asymptotic null distribution corresponds to the chi-squared distribution with a single degree of freedom $\boldsymbol{LR_{ind}}\sim\boldsymbol{\chi 2}$.

4.3.4 Conditional Coverage (CC) Test

We may create a combined test that assesses the correct failure rate and the conditional coverage—the absence of interdependent exceptions—two essential components of a trustworthy VaR model by combining the Conditional Coverage independence statistic with Kupiec's POF-test:

$$LR_{cc} = LR_{POF} + LR_{ind}$$

Since there are two different $LR-s$tatistics in the test, the test statistic LR_{cc} has two degrees of freedom and is distributed according to a chi-squared (χ^2) distribution. The model passes the test if the value of the LR_{cc} statistic is smaller than the critical value of the χ^2 distribution; larger values result in the model's rejection. Christoffersen's framework enables the analysis of whether the failure to pass the test is due to imprecise coverage, concentrated violations, or a combination of both.

To perform this assessment, compute the $LR_{POF}$and $LR_{ind,}$statistics separately. Then, utilize the χ^2 distribution with one degree of freedom as the critical value for both statistics. According to (Campbell, 2005), there exist situations in which a model may pass the joint

test but still fail the independence or coverage tests. Therefore, even if the results of the combined test are positive, it is still advised to conduct separate testing.

4.3.5 The Basel Committee's traffic light test

The Basel Committee on Banking Supervision recommended the traffic light concept in 1996, 2006, and 2011. The goal of the document was to establish a structured plan for conducting backtesting of Value-at-Risk (VaR) specifically for the use of financial institutions. Many banks typically utilize the following methods to assess the effectiveness of their internal models. The Basel Committee 'traffic light' approach is used to assess the frequency of violations (failure rate), like the proportion of failure test which is an unconditional coverage test. This backtesting methodology is mainly utilized for regulatory purposes at the 99% confidence level, and the regulatory framework provides exceptional ranges for this confidence level. Nevertheless, we can compute violation ranges at several confidence levels for the convenience of internal backtesting purposes.

This test relies on the excess ratio, denoted as α:

$$\hat{\alpha} = \frac{1}{N}\sum_{t=1}^{N} I_t \tag{4.15}$$

where N is the total number of forecasts analyzed in the backtest and I_t is either zero or one depending on whether a forecast is a violation or not.

The results of this test have been sorted into three zones: green, yellow, and red. The green zone indicates that the model's predicted accuracy is satisfactory, the yellow zone indicates that there is a probable problem, and the red zone indicates that there is a significant problem. Each of these three regions have been defined and their boundaries selected to achieve a balance between $type\ I$ and $type\ II$ errors. The assumption is that the model

covers 99% of outcomes, and the probabilities of committing a $type\ I$ error (erroneously rejecting an accurate model) have been calculated based on the number of exceptions chosen as a threshold for model rejection. Based on these probabilities, a specific threshold number has been established for each of the three groups.

A binomial distribution is used in the traffic light test. For a viable model, we have $p = 1 - VaRLevel$, where x is the number of failures, N is the total number of observations, and p is the probability of detecting a failure. The cumulative distribution function of the binomial distribution with parameters N and p, where $p = 1 - VaRLevel$, is used to compute the probability of witnessing failures up to x in this test, which is defined by $P(X \leq x|N,p) = F(x|N,p)$, where $F(x|N,p)$ is cumulative distribution function .The following three zones are determined using this cumulative distribution function:

$$\begin{array}{ll} \text{Green} & F(x|N,p) \leq 0.95 \\ \text{Yellow} & 0.95 \leq F(x|N,p) < 0.9999 \\ \text{Red} & F(x|N,p) \geq 0.9999 \end{array} \quad (4.16)$$

By employing the method of binomial distribution, we may assess the precision of the VaR model. Nevertheless, throughout the process of doing a statistical backtest that involves either accepting or rejecting a null hypothesis (regarding the model's quality), there exists a compromise between two categories of errors. $type\ I$ error pertains to the likelihood of erroneously rejecting a valid model, while $type\ II$ error relates to the likelihood of failing to reject an invalid model. A test with high statistical power would effectively minimize both of these probabilities (Jorion, 2001).

4.3.6 Time Until First Failure (TUFF) Test

Extending above the traditional test structure, Kupiec proposed another test called Time Until First Failure (TUFF), which is rather disputed. The TUFF test assesses the temporal

occurrence of the initial rejections. The VaR model is considered to have failed if breaches occur at a rapid rate. By solely examining the initial exception, a significant amount of information is omitted, and any subsequent exceptions are dismissed. The TBFI test builds upon the TUFF method by considering all failures inside an expanded framework of the TUFF test. The TUFF test depends on a geometric distribution and utilizes a likelihood ratio. This distribution is applicable when considering the number of days until the first violations, denoted by n. Asymptotically the test statistic follows a chi-square variable with 1 degree of freedom:

$$\boldsymbol{LR_{TUFF}} = \boldsymbol{-2\log}\left(\frac{\boldsymbol{p(1-p)^{n-1}}}{\left(\frac{1}{n}\right)\left(1-\frac{1}{n}\right)^{n-1}}\right) \tag{4.17}$$

The model is accepted if the test statistic is less than the crucial value; if not, it is rejected. According to (Dowd, Beyond Value at Risk: The New Science of Risk Management, 1998), in situations where a more extensive set of data is unavailable, the TUFF-test should only be utilized as a precursor to the POF-test. Additionally, the test offers a helpful structure for evaluating the independence of violations in mixed Kupiec-test by (Haas M. , 2001).

4.3.7 TBF and TBFI Test

The TBF and TBFI tests stand for time between failures and the time between failures independence test, which incorporates the temporal data for each failure in the sample. While Christoffersen's interval forecast test can be a helpful backtest for investigating VaR violations' independence, it has the limitations of only considering dependency between two consecutive days, so it can't capture all forms of dependence. If a VaR violation

happened yesterday in comparison, say, a week ago may have less of an impact on the likelihood of a violation happening today.

According to (Haas M. , 2001) Christoffersen's interval forecast test is too unstable to yield results that are practical. As a result, he presents an enhanced independence and coverage test that relies on Christoffersen and Kupiec's concepts. Compared to focusing just on whether an exception today is dependent on the outcome of yesterday, Haas suggests a mixed Kupiec-test that calculates the interval between violations. As a result, the test could be able to detect more widespread types of dependence. To create this mixed test, we must calculate the Likelihood Ratio statistic for each exception i.

$$LR_i = -2\,In\left(\frac{p(1-p)^{n_i-1}}{\left(\frac{1}{n_i}\right)\left(1-\frac{1}{n_i}\right)^{n_i-1}}\right)$$

where n_i is the number of days between failures $i-1$ and i (or until the first exception for $i = 1$) and $p = 1 -$ VaR level. Once calculated the LR- statistics, we conduct an independence test assuming that there are x violations.

$$\boldsymbol{LR_{TBFI} = -2\sum_{i=1}^{x}\log\left(\frac{p(1-p)^{n_i-1}}{(\frac{1}{n_i})(1-\frac{1}{n_i})^{n_i-1}}\right)} \tag{4.18}$$

The TBF mixed test is a combination of the TBFI test and the frequency test POF. This type of combination is sometimes referred to as Haas's mixed Kupic Test. Clearly, it follows an asymptotic distribution that can be described as a chi-square variable with $x + 1$ degrees of freedom:

$$\boldsymbol{LR_{TBF} = LR_{POF} + LR_{TBFI}} \tag{4.19}$$

Just like with other likelihood-ratio tests, the statistic is compared to the critical values of χ^2 distribution. If the value of the likelihood ratio test statistic is less than the critical value of the chi-squared distribution, the model is accepted otherwise, rejected. One benefit of

this test is its high level of resilience, as it can detect both problems with dependency and the quantity of violations.

4.3.8 Backtesting Expected Shortfall (ES)

In addition, the following unconditional test statistic of ES backtesting has also been considered in our study:

$$\boldsymbol{Z_{unconditional} = \left(\frac{1}{N_{pVaR}}\right)\sum_{t=1}^{N}\frac{X_t I_t}{ES_t} + 1} \tag{4.20}$$

Here N is the size of the test window and X_t is the portfolio returns for period t. $p - VaR$ is the probability of VaR failure, which is defined as the complement of the VaR level. The term ES_t refers to the predicted expected shortfall for a specific period t. I_t is the indicator variable on period *t* with a value of 1 if $X_t < -VaR_t$, and 0 otherwise.

The expected value of this test statistic is typically 0, and it yields a negative value when there is a danger of underestimating. The crucial values are necessary for determining the threshold at which the model should be rejected based on the magnitude of the test statistic value. Although the test does not rely on any assumptions about the distribution, the critical values can still be found by assuming a distribution for the corresponding outcomes of actual return X_t, for the period t. The expected shortfall backtest has two sets of tables containing important values. The initial table assumes that $X_t$follows a standard normal distribution for doing the unconditional normal test, whereas the second table of critical values assumes that $X_t$follows a t −distribution for conducting the unconditional T test.

In addition, we conduct an investigation that explores the potential correlation between market capitalization and the risk management procedures of banks. This analysis relies on estimates of Normal, Historical, Student-T (location based), and EWMA VaR and ES forecasts. We examine two econometric models that are estimated using various

approaches, including fixed effect, random effect, pooled OLS (ordinary least square), and GLS (generalized least square) methods. We conducted a regression analysis utilizing the logarithm of market capitalization and four VaR forecasts. The data used included daily market valuation data and four daily VaR forecasts produced from four user-friendly models.

Furthermore, we undertake an inquiry that examines the possible relationship between the market capitalization and the risk management protocols of banks. This research is based on estimations of Normal, Historical, Student-T (location based), and EWMA VaR and ES projections. We analyze two econometric representations that are assessed using different methodologies, such as fixed effect, random effect, pooled ordinary least square, and generalized least square (GLS) techniques. We performed a regression analysis using the logarithm of market capitalization and four VaR forecasts. The data utilized consisted of day-to-day market valuation data and four daily VaR projections generated by four manageable algorithms. In this investigation, we will examine the time frame from January 2015 to August 2020 in order to estimate the models. This time period was chosen due to a lack of available market capitalization data.

The table below presents the variables that appear in the models:

Variables	Notation	Measures	Source
Market Capitalization	**MarketCap**	**Log (Number of outstanding share x daily closing share price)**	**Dhaka Stock Exchange, Bangladesh**
Normal-VaR	**Normal-VaR**	**Estimated in empirical section**	**Collected from empirical section**
Historical-VaR	**HS-VaR**	**Estimated in empirical section**	**Collected from empirical section**
Student-T-VaR	**StuT-VaR**	**Estimated in empirical section**	**Collected from empirical section**
EWMA-VaR	**EWMA-VaR**	**Estimated in empirical section**	**Collected from empirical section**
Normal-ES	**Normal-ES**	**Estimated in empirical section**	**Collected from empirical section**
Historical-ES	**HS-ES**	**Estimated in empirical section**	**Collected from empirical section**
Student-T-ES	**StuT-ES**	**Estimated in empirical section**	**Collected from empirical section**
EWMA-ES	**EWMA-ES**	**Estimated in empirical section**	**Collected from empirical section**

Table 4.21: Shows variables that used in the model.

We examine the correlation between market valuation and the risk management procedure of banks using the following econometric models.

$$MarketCap_{it} = \theta_{it} + \sum_{k=1}^{4} \delta_{it} VaR_{itk} + \varepsilon_{it} \quad (4.21)$$

$$MarketCap_{it} = \theta_{it} + \sum_{k=1}^{4} \delta_{it} VaR_{itk} + u_{it} + \varepsilon_{it} \quad (4.22)$$

$$MarketCap_{it} = \theta_{it} + \sum_{k=1}^{4} \delta_{it} ES_{itk} + \varepsilon_{it} \quad (4.23)$$

$$MarketCap_{it} = \theta_{it} + \sum_{k=1}^{4} \delta_{it} ES_{itk} + u_{it} + \varepsilon_{it} \quad (4.24)$$

In this analysis, we will perform a regression of market capitalization against four estimates of value-at-risk and expected shortfall where θ_{it} is a constant, VaR_{it} = VaR forecasts from

Normal, Historical Simulation, $Student-t$ and Exponentially Weighted Moving Average (EWMA) used previous section for daily VaR forecasts. ES_{it} = expected shortfall forecasts from Normal, Historical Simulation, Student-t and Exponentially Weighted Moving Average (EWMA) used previous section for daily expected shortfall forecasts. δ_{it}= coefficient of explanatory variable; ε_{it} = error term / within entity error; u_{it} = between entity error. To ensure the robustness of the models, we have utilized pooled OLS, fixed effect, random effect, and GLS methods to estimate the coefficients of the models based on the four equations (eq.4.21 to eq.4.24). Furthermore, many diagnostic assessments, including tests for multicollinearity, heteroscedasticity, auto-correlation, and model specification, have been conducted to verify the effectiveness of the models.

4.4 Scoring functions for Value-at-Risk (VaR) and Expected Shortfall

We concentrate scoring functions for Value-at-Risk and the pair of Value-at-Risk and Expected Shortfall and identify conditions required for existence of the expectation of these scoring functions. Scoring functions used as a loss function in decision theory to evaluate forecasting of a risk measure of a probability distribution. A scoring function refers to a loss function that can be optimized by the true prediction of some elicitable measures (Tobias Fissler, 2016). VaR is an elicitable measure as the true VaR forecasts can minimize the quantile loss function, while ES is not an elicitable measure. Nevertheless, (Tobias Fissler, 2016) described the joint elicitability of VaR and ES and proposed a general framework of the joint scoring function.

4.5 Elicitability and Scoring functions for VaR

A risk measure is elicitable if the correct forecast of the measure is the unique minimizer of the expectation of at least one scoring function. If such a scoring function exists, it is called strictly consistent for the measure. A strictly consistent scoring function can be used for forecast evaluation and as the loss function in model estimation (Gneiting & Raftery, 2007) VaR is an elicitable measure as the true VaR forecasts can minimize the quantile

loss function, while ES is not an elicitable measure. The consistent scoring function that elicits VaR is

$$S(y,\ VaR) = (I_{VaR \geq y} - \alpha)(VaR - y)$$

Where I is indicator function, S is the loss function score, Y is the out-sample log-returns, $\alpha = 1 - VaR\ Level, I = 1\ when\ y\ < VaR\ and\ 0\ otherwise.$

4.6 Joint Scoring Functions for VaR and ES

(Fissler & Ziegel, 2016) developed a group of scoring functions to assess the accuracy of a model's VaR and ES forecast series. These functions aid in assessing a model's capacity to generate precise VaR and ES predictions, leading to the minimization of loss scores. The loss score values of different new approaches can be used to evaluate the efficiency ranking of models based on their relative efficacy in properly forecasting both value-at-risk and expected shortfall. The loss score value is determined by utilizing a particular model's value-at-risk (VaR) and ES (expected shortfall) forecast series as:

$$\boldsymbol{S}[\mathbf{y},\ \boldsymbol{VaR},\ \boldsymbol{ES}] = (\boldsymbol{I}{-}\propto)\boldsymbol{VaR} - \boldsymbol{I} * \boldsymbol{y} + \mathbf{exp}(\boldsymbol{ES}) * \left(\boldsymbol{ES} - \boldsymbol{VaR} + \left(\frac{\boldsymbol{I}}{\propto}\right)(\boldsymbol{VaR} - \boldsymbol{y})\right)$$

$$- \mathbf{exp}(\boldsymbol{ES}) + \mathbf{1} - \mathbf{log}(\mathbf{1}{-}\propto) \qquad (4.25)$$

Where value of the scoring function denoted by S, and y represents the out-of-sample log-returns, α is the complement of the VaR level (1 minus the VaR level), and I is the indicator variable which takes the value of 1 when y is less than value-at-risk and 0 otherwise.

4.7 Empirical Results with Discussion

This study encompasses the complete banking department of Bangladesh, a developing nation, specifically focusing on the twenty chosen private commercial banks that are now operating in Bangladesh and have their shares listed on the Dhaka Stock Exchange (DSE).

We utilize all of the data available in the electronic database of the securities and exchange commission (SEC) of Bangladesh and carefully choose twenty banks as an appropriate characteristic sample of the economy. These selected banks have superior data records, while only a small number of banks trading their shares on the DSE are excluded. The sample consists of both conventional and Islamic banks, including PCBs and NCBs. We utilize the dataset including the daily closing prices of shares.

Since two out of our four manageable models, namely the HS model and the RiskMetricsTM model, are non-parametric, we do not analyze the results of the other two models, namely the Gaussian model for two-parameters and the student-t-location-scale model for three-parameters using in-sample approximation. The historical simulation model utilizes previous returns, while the RiskMetricsTM model employs a default value for its only parameter as suggested by JPMorgan. Furthermore, since we consider the risk to be in the future rather than in the past, we dedicate our utmost efforts to using backtesting-based deduction to create the most suitable and user-friendly model for predicting future risks that aligns with our manageable design. Our interpretation relies on the collective data obtained from eight value-at-risk and two expected shortfall backtests which consider out-of-sample returns, as well as the corresponding VaR and ES forecasts for each return. By considering all the three elements return, value-at-risk, expected shortfall for out-of-sample data throughout the complete backtest period, we calculate a loss score value using the scoring function developed by (Tobias Fissler, 2016)This score function evaluates the accuracy and adequacy of each VaR and ES forecast over the whole backtest period. A reduce loss score indicates a higher level of accuracy in the model's risk forecasting capacity. The decision to utilize this scoring function to rank the models' performance in risk forecast exactness is also influenced by our strong emphasis on user-friendliness.

4.8 Estimations of value-at-risk and expected shortfall with user-friendly models

In order to create a user-friendly approach, we have implemented a two hundred fifty trading day estimation window each time for the dynamic forecasts of value-at-risk and expected shortfall and takes into account 2,460 returns data for each bank (Mozumder, Abedin, Lalon, & Hossain, 2024). The length of the test windows corresponding to different banks somewhere value-at-risk and expected shortfall forecasts for out-of-sample observations varies between 1200 and 1600. The entire dataset for each bank was processed to remove duplicate closing share prices for successive times, resulting in varying test window lengths across different banks. Therefore, the number of tail events that are anticipated differs from one bank to another. For instance, a bank that has 1,600 predictions can expect 40 tail events, which corresponds to 2.5%, according to the BASEL-III consensus. Figure-4.1 to 4.10 illustrates the day-to-day value-at-risk of twenty commercial banks in Bangladesh for the data spans from January 1, 2013, to August 31, 2020. These banks have consistently traded their equity share prices on the Dhaka Stock Exchange (DSE) since 2010. In accordance with the Basel-III accord, we utilize a 97.5% Value at Risk (VaR) measure. Additionally, we incorporate VaR estimates from four of our user-friendly models for each plot. The VaR values in figure-4.1 to 4.10 are generated using out-of-sample projections. These forecasts are based on a rolling window of one year, using the immediate past return series as the basis for each forecast. This aligns with our user-centric design, recognizing that individual investors at DSE typically have limited access to historical data beyond one year. Furthermore, even if some investors possess such data, they prefer not to incorporate older observations when making dynamic forecasts. Instead, our approach involves utilizing one-year rolling window calibrations for Normal and Student-t models, considering location and scale. It is worth noting that the HS model employs an empirical distribution, while the EWMA model relies on the default estimate provided by JPMorgan, thus obviating the need for calibration. The ES forecasts that correspond to the given information are displayed in Fig-4.11 to 4.20.

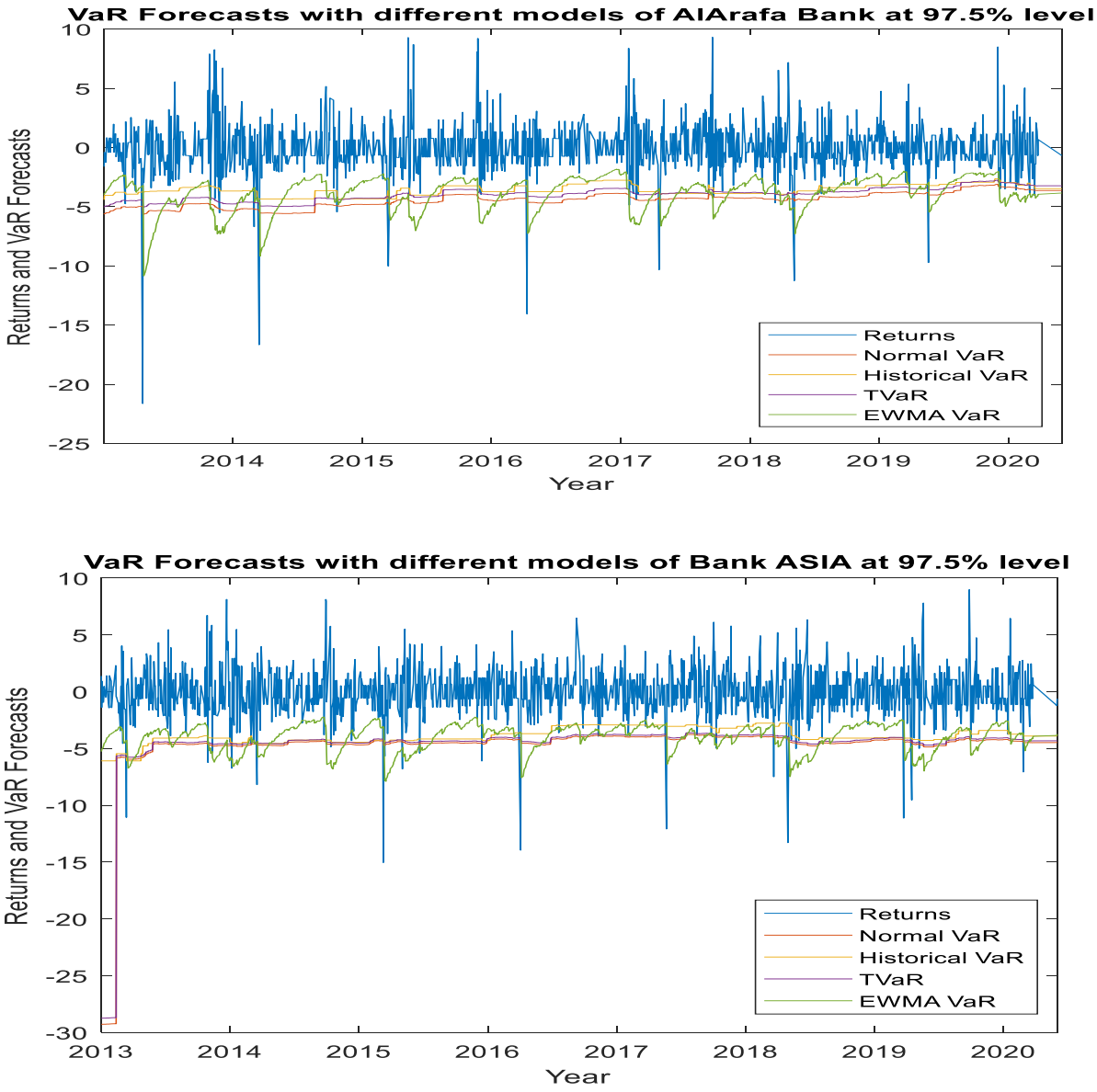
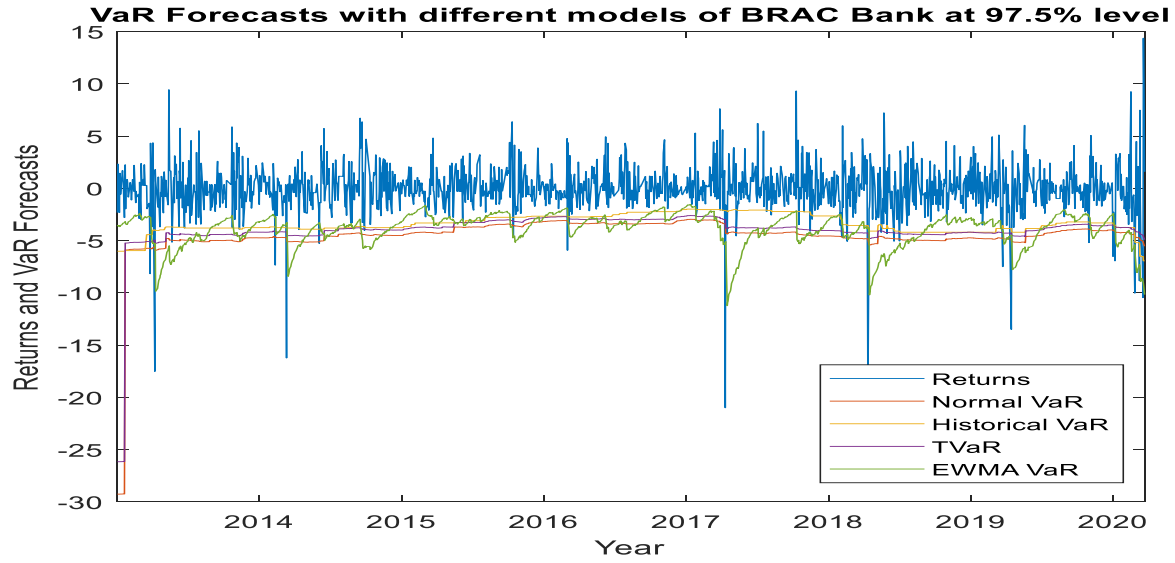


Figure-4.1: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **AL-Arafa banks (Top) and Bank Asia (Bellow)** are available considering four user-friendly models from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.



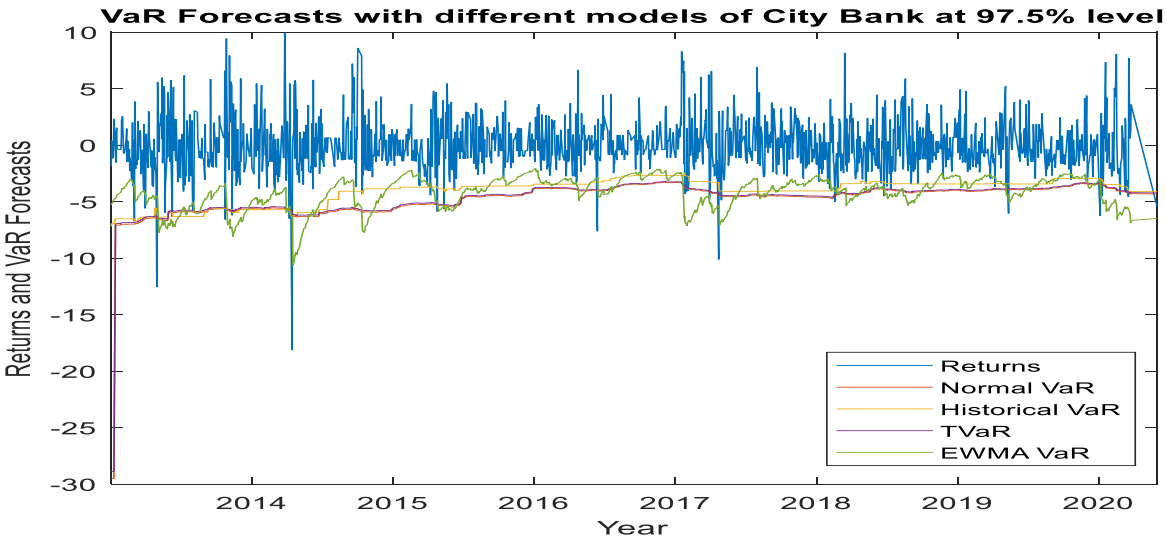


Figure-4.2: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **Brac Bank (top) and City Bank (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

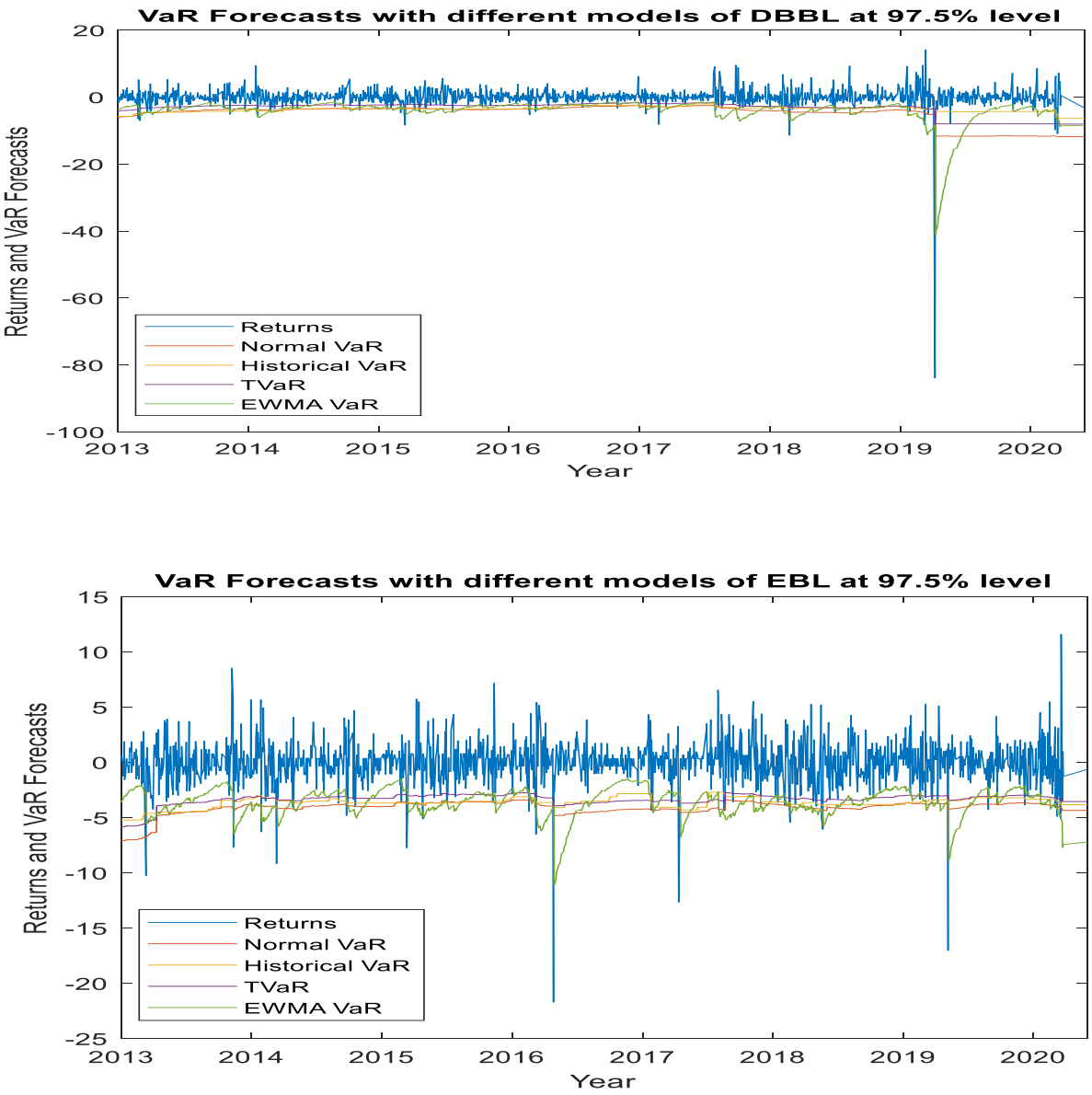


Figure-4.3: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **DBBL (top) and EBL (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

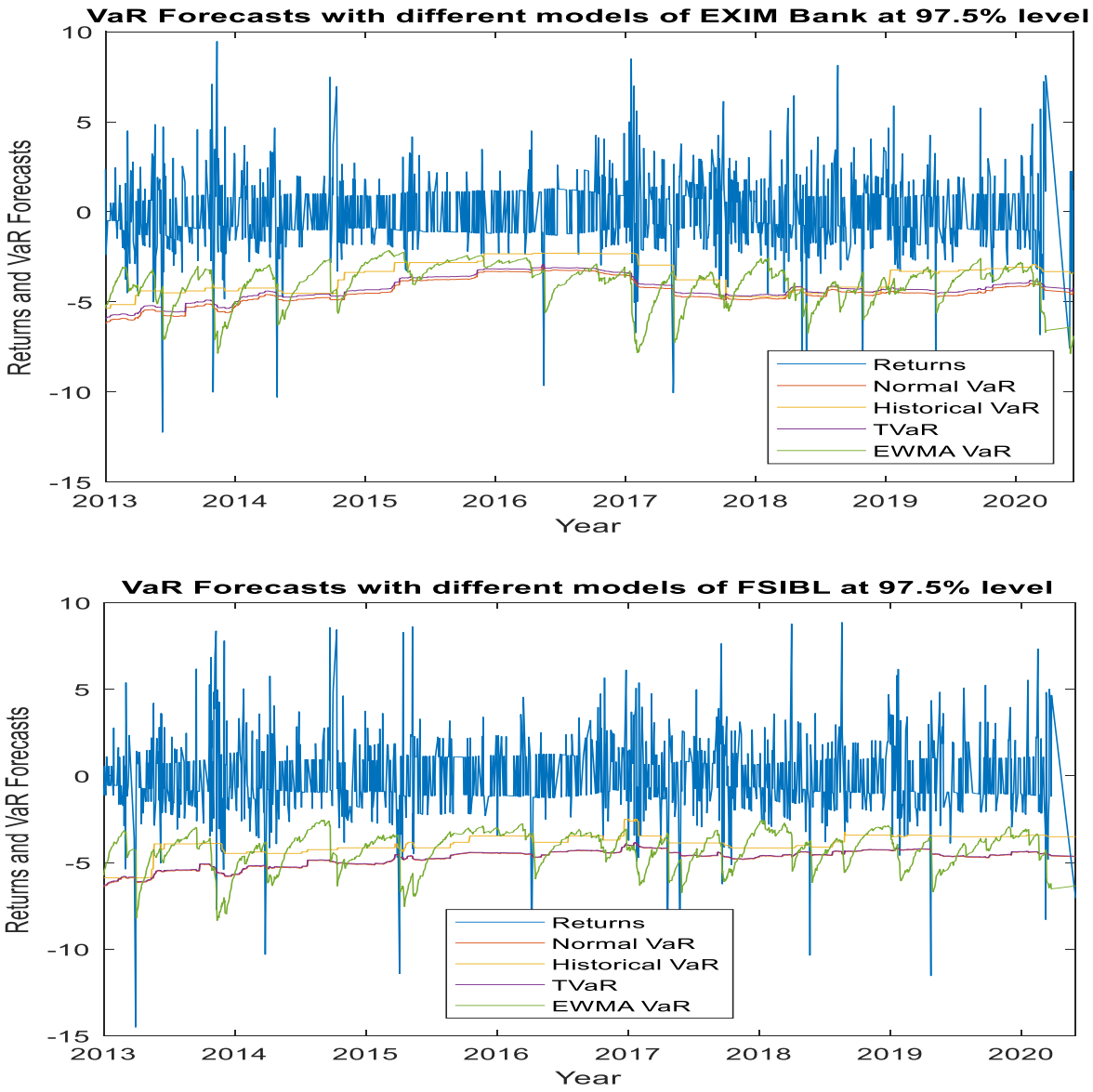


Figure-4.4: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **Exim Banks (top) and FSIBL (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

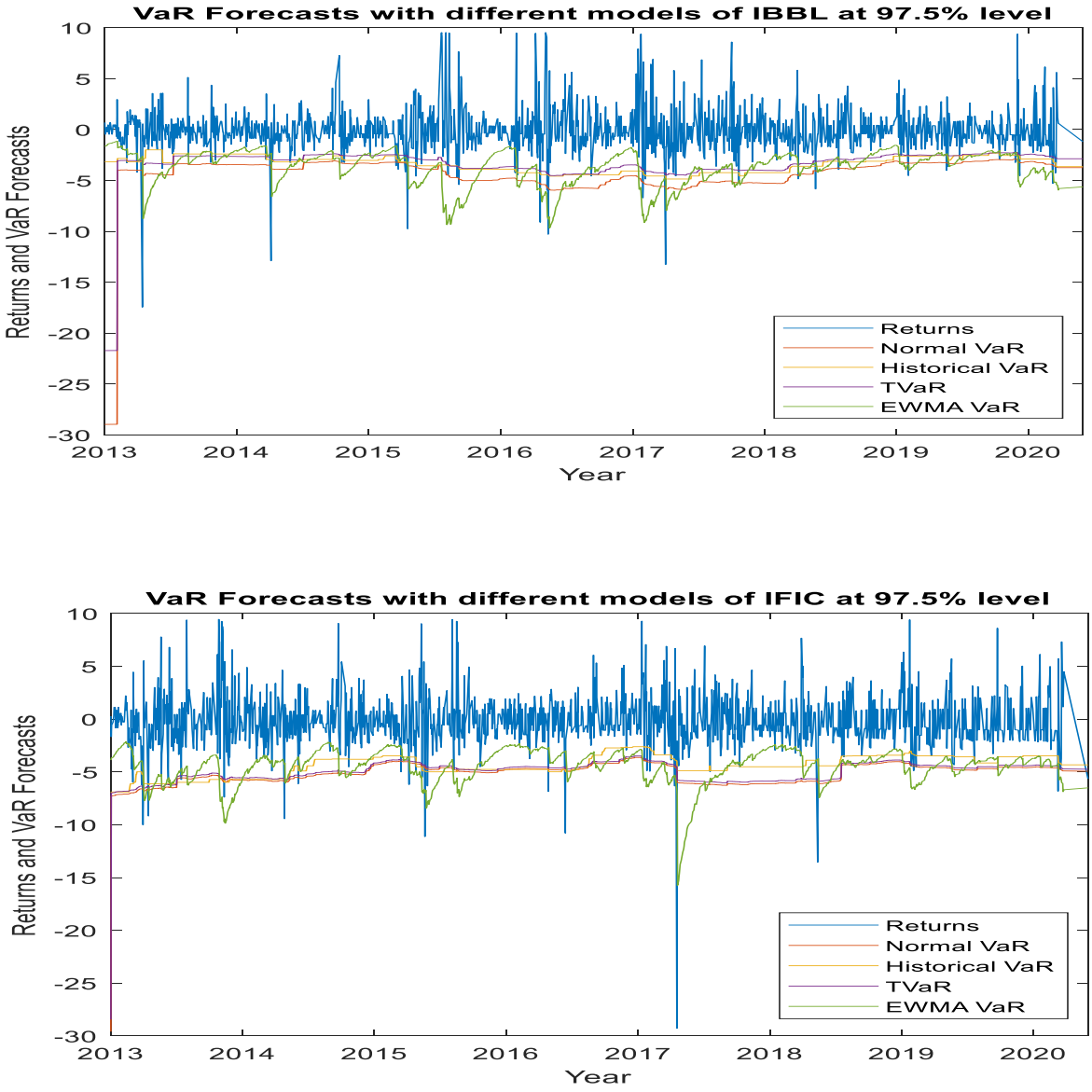


Figure-4.5: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **IBBL (top) and IFIC (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

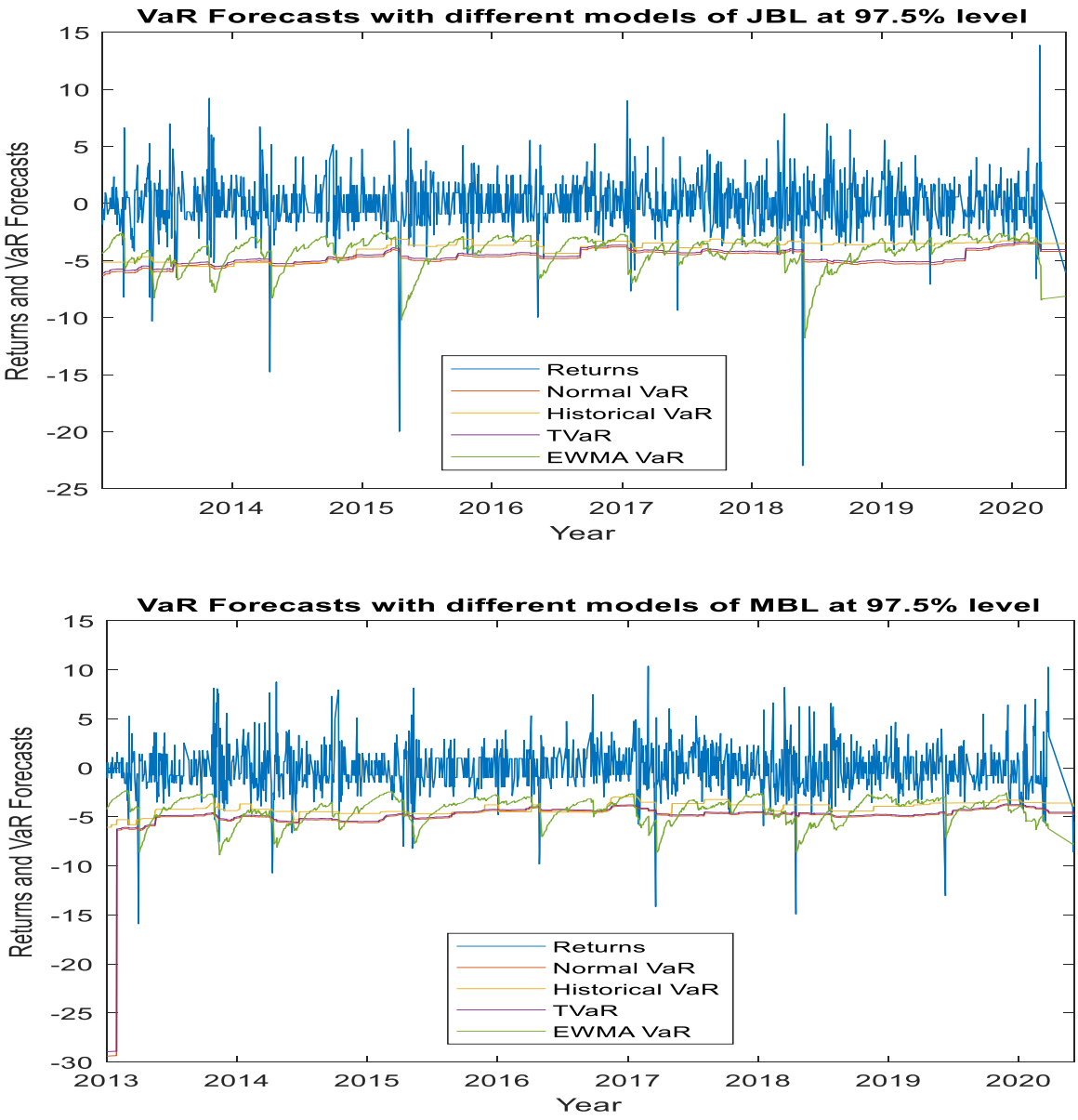


Figure-4.6: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **JBL (top) and MBL (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

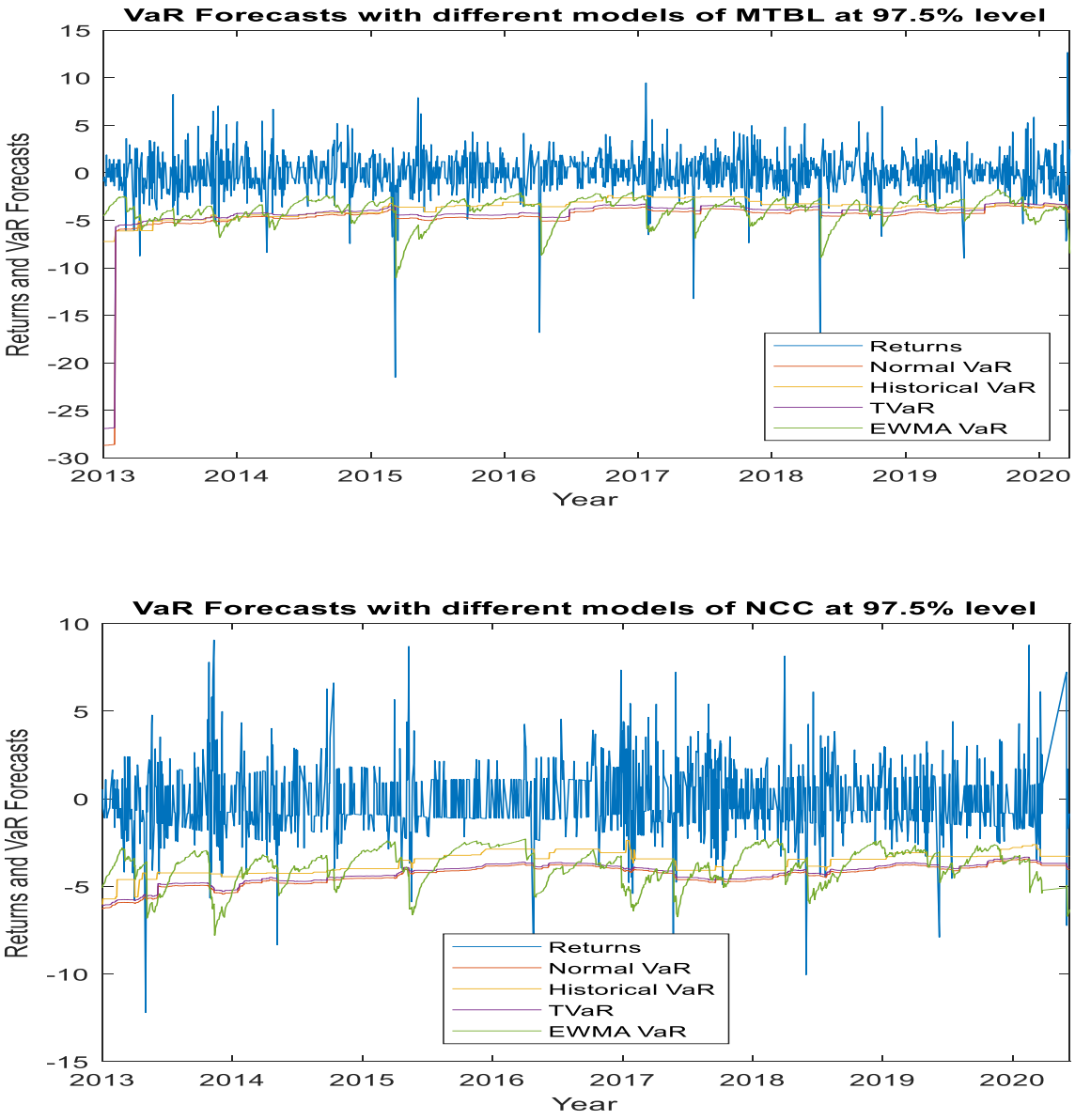


Figure-4.7: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **MTBL (top) and NCC (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

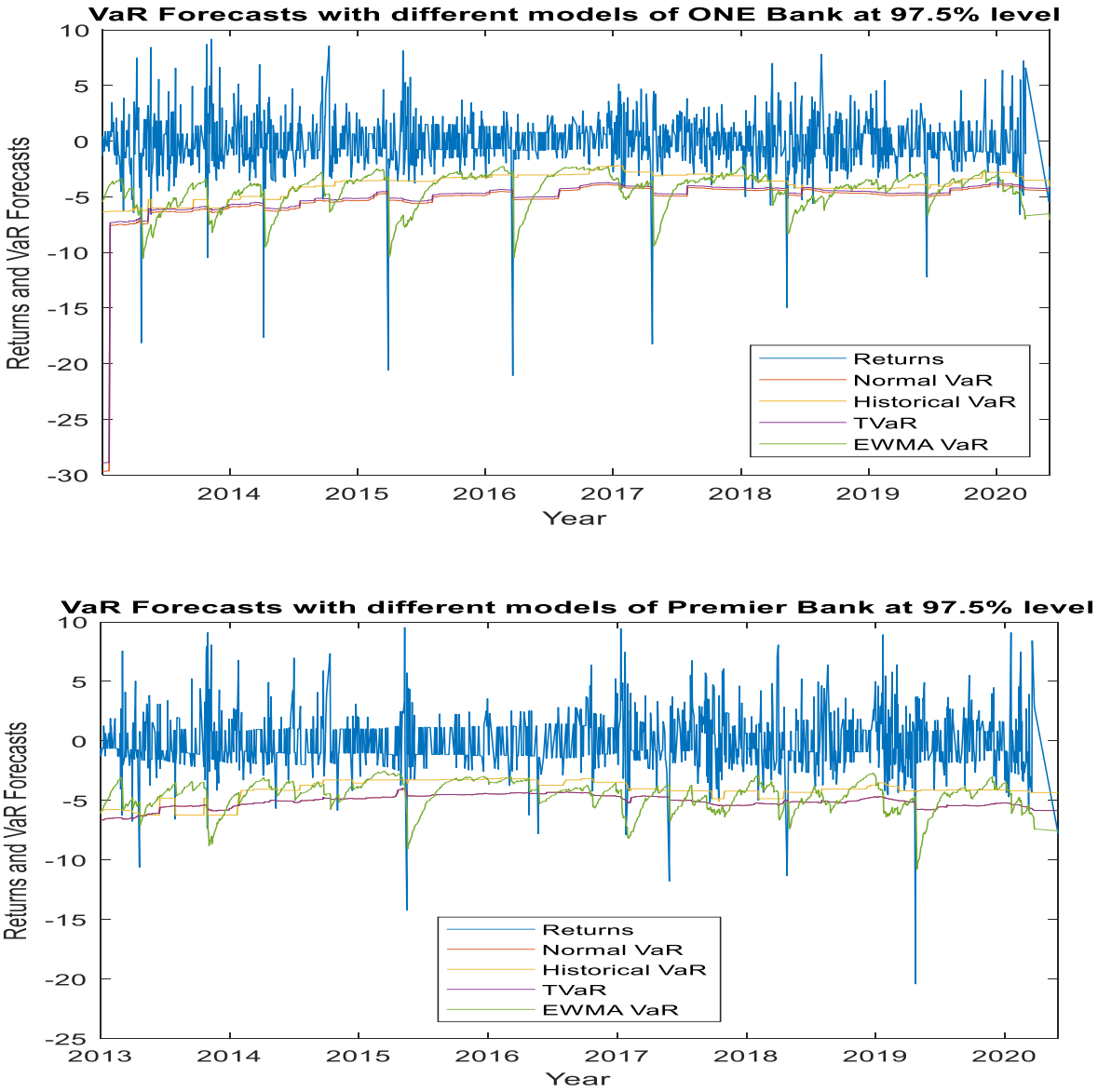


Figure-4.8: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **One bank (top) and Premier Bank (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

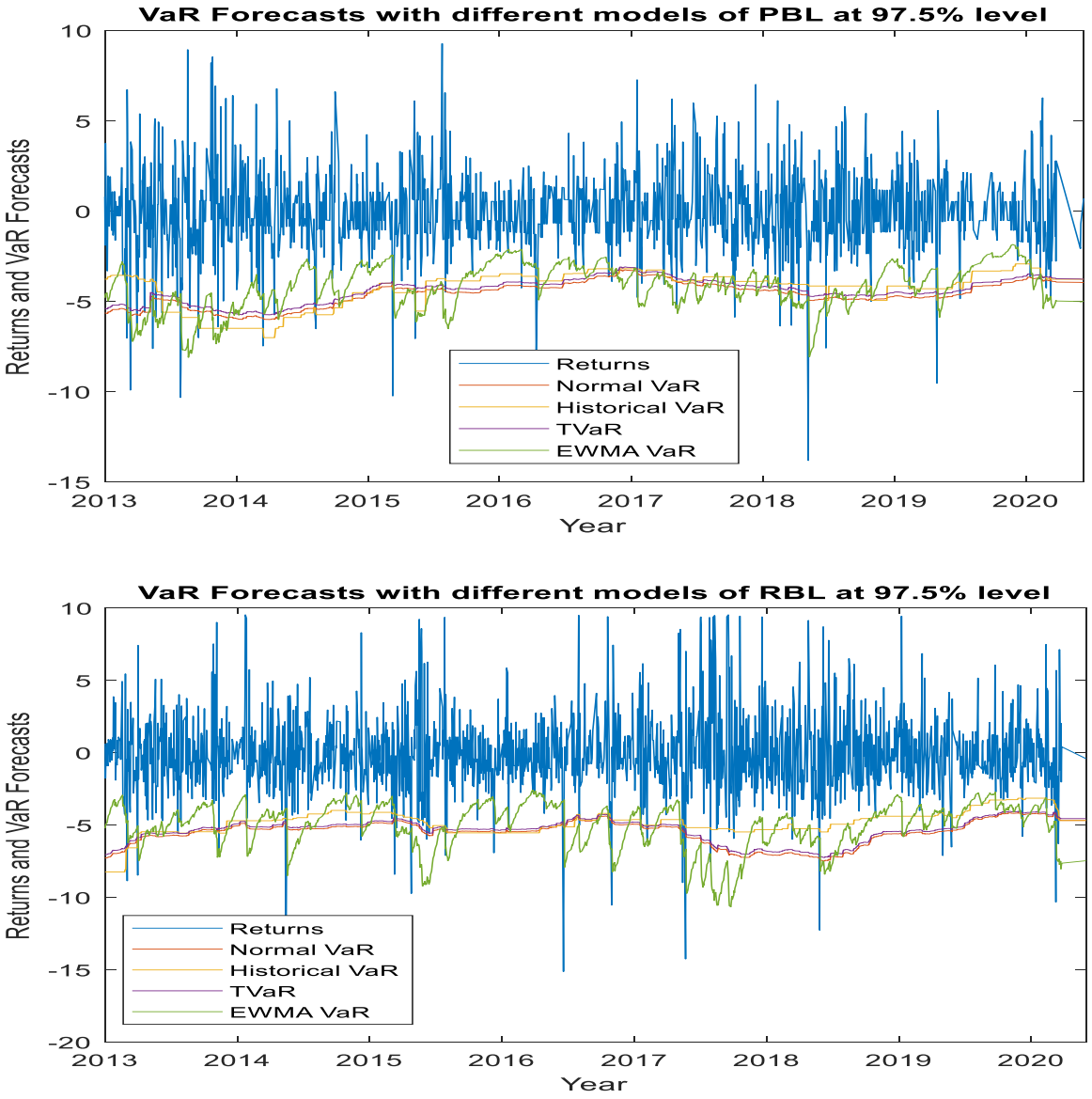


Figure-4.9: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **PBL (top) and RBL (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

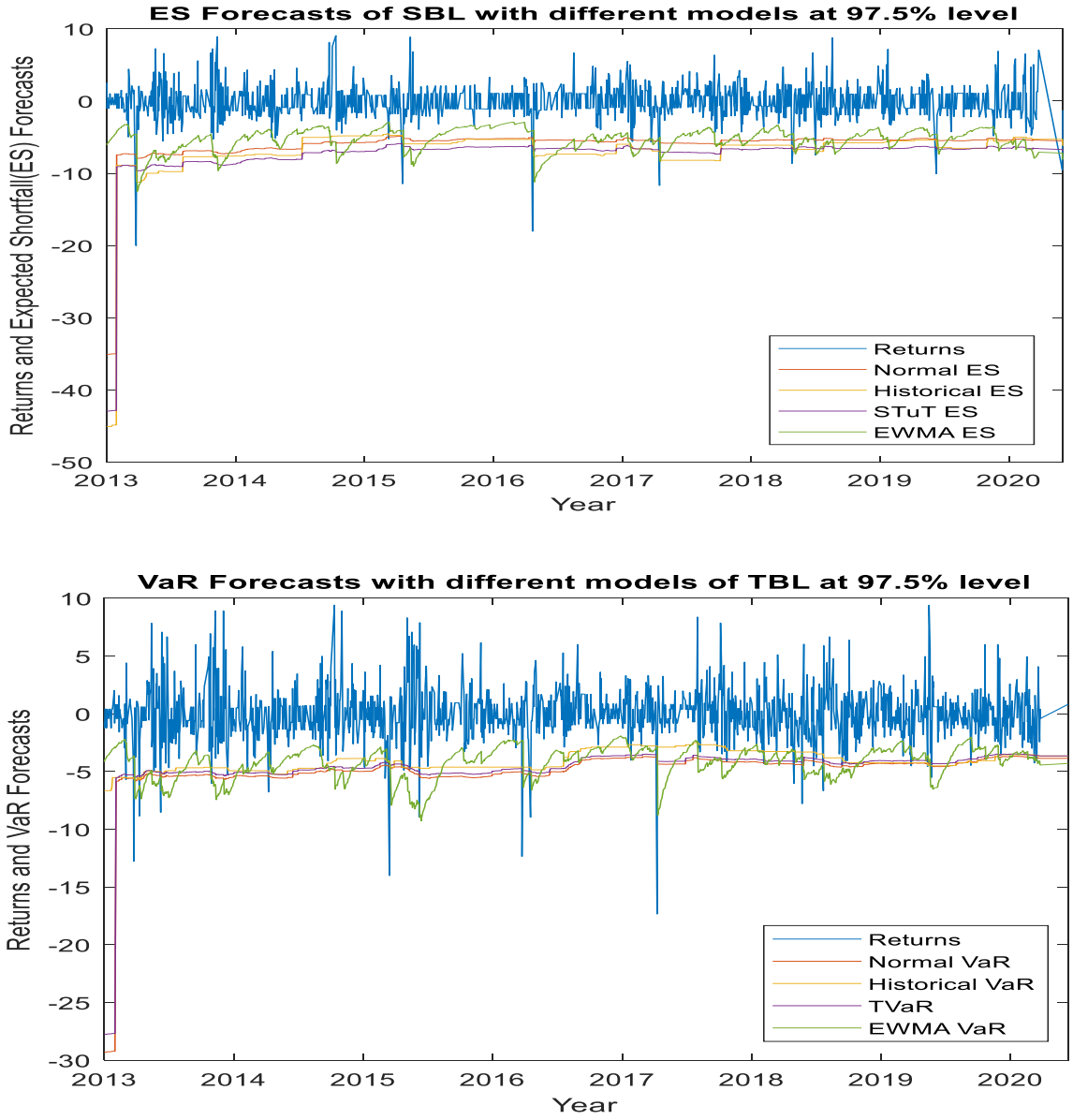


Figure-4.10: The daily Value at Risk (VaR) estimations at the 97.5% confidence level for **SBL (top) and TBL (bottom)** are available considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of projections differs between banks due to data cleansing, especially during the global epidemic period. Implemented inside a specific user-friendly design are risk models from parametric families, including 0, 1, 2, and 3, parameters, all of which are regarded to be user-friendly models.

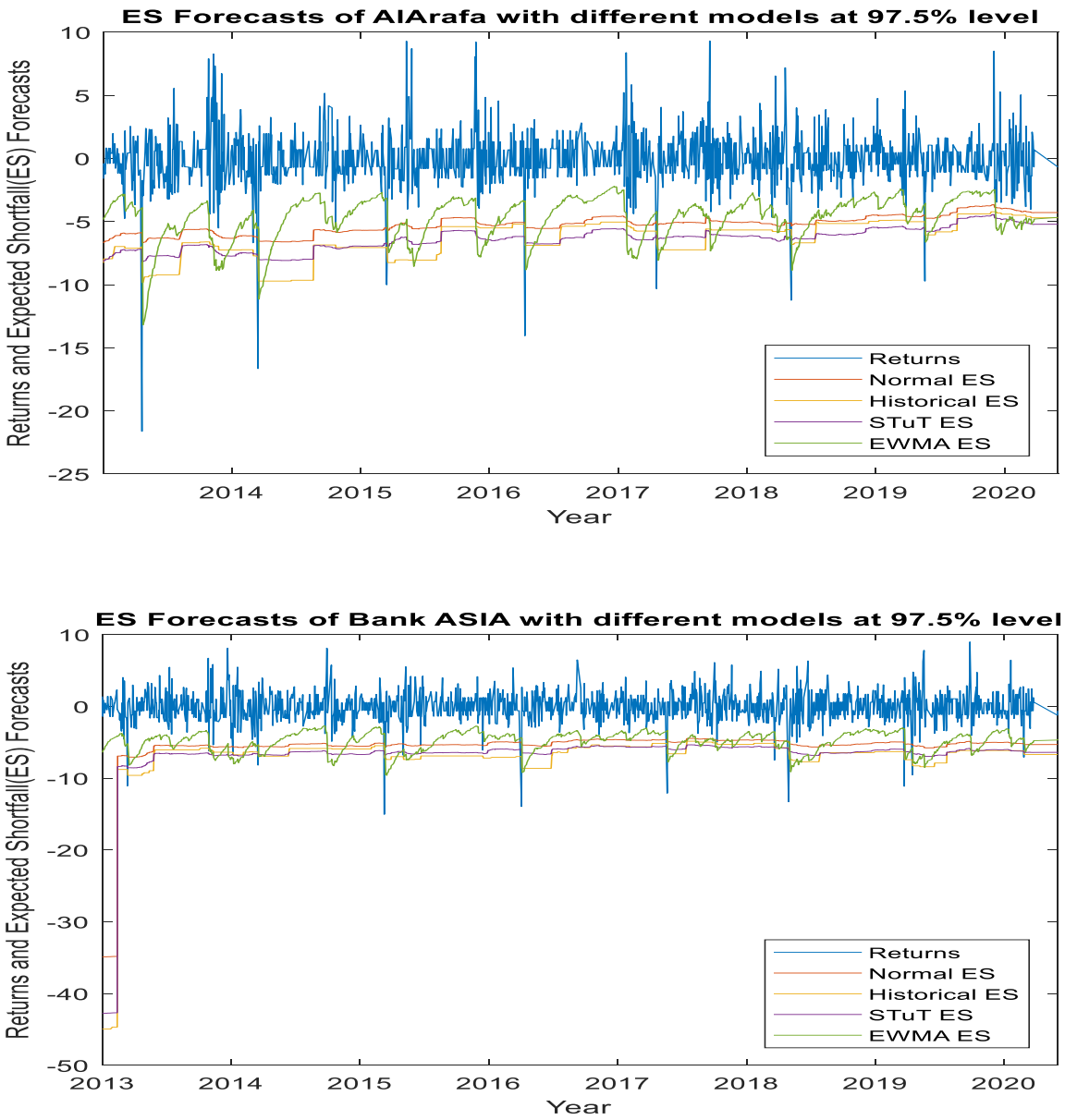


Figure-4.11: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided for **Al Arafa bank (top) and Bank Asia (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

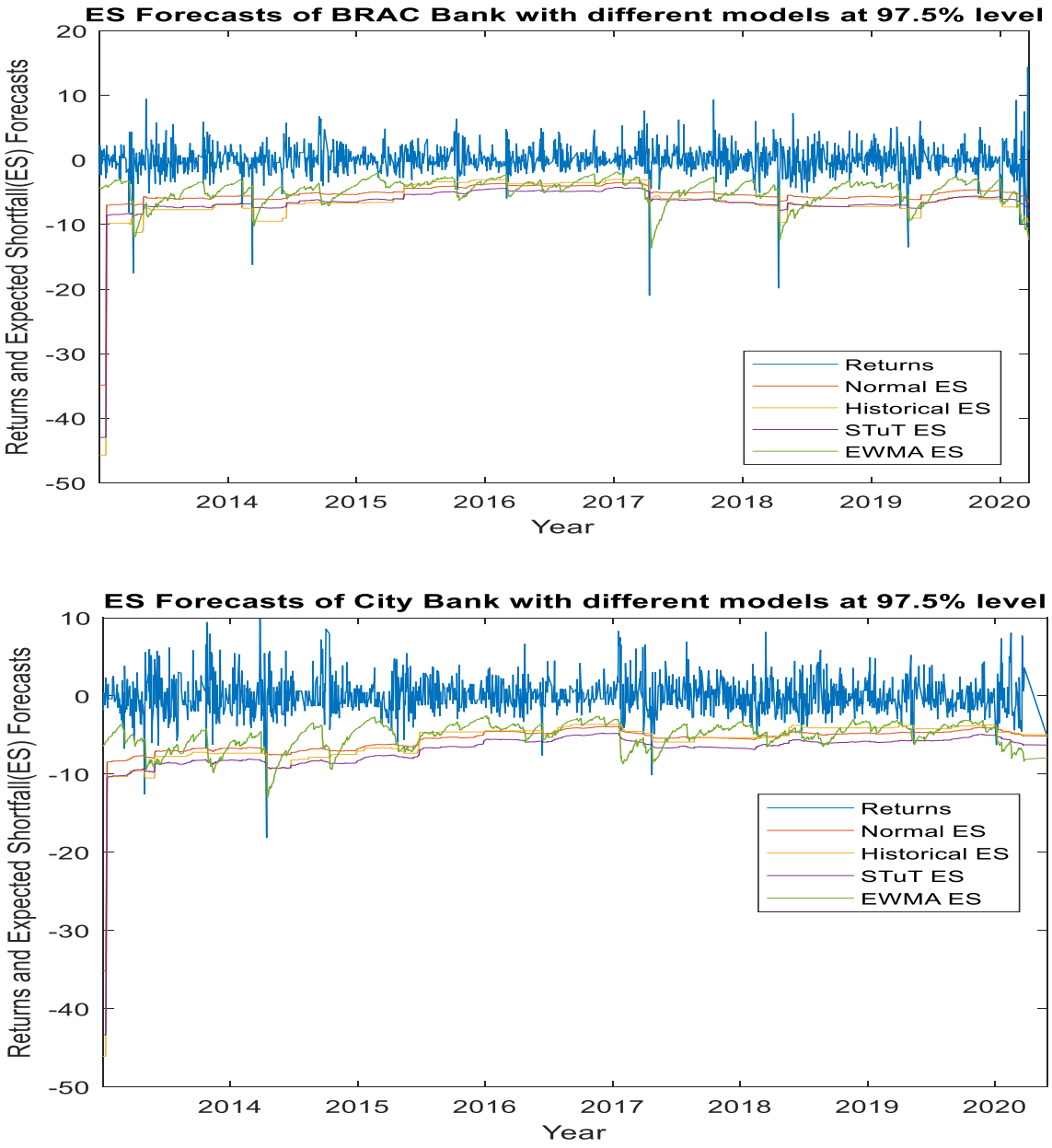


Figure-4.12: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided **BRAC bank (top) and City Bank (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

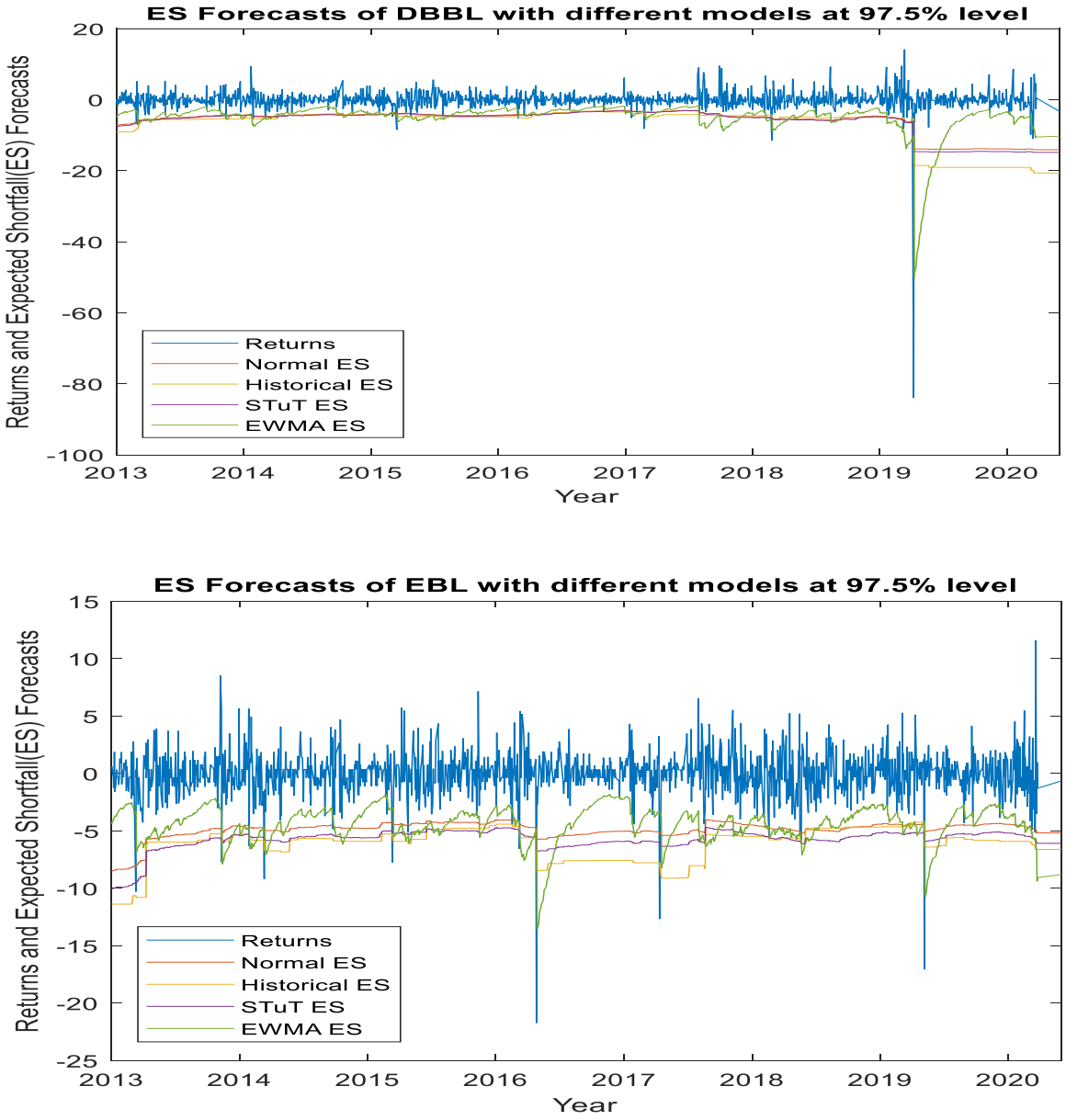


Figure-4.13: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided **DBBL (top) and EBL (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

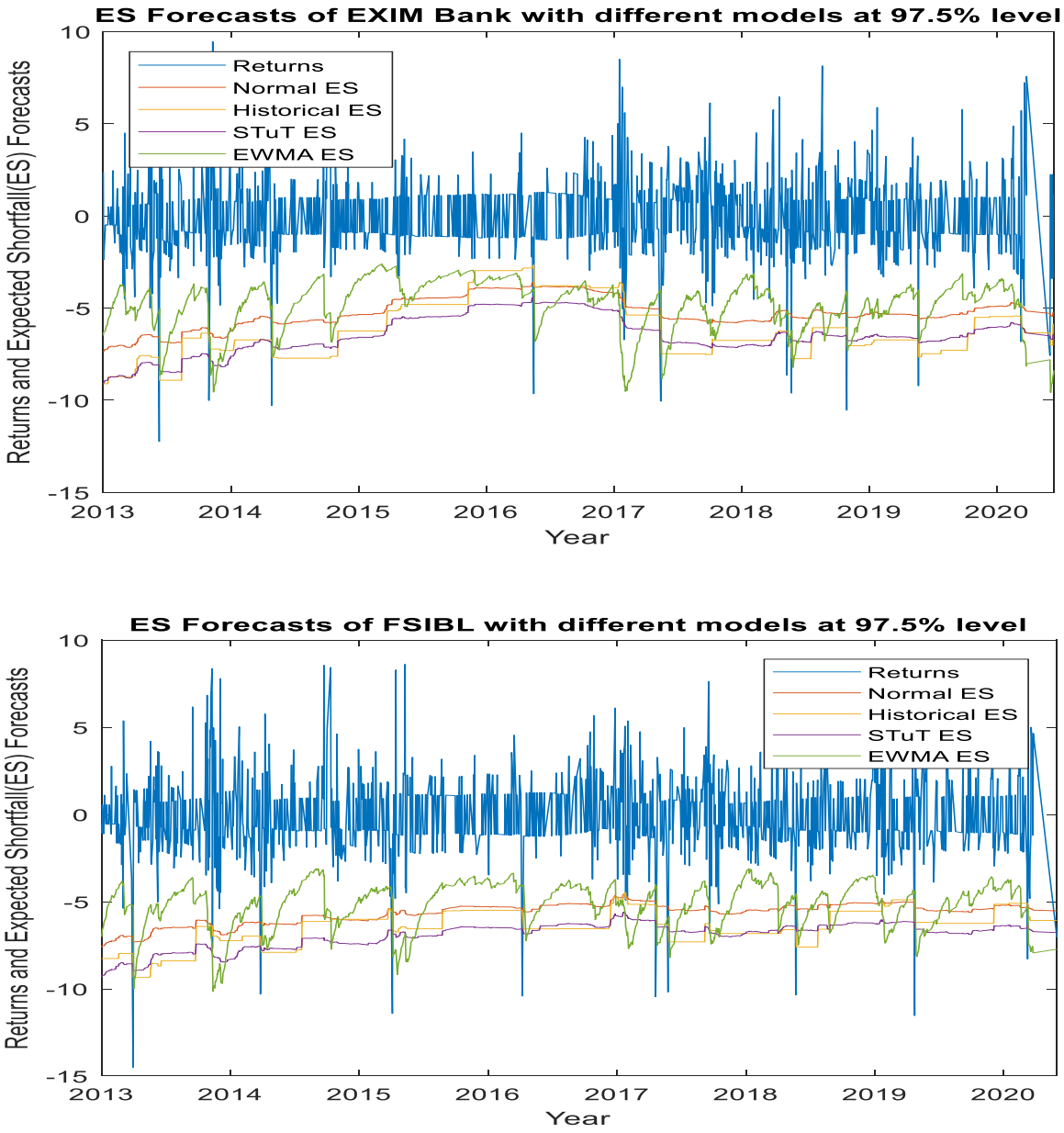


Figure-4.14: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided **Exim bank (top) and FSIBL (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

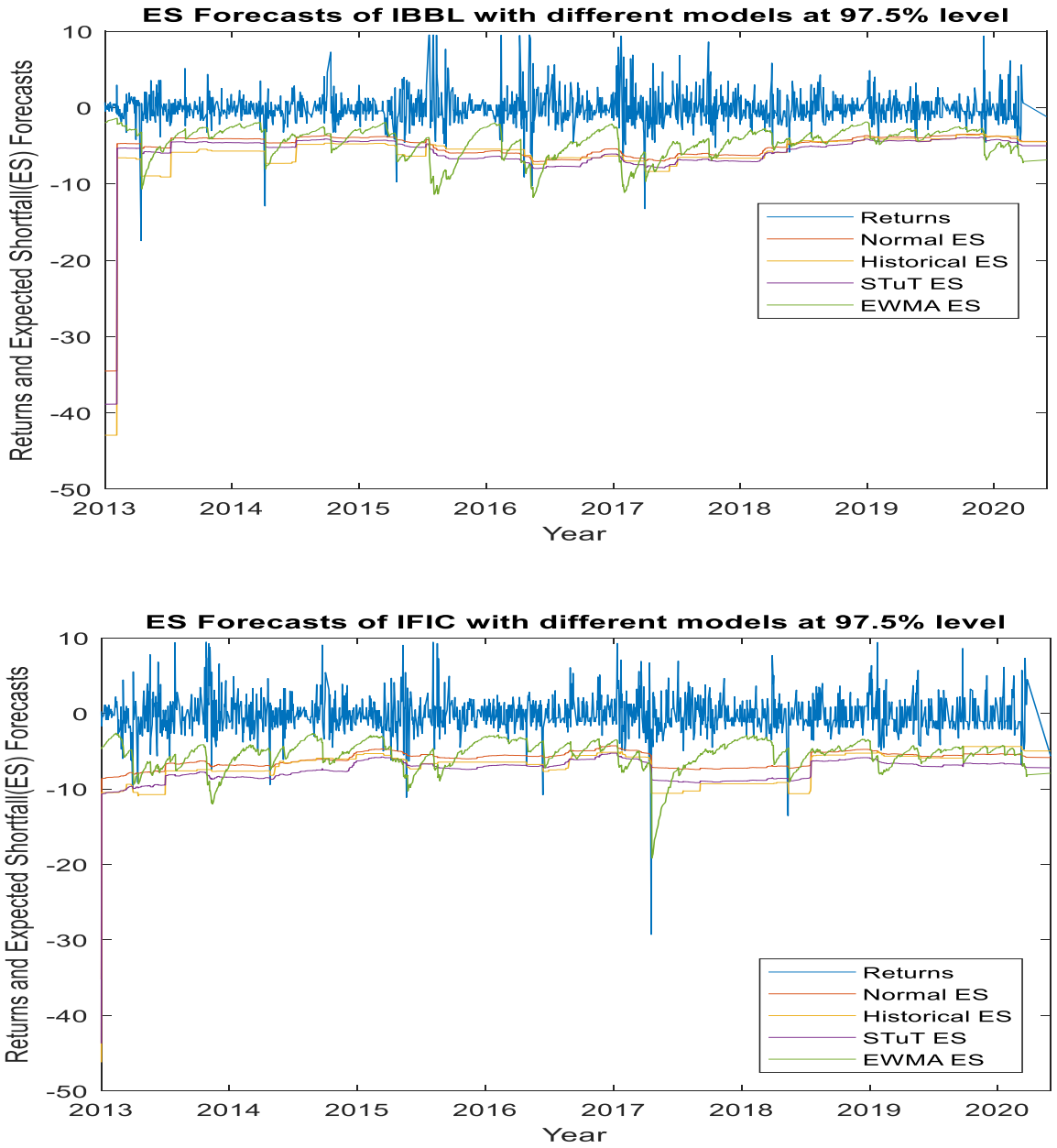


Figure-4.15: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided **IBBL (top) and IFIC (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

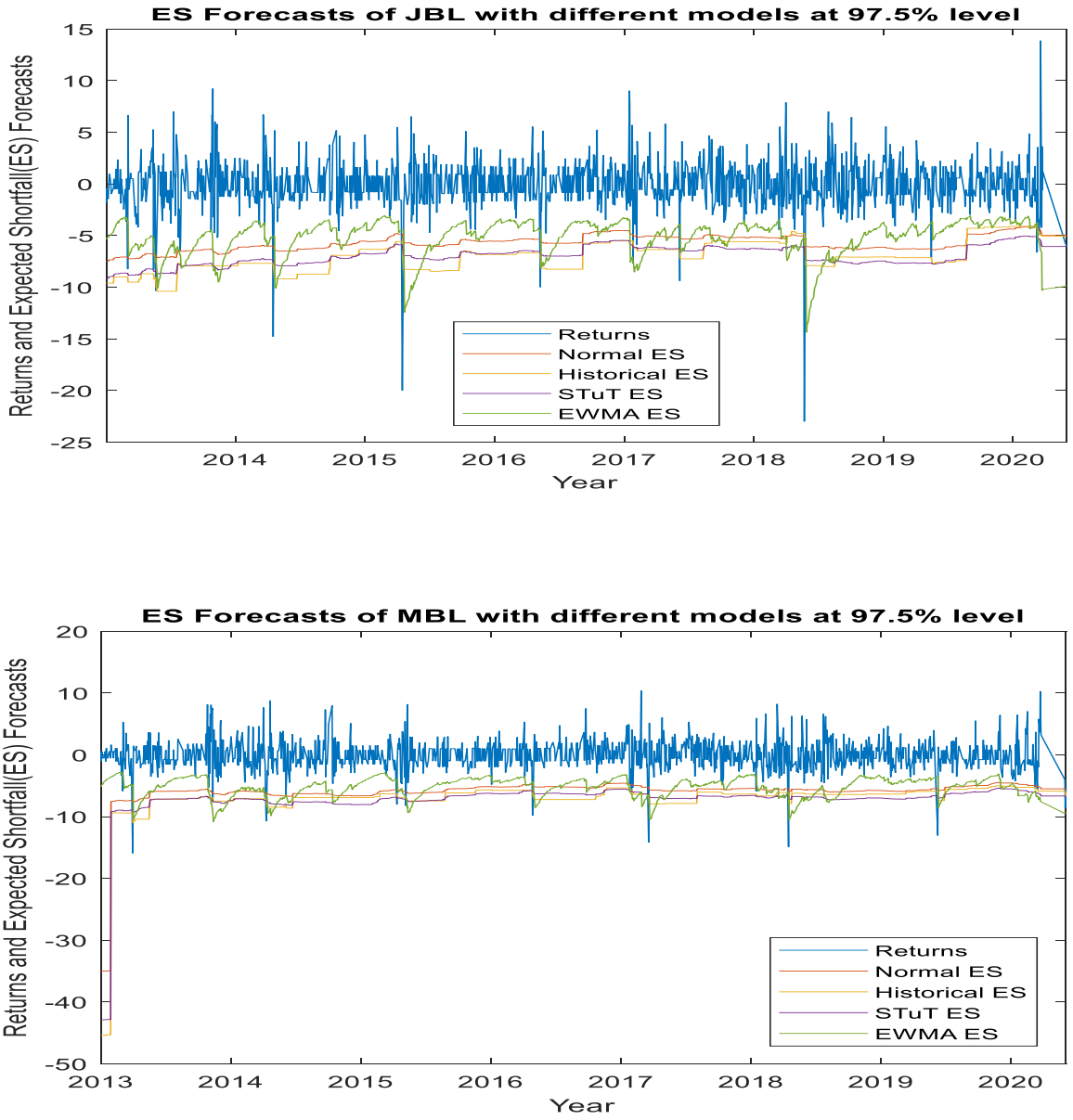


Figure-4.16: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided **JBL bank (top) and MBL (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

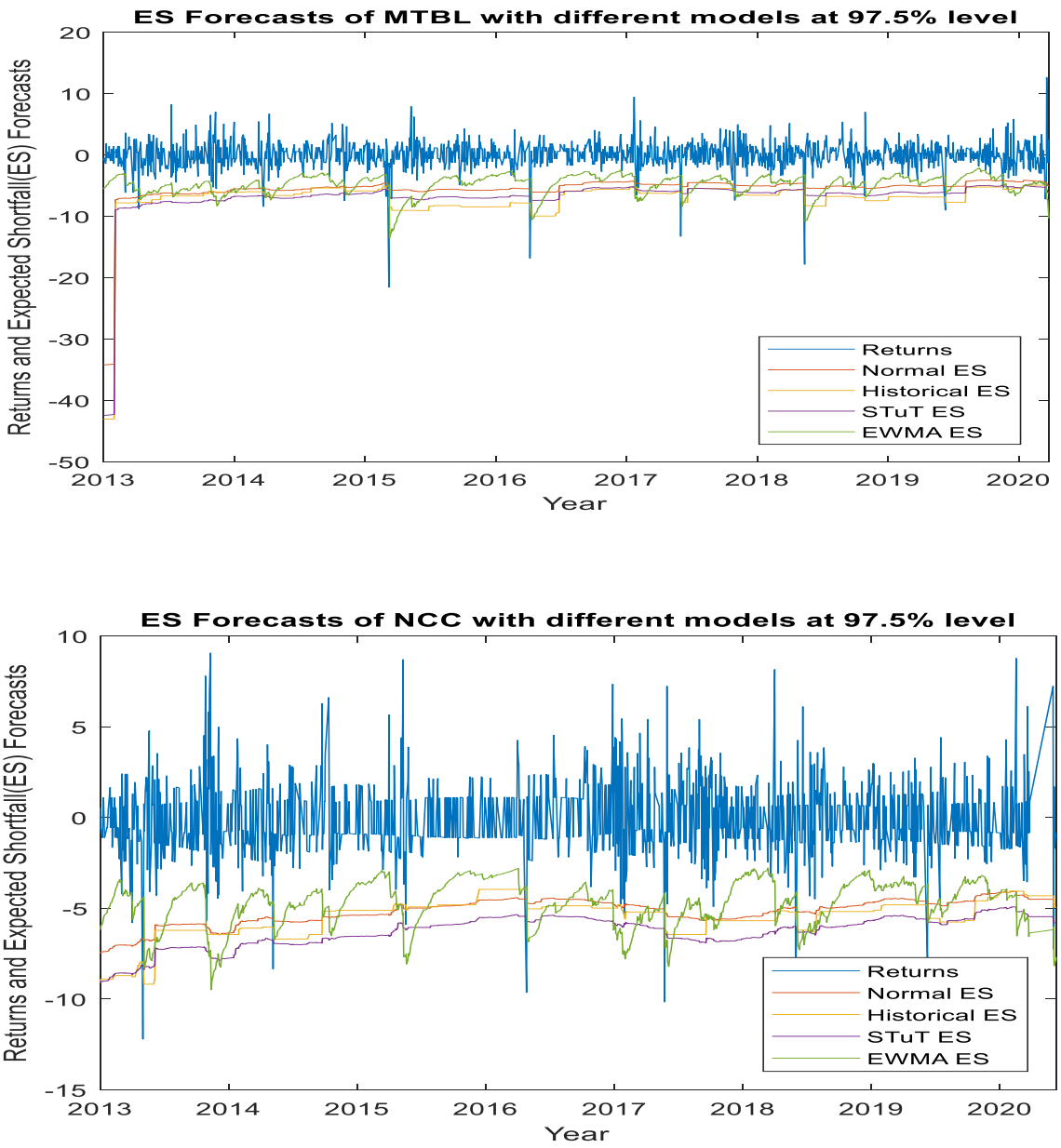


Figure-4.17: The daily **expected shortfall (ES**) estimations at the 97.5% confidence level are provided **MTBL (top) and NCC (bottom)** , considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

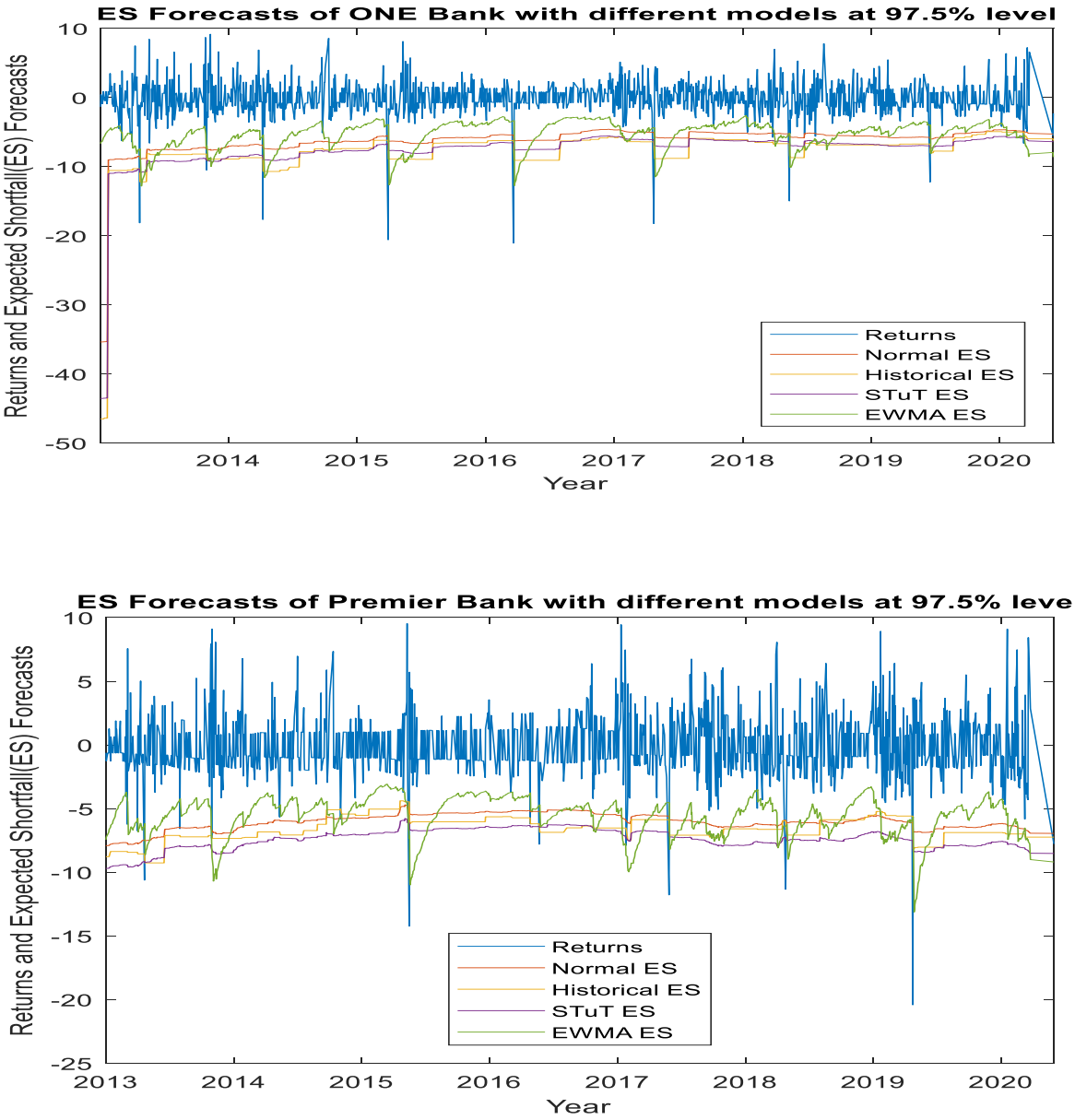


Figure-4.18: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided for **One bank (top) and Premier Bank (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0, 1, 2, and 3, parametric families of all risk models are examined.

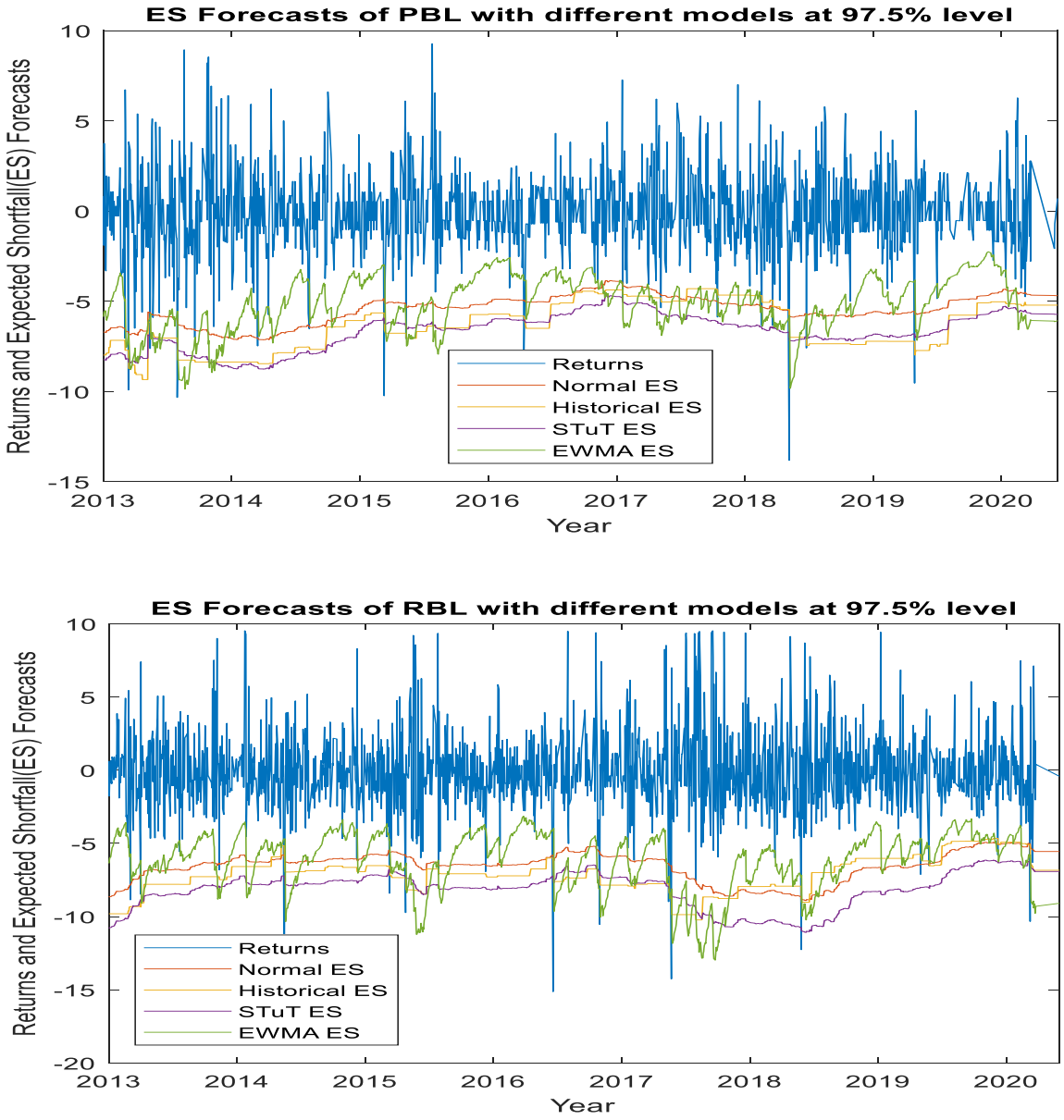


Figure-4.19: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided for **PBL (top) bank and RBL (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0. 1, 2, and 3, parametric families of all risk models are examined.

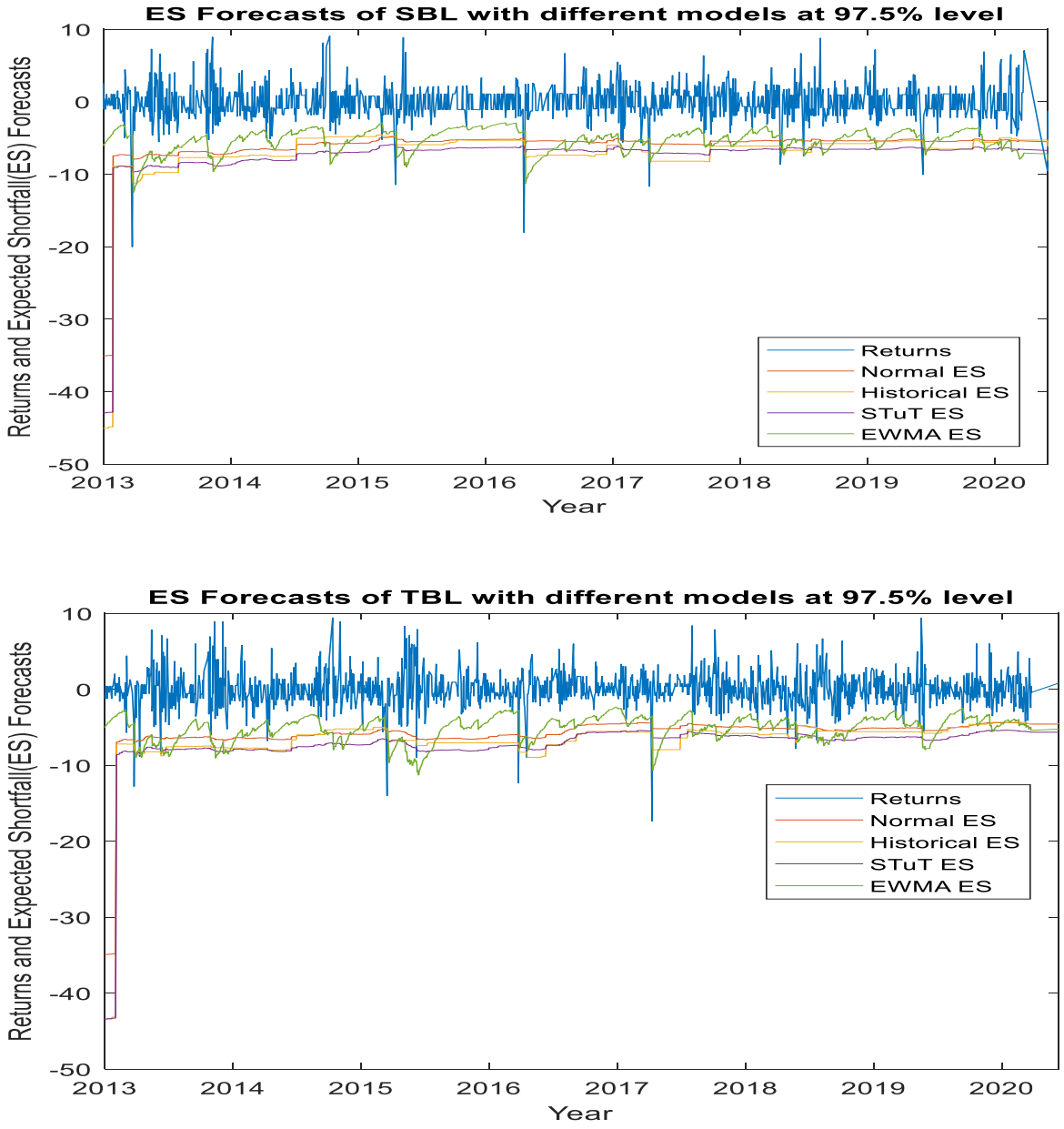


Fig-4.20: The daily **expected shortfall (ES)** estimations at the 97.5% confidence level are provided for **SBL (top) and TBL (bottom)**, considering four user-friendly models for each bank from January 1, 2013, to August 31, 2020. The total number of forecasts differs among banks due to data cleansing, especially during the pandemic period. Implemented in a particular straightforward design, models from the 0. 1, 2, and 3, parametric families of all risk models are examined.

	Estimation Window 250			
Banks	Models			
	Normal	Historical	Student_t	EWMA
AIBL	1609	1599	***1604***	1639
BASIA	1748	1726	***1743***	1767
BRAC	1917	1881	***1910***	1918
CBL	1794	1779	***1795***	1777
DBBL	2013	1982	2015	***2003***
EBL	***1781***	1773	1782	1818
EXIM	1536	1525	***1530***	1525
FSIBL	1510	1502	***1508***	1509
IBBL	1738	1715	***1735***	1780
IFIC	1772	1760	***1770***	1776
JBL	1693	1680	***1689***	1703
MBL	1698	1679	1694	***1687***
MTB	1789	1763	***1780***	1833
NCC	1487	1475	1485	***1484***
ONE	1728	1702	***1723***	1763
PRM	1539	1524	***1538***	1545
PBL	***1745***	1742	1761	1767
RBL	1978	1962	***1974***	1982
SBL	1533	1512	***1530***	1542
Trust	1774	1754	***1771***	1822

Table – 4.22: The values of joint loss function for Value at Risk (VaR) and Expected Shortfall (ES), using Equation (4.23), are calculated across the markets with $\alpha = 0.025$. Here the estimation window is 250.

In table 4.22 and 4.23, the yellow box represents the preferred model for individual models, taking into account each estimation window. The italics box denotes the second-best model. The tables 4.22 and 4.23 displays the loss function values, denoted as S, calculated using Equation (4.25). These values are used to evaluate the exactness of each model's value-at-risk and expected shortfall forecasts. The evaluation is conducted for two different rolling windows: 250 days and 500 days. Table 4.22 represents the values of loss function conducting the 250 trading days as estimation window and table 4.23 represent the same value for estimation window 500 trading days. During the predicting period from 2013 to 2020, the historical simulation model coupled with the EWMA or Risk Metrics model is determined to be the most effective or preferred model for estimating VaR and ES measures.

	Estimation Window 500			
Banks	Models			
	Normal	Historical	Student_t	EWMA
AIBL	1620	1609	***1612***	1639
BASIA	1839	1735	1830	***1767***
BRAC	1998	1906	1976	***1918***
CBL	1888	***1789***	1886	1776
DBBL	2101	2000	2066	***2003***
EBL	1796	1783	***1789***	1818
EXIM	1548	***1540***	1542	1524
FSIBL	1521	1506	1519	***1509***
IBBL	1826	1728	1795	***1780***
IFIC	1864	1764	1855	***1776***
JBL	1696	1681	1691	***1703***
MBL	1775	1682	1770	***1687***
MTB	1879	1775	1861	***1833***
NCC	1509	***1487***	1507	1484
ONE	1801	1712	1792	***1763***
PRM	1544	1532	***1543***	1545
PBL	1765	1746	***1762***	1767
RBL	2068	1971	2060	***1982***
SBL	1618	1521	1613	***1542***
Trust	1859	1765	1848	***1822***

Table-4.23: The values of joint loss function for Value at Risk (VaR) and Expected Shortfall (ES), adopting Equation (4.25), are calculated across the markets with $\alpha = 0.025$. Here the estimation window is 500.

	Estimation Window 750			
Banks	Models			
	Normal	Historical	Student_t	EWMA
AIBL	1391	1379	***1382***	1389
BASIA	1593	1503	1584	***1536***
BRAC	1748	***1662***	1729	1650
CBL	1613	***1532***	1612	1510
DBBL	1831	***1746***	1801	1744
EBL	1554	1540	***1546***	1571
EXIM	1314	***1305***	1309	1287
FSIBL	1307	1278	1305	***1280***
IBBL	1584	1492	1549	***1513***
IFIC	1588	***1503***	1580	1502
JBL	1470	1454	1466	***1460***
MBL	1537	***1447***	1532	1441
MTB	1622	1545	1608	***1591***
NCC	1304	***1269***	1302	1255
ONE	1548	1467	1541	***1505***
PRM	1323	1313	1322	***1314***
PBL	1497	1475	1493	***1484***
RBL	1766	1694	1759	***1710***
SBL	1388	1292	1384	***1296***
Trust	1603	1521	1594	***1560***

Table-4.24: Robustness Check- Calculating the joint loss function values for VaR and ES using Equation (4.25) for different markets, with a significance level of $\alpha = 0.025$. Here the estimation window is 750.

The yellow box represents the model that is most preferred, while the italics box represents the second-best model, when considering each estimation window for individual models. The table above displays values of the loss function obtained by applying the same equation

to evaluate the accuracy of value-at-risk and expected shortfall estimates for each model. The estimation window used was seven hundred fifty days, and the forecasting period spanned from 2014 to 2020 for all 20 commercial banks. We have adjusted the forecasting period by one year because the initial forecasts of VaR and ES necessitate 750 observations. However, using an earlier test window is not feasible due to the need to clean up the entire data set for individual bank, which involves dealing with recurrent closing share prices for some successive days, as previously described. Additionally, modifying the forecasting period allows for a thorough examination of the accuracy of each model's VaR and ES predictions for all 20 banks. During this era of forecasting, both the exponentially weighted moving average and historical simulation models remain the preferred models, as determined by the robustness check.

4.9 Back-Testing VaR and ES forecasts

The out-sample value-at-risk and expected shortfall predictions, along with their respective returns, serve as the basis for conducting and evaluating value-at-risk and expected shortfall backtests. Additionally, they are used to calculate the values of joint scoring function for all models and every bank, considering the methodology proposed by (Tobias Fissler, 2016). The loss score value approach is a simple and effective strategy for identifying the best acting models from a given list in an accessible manner. Table 4.22 to 4.24 contains data regarding the top-performing and the second-highest performing models for enlisted commercial banks observed in this work.

The higher panel is constructed using value-at-risk and expected shortfall predictions, as shown in Figure 4.1 to 4.10 and Figure 4.11 to 4.20 respectively. These predictions are based on our user-friendly design, which utilizes a rolling window-based estimation method over a one-year (250-day) period. However, in the bottom panel in table 4.22, we do the same test as in the higher panel, but this time we use two-year rolling window estimations for the whole backtesting period. The EWMA loss scores stay unchanged in

both the one-year and two-year rolling window situations. This is because JPMorgan's default estimate of λ limits the model to using only the most recent 100 observations for risk forecasts. In table 4.24, we do a robustness analysis by adjusting the initial point of the backtesting window, resulting in a change in the EWMA loss score values. Additionally, we utilize a period of three years (750-days) rolling window-based estimation.

Among all the banks in our survey, historical simulation consistently ranks as the top or second-best performer in terms of rolling calibration and dynamic risk projections, regardless of the window size we evaluate. Furthermore, in the estimation window size 250-days, 500-days and 750-days trial, the percentage of historical simulation being ranked as the best is much greater than its ranking as the second best (0.95 compared to 0.05 for 250-days; 0.85 compared to 0.15 for 500-days; and 0.65 compared to 0.35 for 750-days). The RiskMetricsTM model has odds of 0.10 vs. 0.15 for a duration of 250 days, 0.15 vs. 0.60 for 500 days, and 0.15 vs. 0.45 for 750 days. The reasonable characteristic of historical simulation and RiskMetricsTM models is being examined among the four accessible models under consideration, using our accessible proposal. Although the student-t model has not performed the best in our three different estimation window experiments, its probabilities of being the second-best model (65%) are meaningfully higher than the chances of both the historical simulation (5%) and RiskMetricsTM (15%) models corresponding to our user-friendly approach to executing models with only a 250-day rolling window estimation.

Table 4.25 to 4.28 displays the outcomes of the 10 tests we have taken into account for VaR and ES. All tests, except for the TL test, display results as either a pass ('P') or fail ('F'), along with the associated p-values in parentheses, for all twenty banks. The TL test categorizes each bank's performance zone as Green (G), Yellow (Y), or Red (R) instead of using 'P' or 'F'. This categorization is based on Eq. (2.18), which allocates one of the three zones to each bank depending on the degree of violation represented by the fraction α. Clearly, the value of α is highest (0.999) for the assignment in the red zone and lowest

(0.995) for the assignment in the green zone. There should be no ambiguity regarding the p-values of the TL hypothesis test, which are presented in table 4.25 to 4.28 with 97.5% confidence interval, which stay consistent for all banks. Thus, table 4.25 to 4.28 only includes the TL zone for each bank and does not provide the zone fraction α. Generally, larger p-values correspond to lower α levels.

Table 4.22 to 4.24 display the precision and sufficiency of VaR and ES predictions, as shown in figure 4.1 to 4.10 and figure 4.11 to 4.20, respectively, for each of the twenty banks. The loss scores provided in Table-4.22 to 4.24 incorporate this information. To assess the bank's investor-friendliness in relation to their market risk exposure, it is essential to take into account the information shown in both table 4.22 to 4.24 and table 4.25 to 4.28.

A total of twenty banks actively engages in equity share trading at the DSE, effectively representing the banking sector of Bangladesh. This sector is widely recognized as a growing economy on the global stage, particularly in the past decade. Tests conducted using value-at-risk and expected shortfall backtests, which include both standard and featured methods, indicate that only a small number of commercial banks are truly investor-favorable. Out of the 10 tests analyzed, which included two for expected shortfall and eight for value-at-risk, only TBL, AIBL, JBL, and PRBL banks were able to pass all ten tests using one or two of our commonly-used models. Upon conducting a more thorough examination of these four banks for the top or second-best model out of the four, we find that only TBL and AIBL banks successfully pass complete ten backtests conducted using both value-at-risk and expected shortfall. The selection of the top and second-greatest models is determined by the scoring values acquired from the mutual performance of both value-at-risk and expected shortfall forecasts, which are dynamically generated over an extended period of time. This highlights a discrepancy with the status of the most commonly used model discussed in the literature, specifically the RiskMetricsTM model. The RiskMetricsTM model ranks second least good in terms of performance, surpassing only Gaussian risk projections, when considering the combined performance of both value-

at-risk and expected shortfall with the inclusion of our commonly used features. TBL is the only bank that successfully passes all backtests for value-at-risk and expected shortfall for the best model, as determined by the joint score values. Interestingly, TBL also passes all ten value-at-risk and expected shortfall backtests for the second-best model, which is the student-t based on the joint score values. In addition, this is the sole bank among the twenty banks examined that successfully passes all ten value-at-risk and expected shortfall backtests for both the best and second-top models. Conversely, the RiskMetricsTM model fails half of the value-at-risk backtests and all of the expected shortfall backtests with TBL.

Consequently, with a 15.6 percent share of the total market capitalization in the stock market of Bangladesh, it is essential that the twenty commercial banks in our study receive attention from surveillance groups, such as the Bangladesh Bank and the Bangladesh Securities and Exchange Commission (BSEC), to guarantee the effective implementation of regulatory instructions with regard to capital adequacy based on risk-weighted assets and the preservation of high-quality assets to reduce non-performing loans (NPL).

Moreover, it is crucial to guarantee the precise execution of policy recommendations to effectively address non-performing loans with deferral conditions. This is necessary to stabilize the capital adequacy ratio and meet the requirements for declaring dividends to shareholders. Furthermore, these regulations will also guarantee effective administration with strong corporate governance principles to provide a sustainable ability to generate profits for banks, thereby preserving the value of shareholders' investments and maximizing their wealth. In addition to establishing an effective liquidity management system to prevent unforeseen situations, regulators should also implement strategies to create a dedicated risk management division. This division would specifically address market forces that impact the share price of banks, with the aim of mitigating any periods of instability in capital markets and safeguarding the interests of small-scale investors.

		VaR Back-testing								ES Back-testing	
Banks	Models	TL	BIN	POF	TUFF	CC	CCI	TBF	TBFI	UCN	UCT
AIBL (1362)	Normal	G (0.99)	F (0.01)	F (0.01)	P (0.71)	F (0.00)	F (0.04)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.32)	P (0.30)	P (0.61)	P (0.80)	P (0.50)	P (0.36)	F (0.00)	F (0.00)	P (0.27)	P (0.29)
	Student-t	G (0.83)	P (0.19)	P (0.36)	P (0.80)	P (0.23)	P (0.15)	P (0.07)	P (0.06)	P (0.50)	P (0.50)
	EWMA	G (0.90)	P (0.11)	P (0.20)	P (0.78)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.03)	F (0.06)
BASIA (1464)	Normal	G (0.99)	F (0.01)	F (0.01)	P (0.94)	F (0.03)	P (0.39)	P (0.06)	P (0.15)	P (0.50)	F (0.50)
	Historical	G (0.42)	P (0.40)	P (0.82)	P (0.94)	P (0.23)	P (0.09)	F (0.03)	F (0.02)	P (0.41)	P (0.41)
	Student-t	G (0.98)	P (0.03)	F (0.04)	P (0.94)	P (0.07)	P (0.35)	P (0.11)	P (0.19)	P (0.50)	P (0.50)
	EWMA	G (0.75)	P (0.27)	P (0.54)	P (0.68)	P (0.38)	P (0.21)	P (0.23)	P (0.21)	F (0.02)	F (0.04)
BRAC (1600)	Normal	G (0.94)	P (0.07)	P (0.13)	P (0.83)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.16)	P (0.19)
	Historical	G (0.23)	P (0.21)	P (0.43)	P (0.83)	P (0.09)	F (0.04)	F (0.00)	F (0.00)	P (0.17)	P (0.19)
	Student-t	G (0.80)	P (0.21)	P (0.41)	P (0.83)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.44)	P (0.44)
	EWMA	G (0.95)	P (0.06)	P (0.09)	P (0.87)	F (0.01)	F (0.01)	F (0.00)	F (0.00)	P (0.06)	P (0.09)
CBL (1540)	Normal	G (0.98)	F (0.02)	F (0.03)	P (0.54)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.20)	P (0.18)	P (0.38)	P (0.89)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.19)	P (0.21)
	Student-t	G (0.97)	F (0.03)	F (0.04)	P (0.54)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.91)	P (0.11)	P (0.21)	P (0.85)	P (0.16)	P (0.15)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
DBBL (1670)	Normal	G (0.99)	F (0.01)	F (0.01)	P (0.97)	F (0.03)	P (0.45)	F (0.00)	F (0.00)	P (0.23)	P (0.25)
	Historical	G (0.32)	P (0.31)	P (0.61)	P (0.97)	P (0.32)	P (0.15)	F (0.00)	F (0.00)	F (0.01)	F (0.03)
	Student-t	R (0.00)	F (0.00)	F (0.00)	P (0.97)	F (0.00)	P (0.41)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	EWMA	G (0.79)	P (0.23)	P (0.44)	P (0.94)	P (0.73)	P (0.84)	P (0.13)	P (0.12)	F (0.03)	P (0.06)

Table-4.25 shows that the outcomes of value-at-risk and expected shortfall backtests at 97.5% during the forecasting period of January 1, 2013 to August 31, 2020. In the brackets indicate p-values. Green, yellow and red are denoted by G, Y and R respectively. *P* used for pass and *F* used for fail. Source: authors' contribution based on Matlab (2020a) Output.

Banks	Models	VaR Back-testing								ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	TBF	TBFI	UCN	UCT
EBL (1519)	Normal	G (0.92)	P (0.09)	P (0.17)	P (0.92)	F (0.03)	F (0.02)	F (0.00)	F (0.00)	P (0.29)	P (0.30)
	Historical	G (0.22)	P (0.20)	P (0.41)	P (0.92)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.23)	P (0.25)
	Studen-t	Y (0.00)	F (0.00)	F (0.00)	P (0.92)	F (0.00)	P (0.06)	F (0.00)	F (0.00)	F (0.01)	F (0.03)
	EWMA	Y (0.05)	P (0.05)	P (0.08)	P (0.96)	F (0.00)	F (0.00)	P (0.13)	P (0.12)	F (0.03)	F (0.00)
EXIM (1294)	Normal	G (0.99)	F (0.00)	F (0.00)	P (0.35)	F (0.00)	P (0.21)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.18)	P (0.15)	P (0.32)	P (0.48)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.15)	P (0.18)
	Student-t	G (0.99)	F (0.00)	F (0.01)	P (0.35)	F (0.02)	P (0.28)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.96)	P (0.05)	P (0.07)	P (0.80)	F (0.03)	P (0.06)	F (0.01)	F (0.02)	P (0.34)	P (0.35)
FSIBL (1267)	Normal	G (0.99)	F (0.01)	F (0.01)	P (0.99)	F (0.03)	P (0.45)	F (0.00)	F (0.01)	P (0.50)	P (0.50)
	Historical	G (0.14)	P (0.13)	P (0.26)	P (0.99)	P (0.16)	P (0.12)	F (0.00)	F (0.00)	P (0.11)	P (0.14)
	Student-t	G (0.99)	F (0.01)	F (0.01)	P (0.99)	F (0.03)	P (0.45)	F (0.00)	F (0.01)	P (0.50)	P (0.50)
	EWMA	G (0.98)	F (0.02)	F (0.02)	P (0.80)	F (0.04)	P (0.30)	F (0.00)	F (0.01)	P (0.50)	P(0.50)
IBBL (1481)	Normal	G (0.96)	P (0.05)	P (0.07)	P (0.79)	P (0.05)	P (0.09)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	Y (0.04)	P (0.05)	P (0.08)	P (0.93)	P (0.05)	P (0.08)	F (0.00)	F (0.00)	P (0.08)	P (0.11)
	Student-t	Y (0.01)	F (0.00)	F (0.00)	P (0.93)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.02)	F (0.04)
	EWMA	G (0.39)	F (0.37)	P (0.74)	P (0.75)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.01)
IFIC (1475)	Normal	G (0.96)	P (0.05)	P (0.08)	P (0.78)	P (0.05)	P (0.08)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.17)	P (0.15)	P (0.31)	P (0.81)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.18)	P (0.21)
	Student-t	G (0.92)	P (0.09)	P (0.17)	P (0.78)	P (0.11)	P (0.12)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.58)	P (0.44)	P (0.88)	P (0.75)	P (0.56)	P (0.29)	F (0.00)	F (0.00)	P (0.05)	P (0.08)

Table-4.26 shows that the outcomes of value-at-risk and expected shortfall backtests at 97.5% during the forecasting period of January 1, 2013 to August 31, 2020. In the brackets indicate p-values. Green, yellow and red are denoted by G, Y and R respectively. P used for pass and F used for fail. Source: authors' contribution based on Matlab (2020a) Output.

Banks	Models	VaR Back-testing								ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	TBF	TBFI	UCN	UCT
JBL (1412)	Normal	G (0.99)	F (0.01)	F (0.01)	P (0.87)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.23)	P (0.21)	P (0.43)	P (0.87)	P (0.06)	F (0.02)	F (0.00)	F (0.00)	P (0.17)	P (0.20)
	Student-t	G (0.97)	P (0.05)	P (0.06)	P (0.87)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.87)	P (0.14)	P (0.26)	P (0.82)	P (0.18)	P (0.14)	P (0.05)	P (0.06)	P (0.08)	P (0.10)
MBL (1409)	Normal	G (0.97)	P (0.05)	P (0.07)	P (0.84)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.50)	P (0.48)
	Historical	G (0.18)	P (0.16)	P (0.33)	P (0.85)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.18)	P (0.20)
	Student-t	G (0.97)	P (0.05)	P (0.07)	P (0.84)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.53)	P (0.48)	P (0.97)	P (0.80)	P (0.17)	P (0.06)	F (0.01)	F (0.01)	F (0.03)	P (0.06)
MTB (1483)	Normal	G (0.94)	P (0.06)	P (0.11)	P (0.95)	F (0.01)	F (0.01)	F (0.00)	F (0.00)	P (0.17)	P (0.19)
	Historical	G (0.33)	P (0.31)	P (0.63)	P (0.67)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (0.27)	P (0.29)
	Student-t	G (0.86)	P (0.15)	P (0.29)	P (0.95)	P (0.05)	F (0.02)	F (0.00)	F (0.00)	P (0.47)	P (0.46)
	EWMA	G (0.65)	P (0.36)	P (0.73)	P (0.95)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F(0.00)
NCC (1280)	Normal	G (0.99)	F (0.00)	F (0.00)	P (0.66)	F (0.00)	P (0.52)	F (0.00)	F (0.02)	P (0.50)	P (0.50)
	Historical	G (0.52)	P (0.50)	P (0.99)	P (0.90)	P (0.50)	P (0.24)	F (0.00)	F (0.00)	P (0.45)	P (0.44)
	Student-t	G (0.99)	F (0.00)	F (0.01)	P (0.90)	F (0.03)	P (0.44)	F (0.01)	P (0.06)	P (0.50)	P (0.50)
	EWMA	G (0.97)	P (0.05)	P (0.06)	P (0.75)	P (0.11)	P (0.39)	F (0.04)	P (0.07)	P (0.50)	P (0.50)
ONE (1410)	Normal	G (0.99)	F (0.00)	F (0.00)	P (0.64)	F (0.01)	P (0.44)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.18)	P (0.16)	P (0.33)	P (0.84)	P (0.06)	F (0.03)	F (0.00)	F (0.00)	P (0.18)	P (0.21)
	Student-t	G (0.99)	F (0.00)	F (0.00)	P (0.64)	F (0.02)	P (0.42)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.91)	P (0.11)	P (0.19)	P (0.73)	P (0.13)	P (0.12)	F (0.00)	F (0.00)	F (0.00)	F (0.02)

Table-4.27 shows that the outcomes of value-at-risk and expected shortfall backtests at 97.5% during the forecasting period of January 1, 2013 to August 31, 2020. In the brackets indicate p-values. Green, yellow and red are denoted by G, Y and R respectively. P used for pass and F used for fail. Source: authors' contribution based on Matlab (2020a) Output.

Banks	Models	VaR Back-testing								ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	TBF	TBFI	UCN	UCT
Premier (1288)	Normal	G (0.99)	F (0.00)	F (0.00)	P (0.90)	F (0.00)	P (0.17)	F (0.00)	F (0.01)	P (0.50)	P (0.50)
	Historical	G (0.17)	P (0.15)	P (0.31)	P (0.80)	P (0.17)	P (0.11)	F (0.00)	F (0.00)	P (0.16)	P (0.19)
	Student-t	G (0.99)	F (0.00)	F (0.00)	P (0.90)	F (0.00)	P (0.17)	F (0.00)	F (0.01)	P (0.50)	P (0.50)
	EWMA	G (0.88)	P (0.13)	P (0.25)	P (0.75)	P (0.31)	P (0.31)	P (0.13)	P (0.14)	P (0.33)	P (0.34)
PBL (1476)	Normal	G (0.26)	P (0.24)	P (0.50)	P (0.95)	P (0.25)	P (0.13)	P (0.07)	P (0.06)	P (0.06)	P (0.08)
	Historical	Y (0.06)	P (0.05)	P (0.11)	P (0.95)	F (0.01)	F (0.02)	F (0.00)	F (0.00)	P (0.08)	P (0.11)
	Student-t	G (0.13)	P (0.12)	P (0.25)	P (0.95)	P (0.21)	P (0.19)	F (0.03)	F (0.03)	P (0.27)	P (0.29)
	EWMA	G (0.11)	P (0.09)	P (0.19)	P (0.82)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
RBL (1631)	Normal	G (0.96)	P (0.05)	P (0.07)	P (0.96)	P (0.06)	P (0.12)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.27)	P (0.25)	P (0.50)	P (0.96)	P (0.09)	F (0.04)	F (0.00)	F (0.00)	P (0.26)	P (0.28)
	Student-t	G (0.95)	P (0.06)	P (0.11)	P (0.96)	P (0.08)	P (0.13)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.57)	P (0.45)	P (0.90)	P (0.83)	P (0.06)	F (0.01)	F (0.00)	F (0.00)	F (0.06)	F (0.08)
SBL (1274)	Normal	G (0.99)	F (0.00)	F (0.00)	P (0.94)	F (0.00)	P (0.16)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	Historical	G (0.25)	P (0.22)	P (0.46)	P (0.94)	P (0.19)	P (0.09)	F (0.00)	F (0.00)	P (0.22)	P (0.24)
	Student-t	G (0.99)	F (0.00)	F (0.00)	P (0.94)	F (0.00)	P (0.19)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	EWMA	G (0.98)	P (0.05)	F (0.03)	P (0.85)	P (0.07)	P (0.35)	F (0.01)	F (0.03)	P (0.29)	P (0.30)
TBL (1493)	Normal	G (0.05)	P (0.05)	P (0.07)	P (0.87)	P (0.12)	P (0.31)	P (0.08)	P (0.12)	P (0.50)	P (0.50)
	Historical	G (0.70)	P (0.32)	P (0.66)	P (0.87)	P (0.30)	P (0.13)	P (0.16)	P (0.14)	P (0.35)	P (0.36)
	Student-t	G (0.09)	P (0.08)	P (0.15)	P (0.87)	P (0.20)	P (0.28)	P (0.17)	P (0.20)	P (0.50)	P (0.50)
	EWMA	G (0.52)	P (0.47)	P (0.95)	P (0.82)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	F (0.02)

Table-4.28 shows that the outcomes of value-at-risk and expected shortfall backtests at 97.5% during the forecasting period of January 1, 2013 to August 31, 2020. In the brackets indicate p-values. Green, yellow and red are denoted by G, Y and R respectively. P used for pass and F used for fail. Source: authors' contribution based on Matlab (2020a) Output.

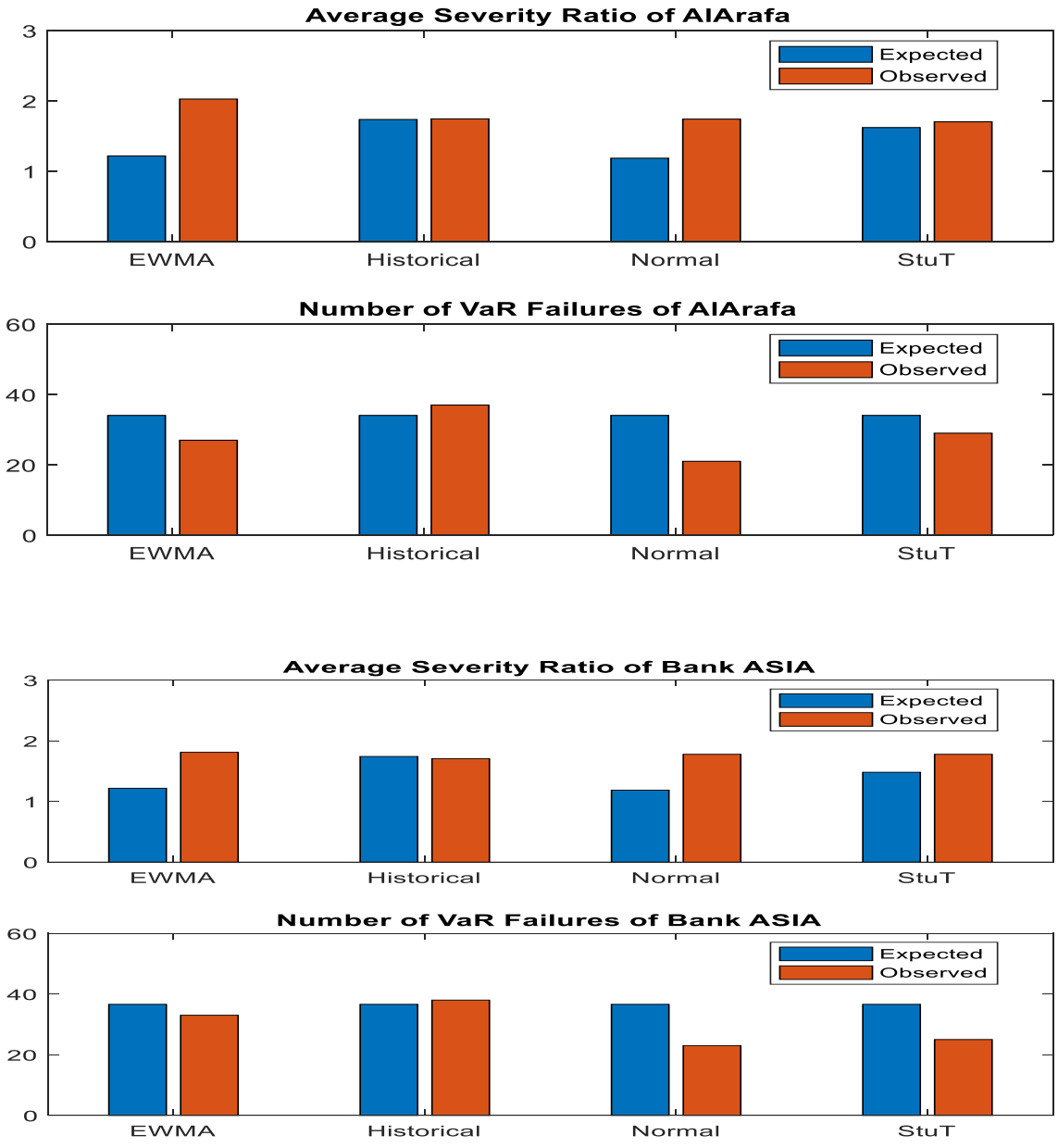


Figure-4.21: The average severity ratio and the number of value-at-risk failures across various models for **AL-Arafa bank (top) and Bank Asia (bottom)**. By examining the average severity ratios, we can gain a clearer understanding of how well all four accessible models execute in backtesting the effectiveness of producing expected shortfall projections in banks.

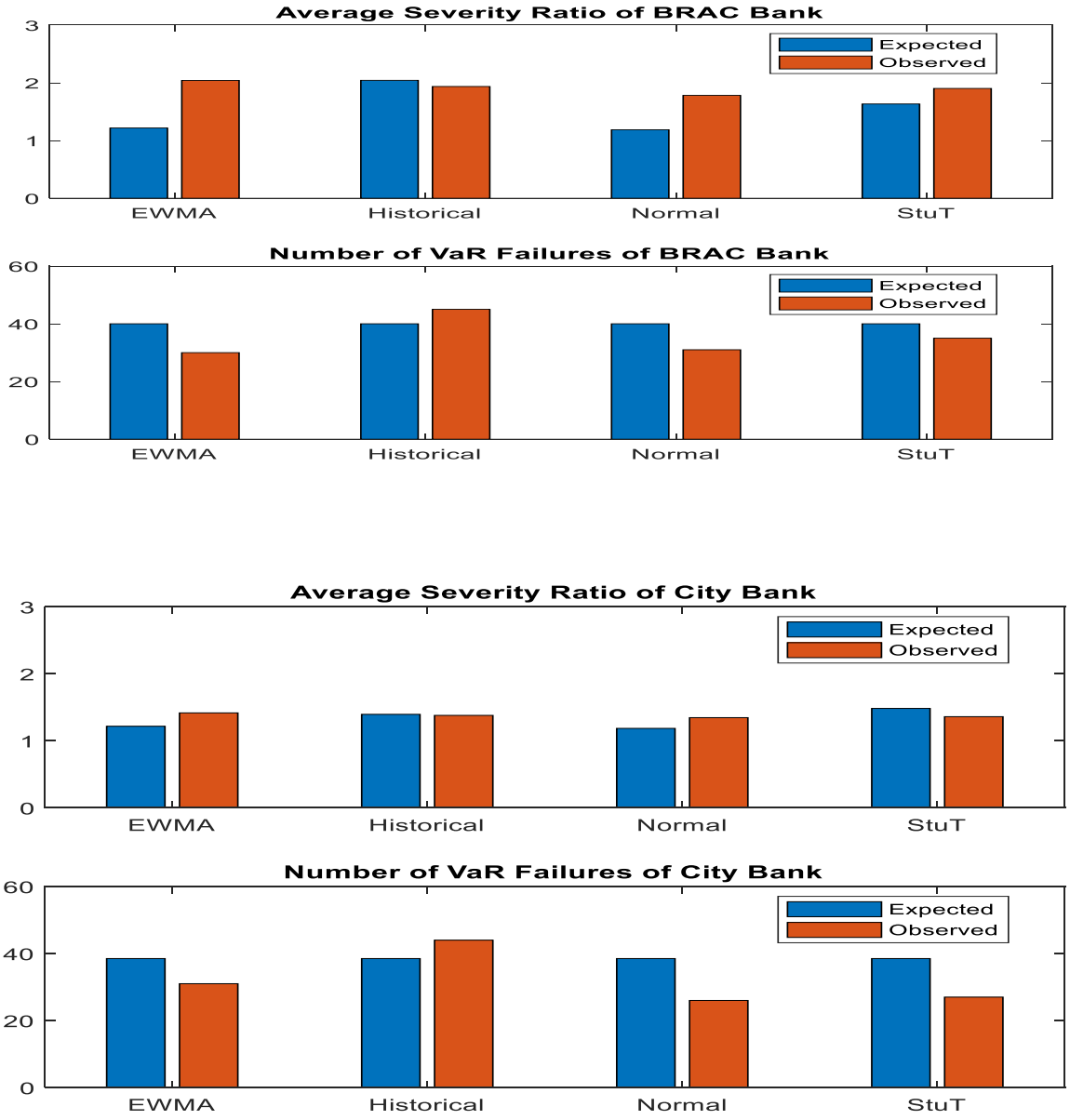


Figure-4.22: The average severity ratio and the number of value-at-risk failures observed throughout various models for **BRAC bank (Top) and City Bank (bottom)**. By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

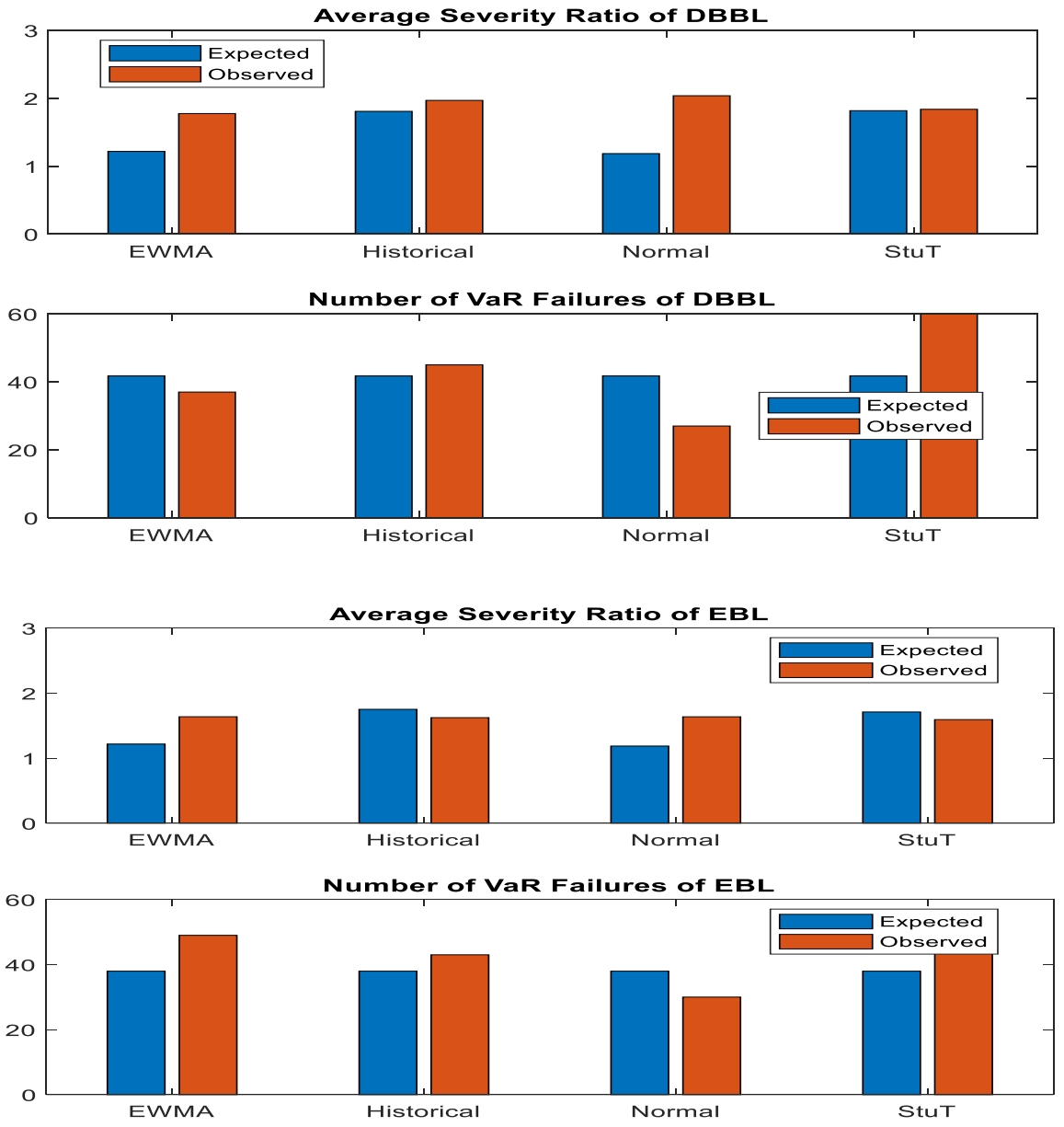


Figure-4.23: The average severity ratio and the number of value-at-risk failures observed throughout various models for **DBBL (top)** and **EBL (bottom).** By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

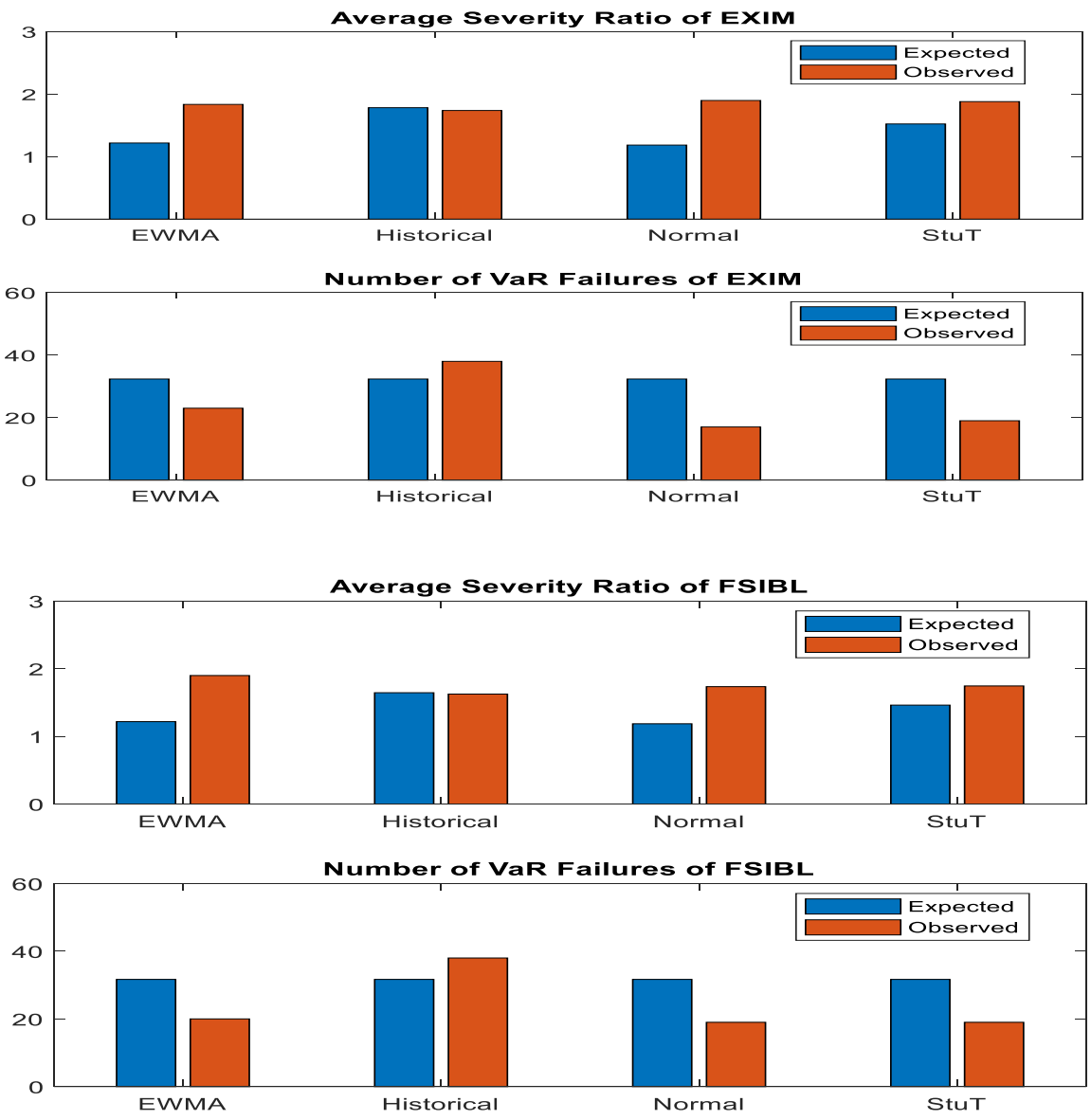


Figure-4.24: The average severity ratio and the number of value-at-risk failures observed throughout various models for **EXIM (top) and FSIBL (bottom)**. By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

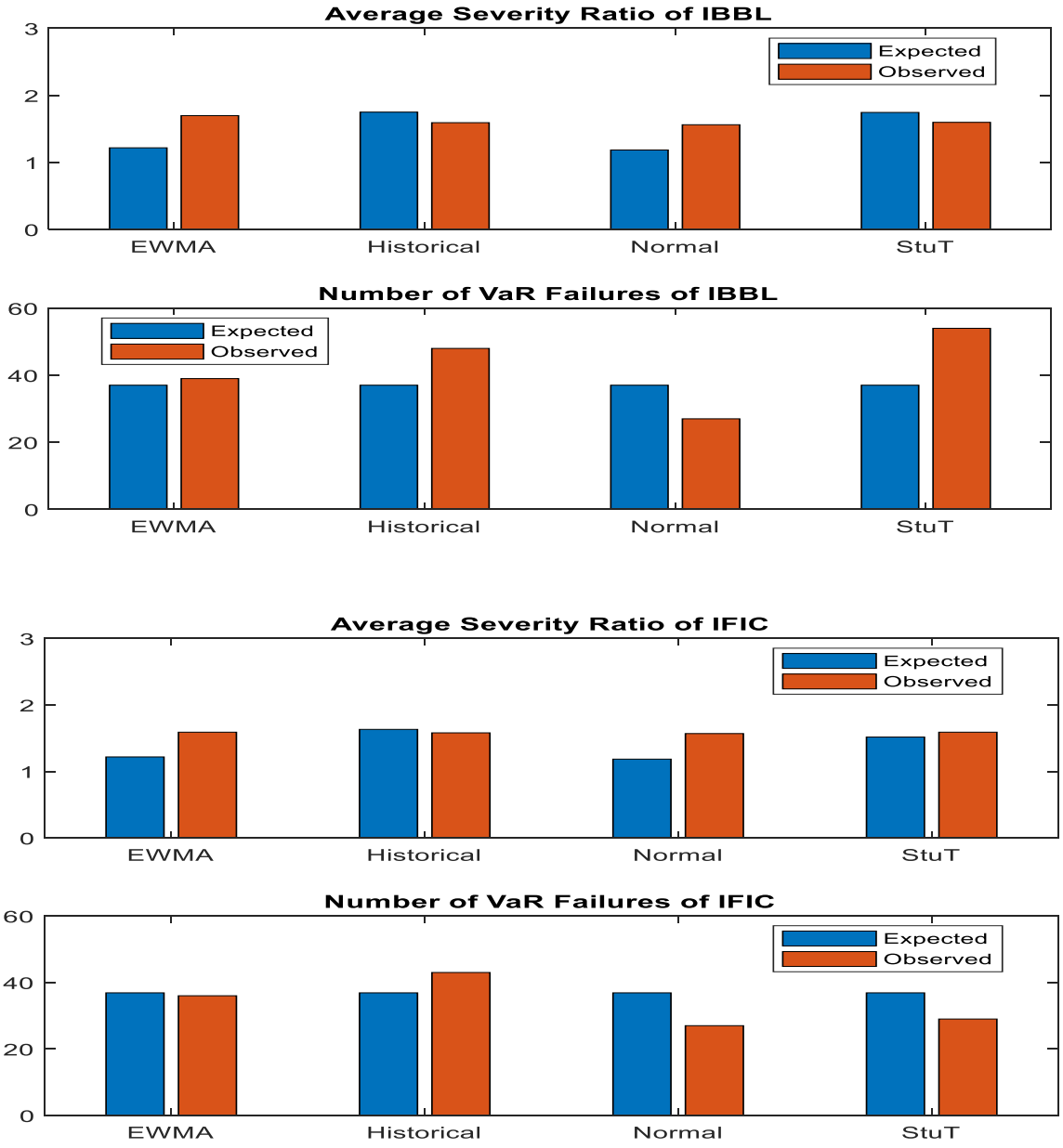


Figure-4.25: The average severity ratio and the number of value-at-risk failures observed throughout various models for **IBBL (top) and IFIC (bottom)**. By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

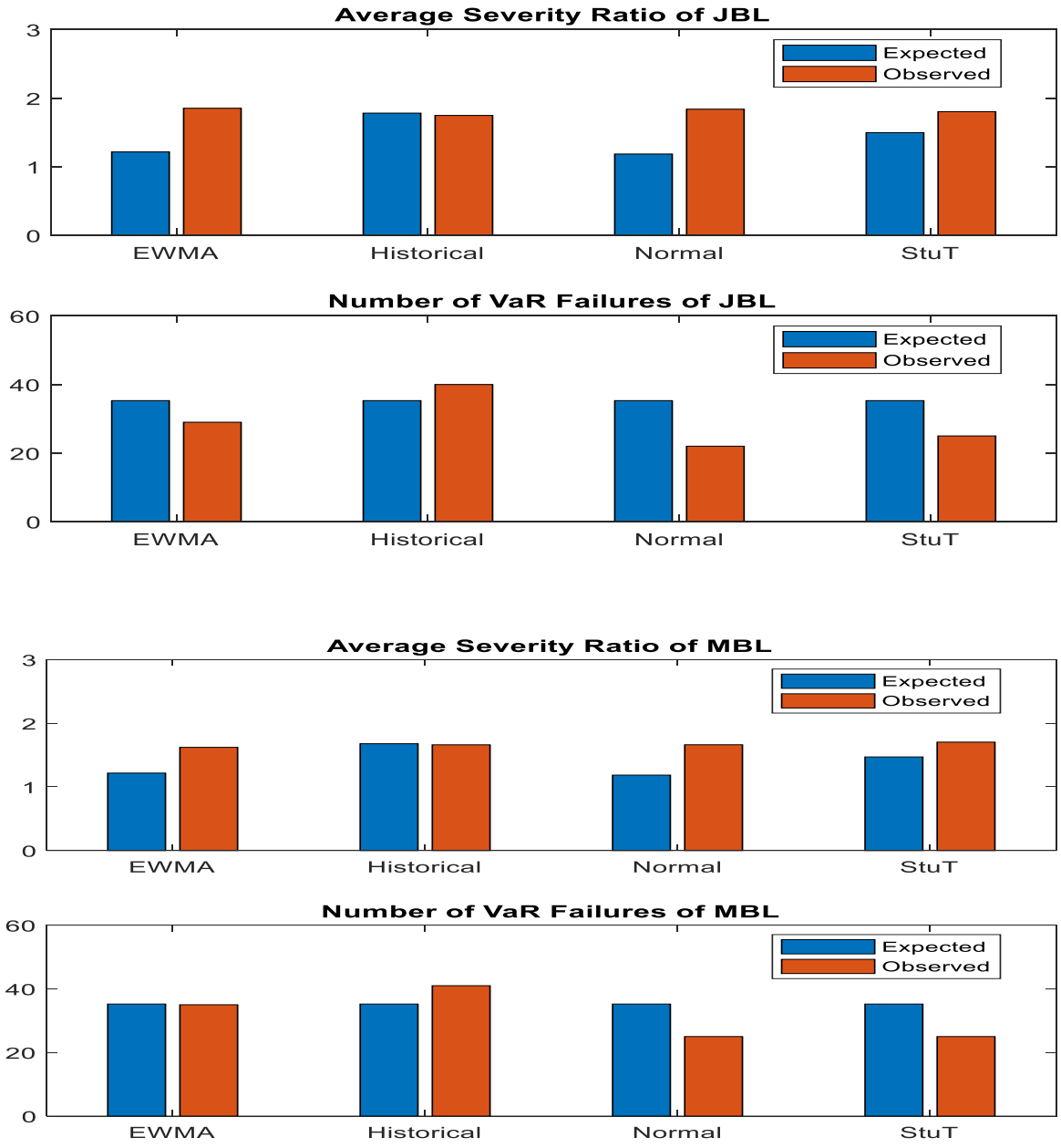


Figure-4.26: The average severity ratio and the number of value-at-risk failures observed throughout various models for **JBL (top) and MBL (bottom)**. By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

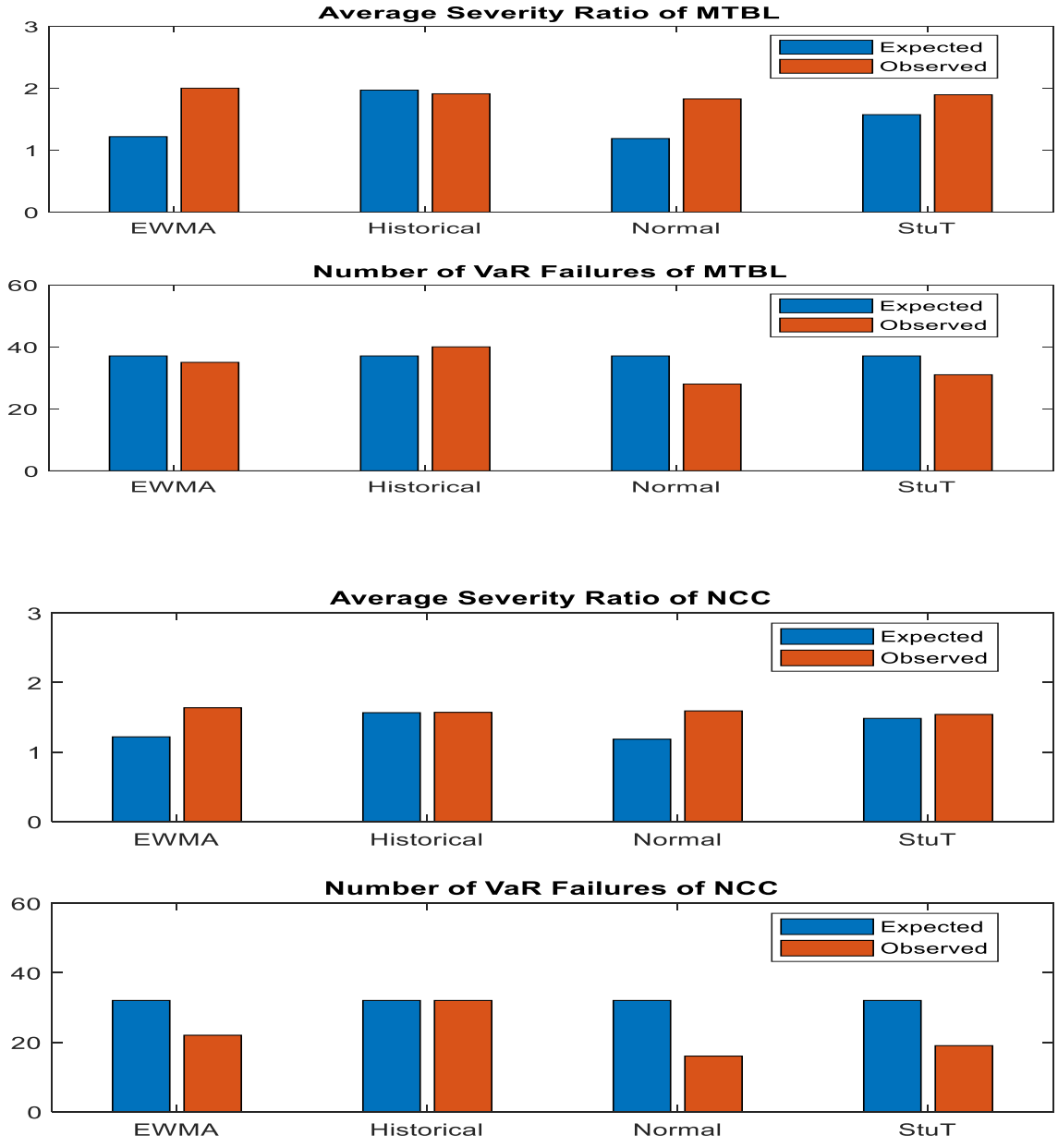


Figure-4.27: The average severity ratio and the number of value-at-risk failures observed throughout various models for **MTBL (top)** and **NCC** (**bottom**). By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

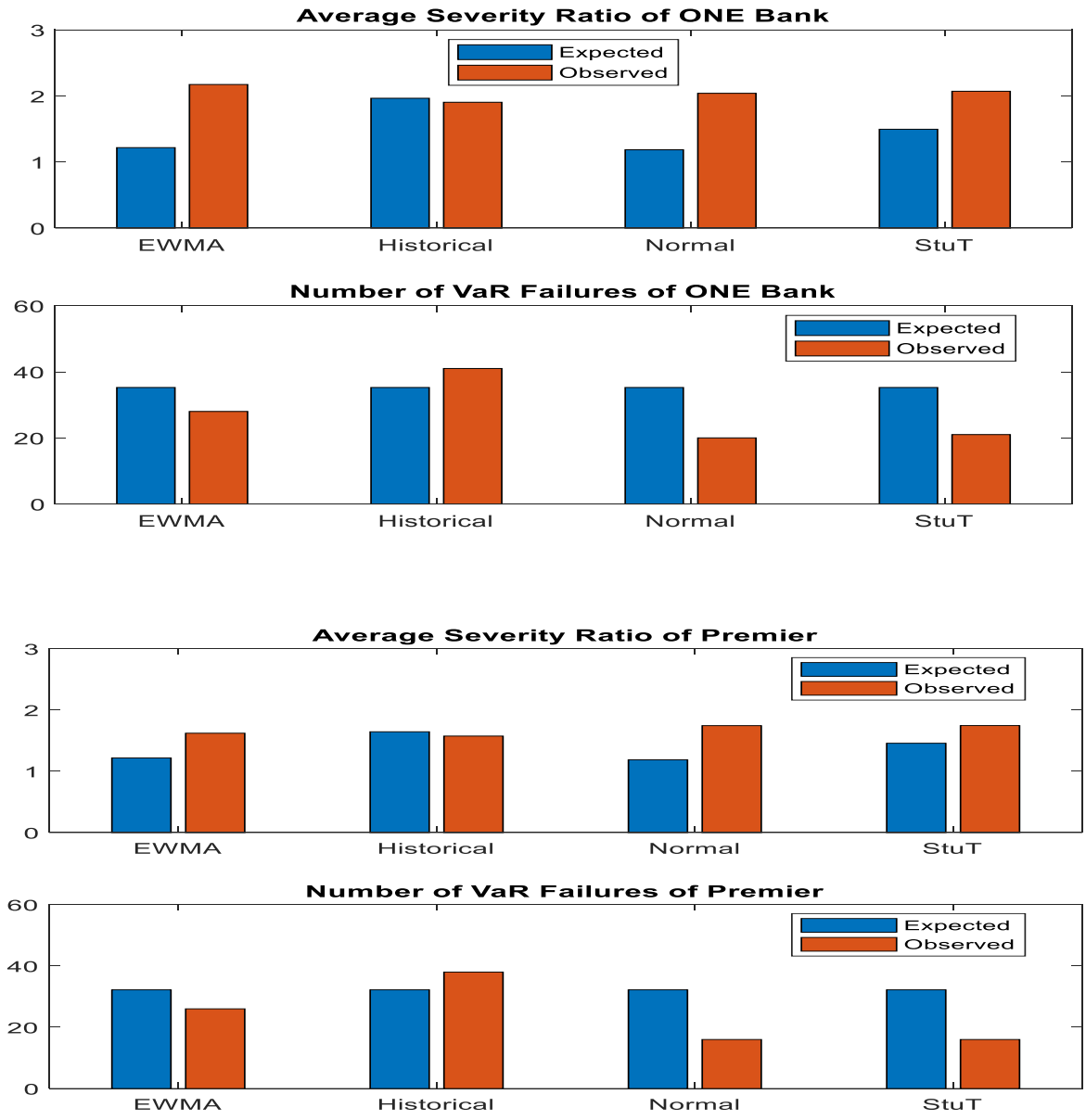


Figure-4.28: The average severity ratio and the number of value-at-risk failures observed throughout various models for **ONE Bank (top) and Premier Bank Ltd (bottom)**. By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

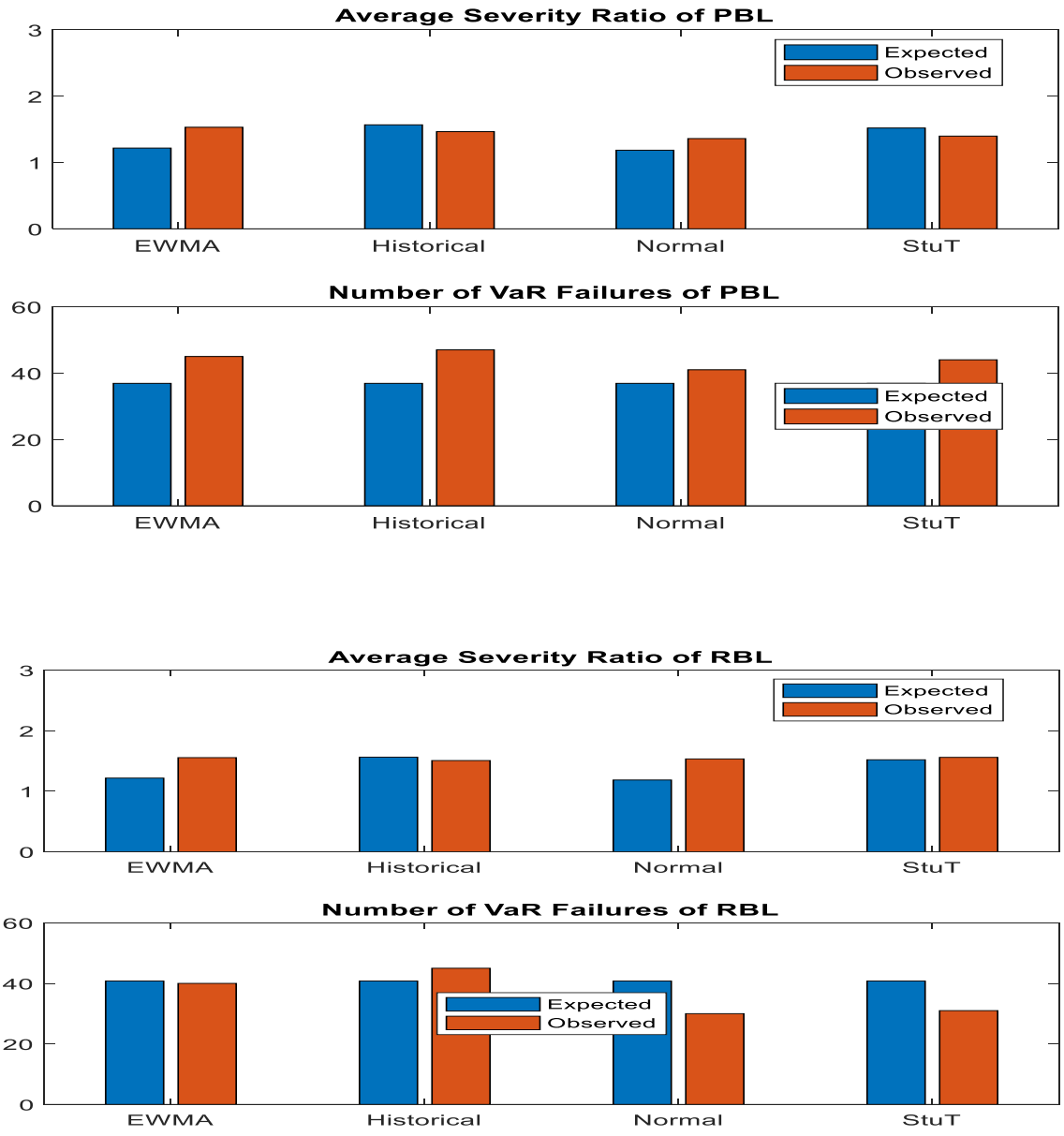


Figure-4.29: The average severity ratio and the number of value-at-risk failures observed throughout various models for **PBL (top) and RBL (below)**. By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

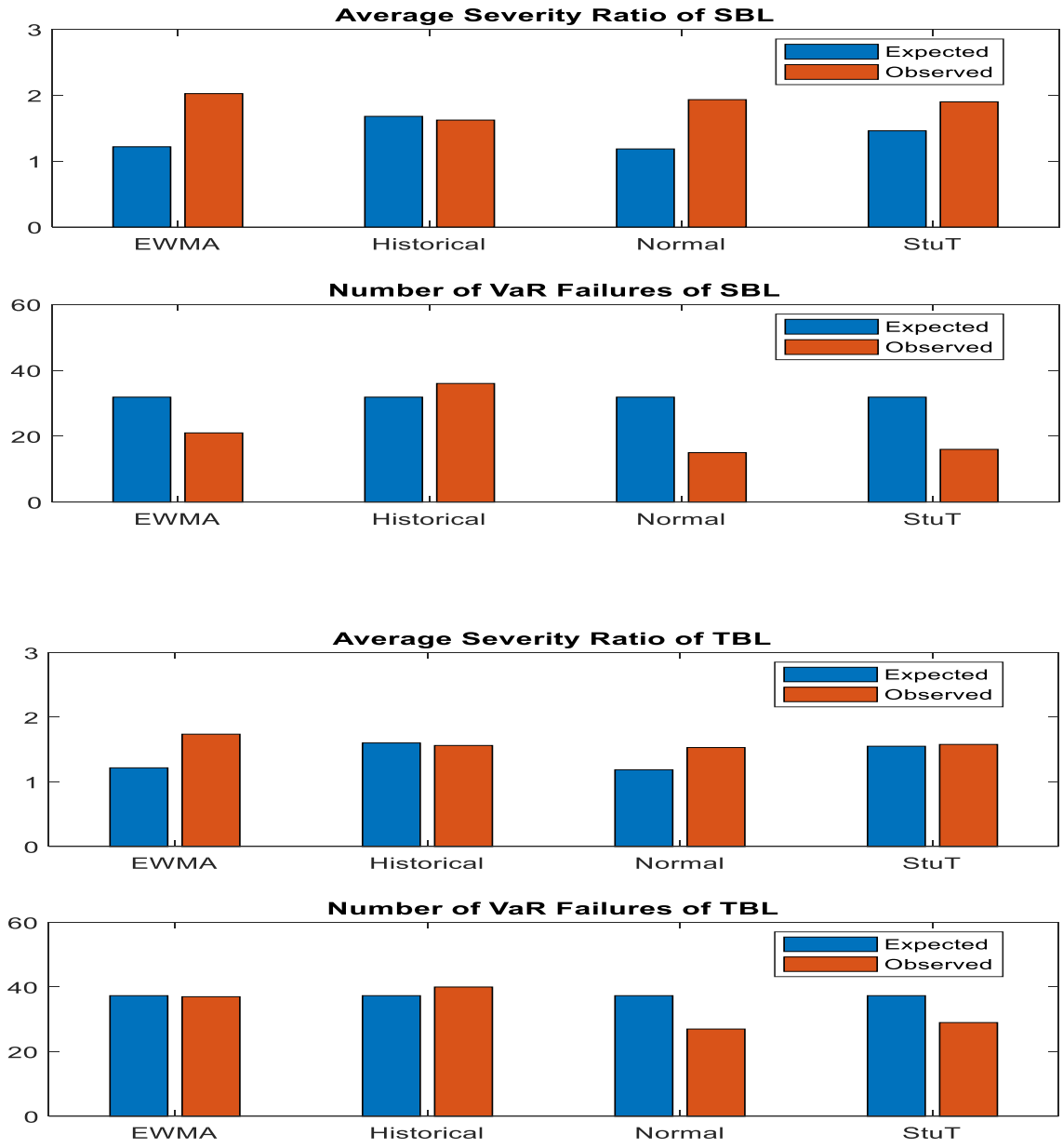


Figure-4.30: The average severity ratio and the number of value-at-risk failures observed throughout various models for **SBL (top)** and **TBL (below).** By examining the average severity ratios, we can gain a clearer understanding of how well all four user-friendly models perform in backtesting the effectiveness of producing expected shortfall forecasts across different banks.

Figure 4.22 to 4.31 displays the 'observed' and 'expected' quantities of value-at-risk violations, as well as the average severity ratios for all violations. This information is encompassed for all twenty banks and all four user-friendly models. This data is of utmost importance in determining the results of ES backtests, regardless of whether we consider the unconditional-N or unconditional-T hypotheses. This feature will protect independent shareholders from the adverse consequences that previous buyers have encountered (Bi & Zhu, 2020). These experiences are known to discourage investors from participating in the money market, including the supposedly robust banking sector. Strategy implementation should aim to assist investors in evaluating their daily investment risk,we may be using commonly used models that is where our work will be useful. We suggest that central bank of Bangladesh and management of TBL, AIBL, JBL, and PRBL banks that successfully passed all backtests for value-at-risk and expected shortfall under a specific model in our study, should organize training sessions to assist non-technical baseline investors in understanding their financial risk.

These sessions should focus on the HS simulation model, and if possible, also cover the student-t with location scale model, despite its challenges in parameter approximation for non-technical stockholders. Assisting novice investors, I offer an Excel spreadsheet that contains ES forecasts based on the HS approach. This prediction is generated using at least one year of historical share price data from the banks they intend to participate in through the DSE/CSE. By utilizing this tool, investors can gain early insight into the potential outcomes of their investments. Even investors with limited or no technological competence can easily undergo such training. Withholding information about investment risk from inexperienced non-technical investors can attract arbitrageurs to take advantage of naive entrepreneurs. These entrepreneurs only realize the consequences when foreign arbitrageurs exploit their success in predatory arbitrage. This results in a significant outflow of cash to foreign destinations and disrupts the personal and family lives of these vulnerable entrepreneurs, leading to social unrest. It is important for investors, regardless of their level of technical knowledge, to understand the true risk-return potential of their investments.

This understanding will contribute to the long-term sustainability of the capital markets. This will generate authentic impetus in the stock markets by providing some protection to inexperienced investors who are often exploited as puppets by gamblers. Similar to our examination of the banking industry, additional research should develop commonly-used risk assessment instruments for investments in extra sectors.

The study examines the correlation between market capitalization and the risk management process. Table 4.28 presents the descriptive statistics of the dependent and independent variables in the model. The data used for the analysis consists of day-to-day observations from January 2015 to August 2020, based on the accessibility of data as previously mentioned.

Variables	Observations	mean	Standard deviation	minimum	Maximum
MarketCap	25,500	13.297	11.902	3.228	29.402
Normal-VaR	25,500	3.982	1.245	2.390	8.711
HS-VaR	25,500	3.306	0.670	1.905	6.941
StuT-VaR	25,500	3.720	1.179	1.342	6.227
EWMA-VaR	25,500	3.598	1.904	1.068	9.029
Normal-ES	25,500	5.730	2.173	3.728	11.711
HS-ES	25,500	6.216	2.057	3.015	12.743
StuT-ES	25,500	6.923	2.815	3.730	10.394
EWMA-ES	25,500	5.829	2.629	3.881	13.592

Table-4.29: Descriptive statistics of variables that are incorporated into the model. Source: Authors' estimates derived from STATA 16.00 output.

Based on the estimated parameters presented in table-4.30, there is a statistically significant correlation between market risk estimates, as represented by four distinct VaR models (explanatory variables), and the market capitalization of banks (explained variable). The main reason for this is that risk management procedures have the power to affect both the perceived and actual stability of a bank, which in turn can have an impact on its stock price and, ultimately, its market capitalization. Furthermore, the implementation of efficient risk management strategies can bolster investor trust, resulting in heightened interest in the

bank's shares and perhaps driving up the stock price. Inadequate risk management may diminish investor trust, potentially resulting in a decline in stock value. This strong statistical correlation suggests that implementing consistent risk management measures can effectively mitigate substantial financial losses, thereby protecting the bank's assets and profitability. These factors, in turn, can have an impact on the stock price and, consequently, the overall market capitalization. Another point to note is that banks that successfully mitigate their risks are less prone to regulatory penalties, which can have adverse effects on their financial performance, stock price, and overall market capitalization. Furthermore, a bank that effectively mitigates its risks is less prone to needing financial assistance or encountering other widespread difficulties, which can directly impact its market worth.

Dependent variable: ln(MarketCap)		**Estimation methods of Models**			
		Pooled OLS	**Fixed Effect**	**Random Effect**	**GLS**
	Normal-VaR	6.129***	-4.107**	-4.101**	6.129***
		(0.278)	(0.113)	(0.113)	(0.278)
	HS-VaR	-10.785***	2.723**	2.715***	-10.785***
Explanatory		(0.493)	(0.201)	(0.200)	(0.493)
Variables	**StuT-Var**	5.147***	3.295***	3.275***	5.147***
		(0.196)	(0.168)	(.166)	(0.196)
	EWMA-VaR	1.485***	1.099**	1.099**	1.485***
		(0.155)	(0.059)	(0.059)	(0.155)
	Constant	19.331***	26.830***	26.834***	19.331***
		(1.344)	(0.535)	(4.552)	(1.344)
N (sample size)		25,500	25,500	25,500	25,500
R^2		0.6140	0.9958		
F		278.1213***	469.8478***		
Rho			0.8831	0.6098	
σ_u			31.3817	14.2763	
σ_e			11.4177	11.4176	
χ^2				1402.4108***	834.6257***

Table-4.30 : Parameter estimations of econometric models. Note: *, ** and *** indicate the level of significance at 5%, 1% and 0.1% level respectively. Standard errors of coefficient are in parenthesis. Source: Authors' estimations based on the output generated from STATA 16.00

The R^2 values of 0.6140 and 0.9958 indicate that 61.40% and 99.58% of the fluctuations in market capitalization can be accounted for by four VaR forecasts, namely Normal-VaR, Historical-VaR, Student-t-VaR (location based), and EWMA-VaR, when using pooled OLS and fixed effect models, respectively. Furthermore, the F-values calculated using pooled OLS and fixed effect application also indicate that all explanatory factors have a significant combined effect on the variations of the market capitalization of banks. The intra-class correlation, as indicated by the 'Rho' values of 0.8831 and 0.6098, explains 88.31% and 60.98% of the changes in market capitalization assessed using fixed effect and random effect approaches, respectively. These variations are attributed to the disparities across the panels.

Various diagnostic assessments of the model have been taken into account to authenticate the findings. The average VIF (variance inflation factor) value indicates a minimal pairwise connection among the explanatory variables in the model. The model is not affected by multicollinearity. In addition, the regression error specification test (RESET) was used to examine whether the model had any specification bias. The test results indicated that the model is free from specification bias, as the null hypothesis of no specification error was not rejected based on the estimated F ratio at a significance level of 5%. Furthermore, we have utilized the Hausman test to determine whether to employ fixed effect or random effect implementations. The test statistic is deemed statistically significant at the selected level of significance (5%), leading to the rejection of the null hypothesis that favors fixed effect over random effect. Therefore, it can be inferred that the random effect approach is superior to the fixed effect method. The BP/LM (Breusch Pagan Lagrangian Multiplier) test was used to determine whether the random effect or pooled OLS method provided better estimates. The test results indicated that the pooled OLS method produced more accurate estimates than the random-effect method. This conclusion was reached because the test statistic did not reach statistical significance at the 5% level, meaning that the null hypothesis of no significant difference across units or no panel effect could not be rejected. Therefore, it is suggested that the pooled OLS method is superior to the random effect

method. The modified Wald test has been utilized to assess if the model exhibits constant error variance, specifically for group-wise heteroscedasticity. The test statistic demonstrates statistical significance, indicating that the model is affected by non-constant error variance. Finally, we have also conducted the serial autocorrelation test using the Wooldridge test statistic and determined that the model exhibits autocorrelation problems. Additionally, we have determined the coefficient of the related explanatory variables using the cross-sectional GLS (generalized least square) approach, which assumes that there is no heteroscedasticity and no autocorrelation in the model.

Explained variable: ln(MarketCap)		Estimation methods of Models			
		Pooled OLS	Fixed Effect	Random Effect	GLS
Explanatory Variables	Normal-ES	15.7130*** (0.6517)	-4.4821** (0.2394)	-4.4821** (0.2395)	15.7130*** (0.6517)
	HS-ES	-15.7241*** (0.1162)	10.5173*** (0.4261)	10.5171*** (0.4262)	-15.7241*** (0.1162)
	StuT-ES	19.2835*** (0.4207)	6.7283*** (0.3902)	6.7284*** (0.3901)	19.2835*** (0.4207)
	EWMA-ES	1.8209*** (0.3620)	9.9741*** (0.1205)	9.9740*** (0.1205)	1.8209*** (0.3620)
	Constant	30.3501*** (3.1721)	30.0175*** (1.1124)	30.0174*** (1.1124)	30.3501*** (3.1721)
N		25,500	25,500	25,500	25,500
R^2		0.5571	0.6127		
F		250.950***	247.45***		
Rho			0.9056	0.8466	
σ_u			0.7396	0.5610	
σ_e			0.2387	0.2387	
χ^2				741.42***	753.10***

Table-4.31: Parameter estimations of Econometric Models. Note: *, ** and *** indicate the level of significance at 5%, 1% and 0.1% level respectively. Standard errors of coefficient are in parenthesis. Source: Authors' estimations based on the output generated from STATA 16.00

Based on the coefficients provided in table 4.30, all four ES measures are independently important in explaining the fluctuations in banks' market capitalization. Furthermore, the F-value and chi-square value calculated for each model confirm the collective relevance of all explanatory factors in describing the market capitalization of banks. The R2 values of 0.5571 and 0.6127 obtained by the pooled OLS and fixed effect technique, respectively, indicate that 55.71% and 61.27% of the fluctuations in market capitalization can be accounted for by the independent variables, namely Normal-ES, Historical-ES, Student-T (location based)-ES, and EWMA-ES forecasts. Furthermore, the intra-class correlation, as evaluated by the 'Rho' values, indicates that 90.56% and 80.66% of the variability in market capitalization under the fixed effect and random effect methods, respectively, can be attributed to changes among panels.

Various technical checks have been implemented to verify the results about the relationship between ES risk procedure and market capitalization. The average VIF (variance inflation factor) result indicates a persistent lack of significant pairwise correlation between the explanatory variables in the model. The model is not affected by the issue of multicollinearity. In addition, the regression error specification test (RESET) was used to examine the presence of specification bias in the model. The test results indicate that the model is free from specification bias, as the null hypothesis of no specification error was not rejected based on the estimated F ratio at a significance level of 5%. Furthermore, we have also taken into account the Hausman test in order to determine whether to use fixed effect or random effect. The test statistic is deemed statistically significant at the selected significance level of 5%, leading to the rejection of the null hypothesis in favor of the fixed effect over the random effect. Therefore, it can be inferred that the random effect approach is superior to the fixed effect method. Subsequently, the BPLM (Breusch Pagan Lagrangian Multiplier) test was conducted to determine the most suitable approach, either random effect or pooled OLS, for the ES explanatory variables. The results indicated that the pooled OLS estimators outperformed the random-effect estimators. The reason for this is

that the test statistic does not reach statistical significance at the 5% level. As a result, we cannot reject the null hypothesis, which states that there is no significant difference between units or no panel effect. This suggests that using pooled OLS is a better approach than using random effects. The modified Wald test has been utilized to assess if the model exhibits constant error variance, which is known as group-wise heteroscedasticity. The test statistic demonstrates statistical significance, indicating that the model is affected by non-constant error variance. Finally, we have conducted the serial autocorrelation test using the Wooldridge test statistic. Similar to the situation with VaR, we have discovered that the model experiences an autocorrelation issue when the explanatory variables are ES. Therefore, we have also used the cross-sectional GLS (generalized least square) approach to calculate the coefficient of the related explanatory variables, given that there is no heteroscedasticity and no autocorrelation in the model.

4.10 Conclusion

The most fundamental and approachable models in the scientific literature include HS, RiskMetricTM, Gaussian (Normal Distribution) model of BS geometric Brownian motion - stochastic differential equation, and t-location-scale distribution models. They belong to the 0,1, 2, and 3, parameterized families of risk models. Non-specialized traders interested in the developing economy of Bangladesh should incorporate HS and/or t-location-scale distribution models, accompanied by monitoring the banks' assets prices for a period of approximately one year. In addition to considering the ES risk measure, rather than relying solely on value-at-risk, these risk models should be included in their accessible toolkit for analyzing market risk. One should not be persuaded by the prevailing belief that RiskMetricsTM is the most accessible methodology for predicting risks. The default parameter of the RiskMetricsTM model is not the most user-friendly model for completing the expected shortfall backtests when assessing the expected shortfall risk measure, especially in Bangladesh.

Both the historical simulation and t-location- scale distribution models have determined that Trust Bank Limited is the most favorable bank for its shareholders and equity investors in terms of minimizing dissatisfaction during an unstable market. This conclusion is based on the analysis of all ten backtests conducted for value-at-risk and expected shortfall. Out of every bank included in this work, only Al-Arafa Islami Bank Limited and Trust Bank Limited passed all 10 backtests of value-at-risk and expected shortfall with historical simulation. Furthermore, AIBL was the only bank to pass all ten tests using the t-location-scale distribution, with the exception of TBL. Using the same criteria as TBL, AIBL is regarded as the second most advantageous model for equity investors. Indeed, only four out of the total of twenty banks successfully cleared all backtests, despite the fact that completing all ten assessments is mandatory according to a minimum one of the four risk models that were examined in the current research.

The study conducted by (H. P & Hotta, 2006) supports the prevalence of the HS model, demonstrating how the past simulation may be customized to accurately represent the unique features of the Brazilian Market. In addition, (Gencay & Selcuk, 2004) discovered in a separate study that the HS model demonstrated a favorable combination of simplicity and effectiveness when used to forecast market risk in the Turkish market. In a separate study conducted by (Kenourgios, Samitas, & Paltalidis, 2011) it was discovered that the Past Simulation framework well represented the spread of financial distress during periods of economic crisis in the Greek financial sector. Though Greece is not categorized as a developing market, this work could be applicable to developing nations that possess comparable financial attributes. Another study conducted by (Diebold, Gunther, & Tay, 1998) utilized student's t-distribution to predict the density of U.S. returns on stocks and emphasized the effectiveness of the student-t distribution model in accurately representing the extreme behavior of the return's distribution. In addition, (Haas, Mittnik, & Paolella,

2004) utilized the t-location- scale distribution to identify the characteristic of having a higher peak and fatter tails, which is often seen in the financial returns within the German stock market. Performing these audits would confirm that banks are efficiently handling their hazards and that their market capitalization precisely reflects their worth.

We recommend that Bangladesh Bank and the senior management of each of these financial institutions, who have passed the expected shortfall backtests in our analysis, work together to develop a policy that educates investors about the consequences of the available models utilized in our study. We argue that conducting comparative efficacy tests using easily accessible methods could assist generic traders in properly predicting the maximum probable loss that may arise when owning assets of registered commercial banks during unfavorable market conditions. This is possible if these investors utilize these models in conjunction with standard backtesting guidelines for expected shortfall, which serves as a tool for quantifying their market risk exposure. Additionally, it is crucial for the central bank of Bangladesh to implement policies that promote ordinary investors to undergo a brief risk-management instruction, as per our suggested approach. These measures may also reduce the probability of a capital market bubble leading to a stock market catastrophe due to an unexpected fraud in the future.

Chapter-5

Markovian and non-Markovian conditioning analysis for multi-period ahead risk forecasts with mean reversion and leverage effect.

5.1 Introduction

One of the goals of Basel committee on banking supervision is to popularize the practice of risk management in banks worldwide. However, the complicated implementation procedure often doesn't encourage banks in developing and underdeveloped countries to report investor's investment risk on a regular basis. Investors themselves do not dare implementing risk models on their own due to a lack of technical know-how; even when they have the long time series of banks share price data available to their exposure. Analytic formulas for risk measures Value-at-Risk (VaR) and / or Expected Shortfall (ES)[3] are very rare in the literature that pushes investors further away from implement any risk model. Benchmark Black and Scholes (BS) model has analytic formulas for VaR and ES but these are inadequate (McNeil & Frey, Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach, 2000). Among other sources of inadequacies most pronounced are BS doesn't allow updating risk forecasts on varying background like varying volatility or varying current asset level, it doesn't allow modelling a possible revert back for the asset to any level. In other words, it doesn't allow time varying drift and diffusion; as a consequence, it can't capture any mean reversion, as well, in assets movement.

Conditional models for risk forecasts have been proposed in the literature to deal with inadequacies mentioned above (McNeil & Frey, Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach, 2000). These models mostly update the risk forecasts on time varying volatility background and thus they are non-Markovian in nature and often do not have ease of implementation as it is often impossible to provide any analytic formula for VaR and/or ES. In fact, conditional risk forecasts with Markovian structure probably doesn't have any trace in the literature. This chapter proposes a way to implement analytic risk assessment/forecasts conditional on

[3] Not to talk about other risk measures like Spectral Risk Measures (SRM), Distortion Risk Measure etc.

assets current level only, and not on current volatility[4], and thus preserves the structure of Markov. In so doing it allows both time varying diffusion as well as mean reversion while keeping the ease of implementation to the level of applying analytic BS risk measures VaR as well as ES. Moreover, unlike any conditional risk model it can be applied without carrying out any optimization or numerical search procedure and thus have the potential to popularize risk management to the individual level in many developed, developing and/or under developed countries[5]. We name the implementation as conditional analytic Markov VaR (CAM-VaR) and conditional analytic Markov ES (CAM-ES).

CAM implementation's most significant attraction is multi-period parameterization yielding multi-period risk forecasts, as well, in analytic fashion. For any traditional conditional risk model on volatility varying background, multi-period forecasts are a challenging task as the multi-period ahead distribution of innovations is not analytically traceable, further complicating and discouraging the implementation to individual level. As explained in (McNeil & Frey, Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach, 2000) simulating the multi-period innovation distribution is the only way forward and in case of conditional extreme value model that requires a second-round application, and hence optimization, of extreme value theory (EVT). We compare our CAM-VaR and CAM-ES with conditional-EVT model of (McNeil & Frey, Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach, 2000) as that is one of the most attractive primary conditional models proposed in the literature[6]. So, while in GARCH-EVT model one needs to carry out two optimizations for single period (daily in our case) risk forecasts

[4] Saving one from relying on past asset prices except the current level.

[5] The only requirement of implementation is the exposure to long time series of daily share price data of the bank in hand; and not even the log returns.

[6] McNeil et al (2000) shows that their conditional approach performs better those other conditional approaches like GARCH-N, GARCH-t. So, we chose to compare our CAM method with the GARCH-EVT conditional method of McNeil et al (2000).

and three for multi-period (two-days, weekly and bi-weekly in our case) forecasts, for CAM implementation there is none and even then, the CAM approach performs way better than GARCH-EVT for both single and multi-period forecasts.

The rest of the chapter is structured in the following way. Section 5.2 briefly discusses the analytic forecasts of VaR and ES under BS model, with a discussion on square root law and analytic formulas to make multi-period forecasts; it presents estimation of parameters as well. It then introduces our CAM approach and revisits the conditional GARCH-EVT approach of (McNeil & Frey, Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach, 2000). Section 5.3 briefly introduces back-testing tests considered for VaR and ES. Section 5.4 presents the data used for both main calibration and robustness studies. Section 5.5 presents the results and some standalone empirical comments to show what can happen in case there is no evidence of mean reversion. Section 5.6 concludes the chapter.

5.2 Research Methodology

5.2.1 Analytic Multi-period Forecasts Under BS Model

The celebrated BS model is premised on the following geometric Brownian motion SDE:

$$dS_t = \mu S_t dt + \sigma S_t dB_t$$

$$S_{t+dt} = S_t + S_t\mu dt + S_t\sigma\sqrt{dt}\; z \sim N(S_t + S_t\mu dt, S_t\sigma\sqrt{dt}) \qquad (5.1)$$

where both drift parameter μ and the diffusion parameter σ are constants[7] and $\sigma > 0$. The fact that these two parameters solely characterize the risk measures VaR and ES, at any coverage level α, in this model, leaves us with no possibility to see the forecasts adjusted or updated promptly when either diffusion or both drift[8] and diffusion[9] enter in new phases

[7] B_t is a standard Brownian motion and so $dB_t = z\sqrt{dt}$, z being a standard normal random number.

[8] e.g. in of mean-reversion case

[9] e.g. when one wants to make risk forecasts based on current level of volatility which is not constant; but that requires relying on certain number of past returns data to estimate

across time. The following analytic estimates of VaR and ES in this model are possible[10] only due to not allowing the drift and diffusion coefficients to change over time (McNeil, Frey, & Embrechts, Quantitative risk management: concepts, techniques and tools, 2015).

$$VaR(\alpha) = -(\mu - \sigma\Phi^{-1}(\alpha)) \tag{5.2}$$

and

$$ES(\alpha) = \frac{-(\mu - \sigma\varphi(\ \Phi^{-1}(\alpha)\ ))}{1-\alpha} \tag{5.3}$$

where φ and Φ are density and distribution, respectively, of a standard normal variate. The elegance and simplicity of using $eq.5.2$ and $eq.5.3$ for risk forecasts are beyond questions but the question arises only with the adequacy. There are no direct efforts in the literature to keep the elegance and simplicity of $eq.5.2$ and $eq.5.3$ intact while increasing the adequacy, CAM-VaR and CAM-ES strive to do so. Once drift and diffusion parameters are estimated, most often from daily log-returns of an asset, $eq.5.2$ and $eq.5.3$ can be directly applied to obtain the daily VaR and ES forecasts and the forecasts themselves remain constant across time. Popular, but questionable, square root rule can then be applied to obtain multi-period forecasts in this model, e.g. for h −day forecasts from one day:

$$VaR^h(\alpha) = \sqrt{h}\ VaR(\alpha). \tag{5.4}$$

Similarly for ES, an alternative and more methodological approach to make multi-period ahead forecasts from single period estimates of drift and diffusion. Using $eq.5.2$ and $eq.5.3$ we estimate h −day of drift and diffusion:

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current volatility and hence model forecasts no-longer have Markov property (though Markov property has strong appeal in general). This is a common feature of conditional approach. However, CAM approach introduces conditioning on only current level of asset that quantifies current levels of drift and diffusion hence forecasts possess the Markov property.

[10] If daily log-returns are used to estimate μ and σ and $dt = \frac{1}{252}$ is used in estimation, instead of $dt = 1$, then μ and σ should be replaced by μdt and $\sigma\sqrt{dt}$, in the VaR and ES formulas.

$$\mu^{h-day} = \frac{h}{252}\,\mu \text{ and } \quad \sigma^{h-day} = \sqrt{\frac{h}{252}}\,\sigma \tag{5.5}$$

when μ and σ are estimated from daily log-returns with a daily time resolution of $\frac{1}{252}$. We refer to these later estimates as analytic multi-period ahead forecasts while the former estimates as square root rule multi-period ahead forecasts.

An application of Ito's formula in GBM set up yields the distribution of log-return in the BS model as:

$$\log\left(\frac{S_{t+dt}}{S_t}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma\sqrt{dt}\,z$$

$$S_{t+dt} = S_t e^{\left(\mu-\frac{1}{2}\sigma^2\right)dt+\sigma\sqrt{dt}\,z} \tag{5.6}$$

We use eq.5.6 to carry out our simulation studies[11] to establish the relative performance of back-testing for square root rule based as well as analytic formula based multi-period forecasts of VaR and ES under the dynamics in $eq.5.1$. However, $eq.5.6$ is more significant for our CAM approach and its simulation studies than the role its play in studying the model in $eq.5.1$. Namely the simulation can still be conducted on any interval $[t, t+dt]$ when μ and σ are not exactly constant but measurable when S_t is given. With this note we are in position to introduce the CAM-VaR and CAM-ES.

5.2.2 Multi-period Conditional Analytic Markov (CAM) VaR and ES

Our CAM approach utilizes the popular the Cox-Ingersoll-Ross model or CIR dynamics which is renowned for modelling mean reversion (Cox, Ingersoll, & Ross, 1985). The unconditional distribution of the following CIR dynamics is non-central Chi-square but the conditional distribution can still be seen as normal:

$$dS_t = \kappa(\tilde{\mu} - S_t)dt + \tilde{\sigma}\sqrt{S_t}dB_t$$

[11] The simulation studies are to confirm the relative back-testing based preference obtained with original data.

$$S_{t+dt} = S_t + S_t\{\kappa(\frac{\tilde{\mu}}{S_t} - 1)\}dt + S_t\{\frac{\tilde{\sigma}}{\sqrt{S_t}}\}\sqrt{dt}\, z \tag{5.7}$$

Comparing equation (5.7) with equation (5.1) we can see that

$$S_{t+dt} = S_t + S_t\, \mu_t^{cam} dt + S_t \sigma_t^{cam} \sqrt{dt}\, z \tag{5.8}$$

where $\mu_t^{cam} = \kappa(\frac{\tilde{\mu}}{S_t} - 1)$ and $\sigma_t^{cam} = \frac{\tilde{\sigma}}{\sqrt{S_t}}$ are measurable given S_t.

Here equation (5.1) and equation (5.7) represent quite different evolution of assets prices. On a fixed interval of length 'dt' the drift of '$\frac{S_{t+dt}-S_t}{S_t}$' in equation (5.1) remains same everywhere but the drift of '$\frac{S_{t+dt}-S_t}{S_t}$' in equation (5.7) varies over time. On occasions when 'S_t<$\tilde{\mu}$' the drift is positive, then 'S' on average increases and reverts back to the level '$\tilde{\mu}$' at a rate κ; but on occasions when 'S_t>$\tilde{\mu}$' the drift gets negative, then 'S' on average decreases and reverts back to the level '$\tilde{\mu}$' at a rate κ. So while dynamics in equation (5.1) exhibits no mean reversion, dynamics in equation (5.7) exhibits a time varying drift that always pushes asset 'S' to revert back to the level '$\tilde{\mu}$' at a rate κ. Similarly on any fixed interval of length 'dt' the variance of '$\frac{S_{t+dt}-S_t}{S_t}$' in equation (5.1) always remains same; but the variance of '$\frac{S_{t+dt}-S_t}{S_t}$' in equation (5.7) changes over time. On occasions with high $'S_t'$ values the near future variances get lower; whereas on occasions with low $'S_t'$ values the near future variances get higher- a phenomenon commonly known as leverage effects. However, given S_t , $S_{t+dt} \sim N(S_t + S_t\mu_t^{cam}dt,\ S_t\sigma_t^{cam}\sqrt{dt})$. Hence given S_t, the CAM-VaR and CAM-ES forecasts on $'t + dt'$ can be obtained by equation (5.2) and equation (5.3) using μ_t^{cam} and σ_t^{cam} replacing μ and σ respectively. Moreover, given S_t simulations can be carried out on $[t, t + dt]$ using eq.5.6 to obtain simulated CAM-VaR and CAM-ES forecasts on $'t + dt'$, in much the same way they are obtained for the BS dynamics in equation (5.1).

5.2.3 Extreme Value Theory

Extreme Value Theory (EVT) is a statistical framework designed to analyze and model extreme events occurring in the tails of probability distributions, making it particularly well-suited for assessing financial risk**.** Originally developed by Fisher**,** EVT comprises two main approaches: the block maxima method and the peaks-over-threshold (POT) method. In the block maxima approach, the data is divided into non-overlapping blocks of equal size, with the maximum value from each block extracted and modelled using the Generalized Extreme Value (GEV) distribution. Conversely, the POT method focuses on observations that exceed a predefined threshold, modelling these excesses using the Generalized Pareto Distribution (GPD)**.** A key challenge in the POT method lies in selecting an appropriate threshold to accurately capture extreme values. Despite this, the POT approach is widely recognized for its effectiveness in modelling rare, extreme events within financial datasets.

5.2.4 The Peaks Over Threshold (POT) Method

The distribution of excesses over high threshold is defined as

$$F_u(y) = P(X - u \leq x | X > u)$$
$$= \frac{F(y+u)-F(u)}{1-F(u)} \quad \text{for } 0 < y < x_0 - u. \tag{5.9}$$

The cumulative distribution function of the GP distribution with shape parameter ξ scale parameter β has the following representation.

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-1/\xi} & if\ \xi \neq 0, \\ 1 - \exp\left(-\frac{y}{\beta}\right) & if\ \xi = 0, \end{cases} \tag{5.10}$$

Where $\beta > 0,$ and the support is $y \geq 0$ when $\xi \geq 0$ and $\xi \leq y \leq -\beta/\xi$

When $\beta < 0$. Rearranging equations (5.9) and (5.10) we obtain a cumulative distribution function of returns:

$$F(x) = \big(1 - F(u)\big)\, G_{\xi,\beta}(y)(x-u) + F(u), x > u. \tag{5.11}$$

The function $F(u)$ can be estimated non parametrically by $\frac{n-N_u}{n}$ where n is the total number of observations and N_u represents the number of exceedances over the threshold u. after replacing $F_u(y)$ by $G_{\xi,\beta}(y)$, we get the following estimate for $F(x)$:

$$\hat{F}(x) = 1 - \frac{N_u}{n}\left(1 + \hat{\xi}\frac{x-u}{\hat{\beta}}\right)^{-1/\hat{\xi}} \tag{5.12}$$

By inverting this expression, we get VaR for a short position (right tail of distribution) is given by the x value in equation 5.12 and the expression for VaR becomes

$$VaR_p = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}\, p\right)^{-\hat{\xi}} - 1\right) \tag{5.13}$$

Where p is th confidence level of VaR. In order to calculate VaR for long position (left tail of distribution) it is necessary to carry out the calculation for negative returns.

5.2.5 The Conditional EVT (GARCH-EVT) Model

The conditional Extreme value theory (GARCH-EVT) is a two-stage hybrid model that combines a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) filter with the extreme value theory. The GARCH-EVT model was introduced by (McNeil & Frey, Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach, 2000) to VaR modeling by extending the EVT framework to dependent data. This model uses the EVT to model the tails of standardized residues ϵ_t obtained from the GARCH model. Despite the growing use of EVT in Value-at-Risk estimation, the assumption that observations are identically and independent distributed (iid) does not sit well with the reality of financial series data. The immediate solution is to use filtered data, the conditional EVT. That is, we fit a time-varying volatility model to the data, then estimate the tails of the standardized residuals retrieved from the fitted model using EVT. The advantage of conditional EVT is that it not only captures volatility clustering within the GARCH framework, but also explores the tail behavior with an EVT scheme simultaneously.

The conditional EVT is described as follows:

(i) The AR (1)-GARCH (1,1) model is fitted to the negative logarithm of returns using quasi maximum likelihood estimation. Then we get a one step ahead forecast of conditional mean and conditional standard deviation.

(ii) Apply EVT to the standardized residual retrieved from Step1 to get the estimate VaR^{α}.

In this study we assume that returns are modelled using the most popular GARCH (1,1) model:

$$\mathrm{r_t} = \mu_t + \epsilon_t$$
$$\sigma_t^2 = \kappa + \varphi r_{t-1}^2 + \alpha \sigma_{t-1}^2 \tag{5.14}$$

Where $\kappa, \varphi, \alpha > 0,\ \alpha + \beta < 1$ for stationary.

We assume that the return series is decomposed into two parts, the predictive and unpredictive component where μ_t is the conditional mean and ϵ_t is the unpredictive part or innovations process. The conditional mean returns can be expressed as a q-order autoregressive process AR(q):

$$\mu_t = c + \sum_{i=1}^{q} \theta\, \mathrm{r_{t-i}} \tag{5.15}$$

The unpredictable component ϵ_t can be express as an ARCH process as follows: $\epsilon_t = z_t\, \sigma_t$ where z_t is a sequence of iid random variables with mean zero and variance one. The conditional variance on information at time $\mathrm{t-1}$ of innovation ϵ_t is ${\sigma_t}^2$.

Therefore, the value-at-risk derived from conditional EVT can be expressed as:

$$VaR_{t+1}^{\alpha} = \mu_t + \sigma_{t+1} VaR_t(Z)$$

where VaR(Z) the unconditional VaR^{α}. If we substitute VaR(Z) with VaR^{α}, then the GARCH-EVT based value-at-risk is

$$VaR_{t+1}^{\alpha} = \mu_t + \sigma_{t+1}\left[u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}\, p\right)^{-\hat{\xi}} - 1\right)\right] \tag{5.16}$$

5.2.6 Parameter Estimations of GBM and CIR Dynamics

The maximum likelihood estimates (mle) of μ and σ for GBM dynamics are known in closed form so are the multi-period characterizations of parameters. The same holds true for CIR parameters on which CAM risk measures are based. If $r_i = \log\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right)$; then with an application of Ito's lemma one obtains $E[r_i] = \left(\mu - \frac{1}{2}\sigma^2\right)dt$ and $Var[r_i] = \sigma^2 dt$; where $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} r_i$; and with $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \mu)^2$, $\hat{\sigma} = \frac{S}{\sqrt{dt}}$.[12]. The analytic MLE for CIR dynamics can also be found in the literature e.g. in (Tang & Chen, 2009). If[13] $\widehat{\beta_1} = \frac{\frac{1}{n^2}\sum_{i=1}^{n} S_i \sum_{i=1}^{n}\frac{1}{S_{i-1}} - \frac{1}{n}\sum_{i=1}^{n}\frac{S_i}{S_{i-1}}}{\frac{1}{n^2}\sum_{i=1}^{n} S_{i-1}\sum_{i=1}^{n}\frac{1}{S_{i-1}} - 1}$; $\widehat{\beta_2} = \frac{\frac{1}{n}\sum_{i=1}^{n}\frac{S_i}{S_{i-1}} - \widehat{\beta_1}}{\frac{1-\widehat{\beta_1}}{n}\sum_{i=1}^{n}\frac{1}{S_{i-1}}}$ and $\widehat{\beta_3} = \frac{1}{n}\sum_{i=1}^{n}\left\{S_i - S_{i-1}\widehat{\beta_1} - \widehat{\beta_2}\left(1 - \widehat{\beta_1}\right)\right\}^2 \frac{1}{S_{i-1}}$, then $\hat{\kappa} = \frac{-log(\widehat{\beta_1})}{dt}$; $\hat{\hat{\mu}} = \widehat{\beta_2}$ and $\hat{\hat{\sigma}} = \sqrt{\frac{2\hat{\kappa}\,\widehat{\beta_3}}{1-\widehat{\beta_1}^2}}$. Given S_{i-1} since r_i in CIR dynamics conditionally follows $N\left(\left(\mu_{i-1}^{cam} - \frac{1}{2}\sigma_{i-1}^{cam\,2}\right)dt,\ \sigma_{i-1}^{cam}\sqrt{dt}\right)$; with $\mu_{i-1}^{cam} = \hat{\kappa}\left(\frac{\hat{\hat{\mu}}}{S_{i-1}} - 1\right)$and $\sigma_{i-1}^{cam} = \frac{\hat{\hat{\sigma}}}{\sqrt{S_{i-1}}}$, so we obtain the standard error (SE) of the CIR, as well as GBM, parameters by numerically inverting the hessian of the normal likelihoods[14]. We present the optimum likelihoods corresponding to the analytic MLE estimations for GBM and CIR (as conditional normal) calibrations; however, these likelihoods remain stand alone as EVT, by definition, doesn't consider the likelihood of entire data[15].

Using the BS, Conditional Analytic Markov (CAM) and GARCH-EVT models, we calibrated the model's parameters from the JPM, BAC, AIG and PRU index price series log-returns. The calibration result of the models' parameters is given in table 5.1 to 5.6. After that we computed model's price using parameters of this tables.

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[12] These estimates of μ and σ are the analytic MLE for the GBM SDE.

[13] Here S_i stands for S_{t_i} and so on.

[14] For GBM, r_i unconditionally follows $N\left(\left(\hat{\mu} - \frac{1}{2}\hat{\sigma}^2\right)dt,\ \hat{\sigma}\sqrt{dt}\right)$.

[15] It is worth remembering that these likelihoods are obtained without running any optimization.

Table 5.3 and table 5.6 report the estimated parameters of the general Pareto distribution (GPD) as well as their estimated standard errors resulting from applying the POT approach to the filtered standardized innovations. For all the returns series except BAC (lower tail) , GE (lower tail) and BMW (upper tail), the shape parameter is found to be positive and significantly different from zero, indicating heavy-tailed distributions of the innovation process characterized by the Frechet distribution. The estimated tail index values range between -0.14 (Bank of America, lower tail) and 0.134 (JP Morgan, lower tail).

Data	BS Model Parameters			CAM Model parameters			
	LL	μ	σ	LL	κ	μ	σ
JPM	25940	0.000188 (1.01e-06)	0.27773 (0.000138)	27361	0.73369 (0.000650)	51.921 (0.018238)	1.8155 (0.000927)
BAC	19743	-0.000159 (1.92e-06)	0.37188 (0.000202)	21437	0.74587 (0.000688)	12.361 (0.01178)	1.2093 (0.000698)
AIG	20872	0.000416 (2.04e-06)	0.35963 (0.000173)	22434	0.64626 (0.000541)	49.533 (0.010498)	2.1556 (0.001061)
PRU	23660	0.000174 1.61e-06	0.30247 (0.000168)	24577	1.041 (0.000708)	70.513 (0.009189)	2.3761 (0.001354)

Table-5.1: Parameters estimation of BS model and CAM model for financial data such as JP Morgan (JPM), Bank of America (BAC), American Investment Group (AIG), Prudential Capital (PRU) over the period 2010-2015. Each data set contains 1512 number of observations. The numbers in parentheses represent standard errors and LL gives the Log-likelihood values of the models.

Data	GARCH-EVT Model Parameters				
	c	θ	κ	ARCH coefficient (α)	GARCH coefficient (φ)
JPM	-0.000820 (0.000380)	-0.03436 (0.02911)	5.46e-06 (1.79e-06)	0.073704 (0.008498)	0.90838 (0.010822)
BAC	-0.000522 (0.000453)	0.011976 (0.028642)	9.20e-06 (2.10e-06)	0.10396 (0.007622)	0.88071 (0.010742)
AIG	-0.000578 (0.000398)	-0.014979 (0.028503)	3.06e-06 (1.11e-06)	0.08684 (0.007699)	0.91189 (0.007724)
PRU	-0.000722 (0.000447)	-0.023057 (0.028018)	1.53e-05 (3.56e-06)	0.085177 (0.010842)	0.87074 (0.018959)

Table-5.2: Parameters estimation of GARCH-EVT Model for financial data such as JP Morgan (JPM), Bank of America (BAC), American Investment Group (AIG), Prudential Capital (PRU) over the period 2010-2015. Each data set contains 1512 number of observations. The numbers in parentheses represent standard errors and LL gives the Log-likelihood values of the models.

Financial Assets	Upper Tail			Lower Tail		
	Threshold (u)	ξ	β	Threshold (u)	ξ	β
JPM	1.233	0.13427 (0.089282)	0.065262 (0.065262)	1.1632	0.059841 (0.089535)	0.48098 (0.057886)
BAC	1.1949	0.088892 (0.085992)	0.56375 (0.066203)	1.1878	-0.14775 (0.079975)	0.67203 (0.07755)
AIG	1.1913	0.12243 (0.097229)	0.59154 (0.075609)	1.1568	0.13182 (0.084748)	0.48851 (0.057912)
PRU	1.2341	0.073205 (0.072178)	0.61306 (0.06529)	1.151	0.017124 (0.082372)	0.45701 (0.051572)

Table-5.3: Shows threshold values and maximum likelihood GPD parameters values for both positive and negative tails of the distribution of the financial mainly bank data, the numbers in parentheses represent standard errors.

Data	Number Of Observations	BS Model Parameters			CAM Model parameters			
		LL	μ	σ	LL	κ	μ	σ
GE	3780	65953	0.000480 (2.88e-06)	0.27117 (9.28e-05)	54690	0.015216 (1.34e-05)	1143 (0.008011)	4.0317 (0.001316)
IBM	3780	56636	0.000289 (6.56e-07)	0.32252 (0.000107)	49889	0.0437 (0.000134)	126.09 (0.021335)	2.5415 (0.000824)
COCA-COLA	3780	68311	0.000395 (7.16e-07)	0.26118 (8.91e-05)	63460	0.16343 (4.63e-05)	27.453 (0.017485)	1.2312 (0.000405)
BMW	2016	23135	0.000229 (1.15e-06)	0.41294 (0.000218)	23671	2.7438 (0.001069)	33.098 (0.047129)	2.2932 (0.001247)

Table-5.4: Parameters estimation of BS model and CAM model for non-financial data. Models are calibrated over 1990-2005 for GE, IBM and COCA-COLA and over 1996-2005 for BMW (as data prior to 1996 are not available for BMW). The numbers in parentheses represent standard errors and LL gives the Log-likelihood values of the models.

Data	Number Of Observations	GARCH-EVT Model Parameters				
		c	θ	κ	ARCH Coefficient (α)	GARCH Coefficient (φ)
GE	3780	-0.000756 (0.000222)	-0.007972 (0.016068)	9.18e-07 (5.04e-07)	0.041964 (0.003869)	0.95531 (0.003740)
IBM	3780	-0.000496 (0.000265)	0.006433 (0.016183)	4.65e-06 (6.01e-07)	0.063927 (0.004339)	0.92742 (0.004594)
COCA-COLA	3780	-0.000655 (0.000229)	0.015223 (0.015642)	1.91e-06 (4.91e-07)	0.039852 (0.003071)	0.95327 (0.003772)
BMW	2016	-0.000303 (0.000455)	0.037352 (0.023638)	3.90e-06 (1.41e-06)	0.084232 (0.008267)	0.91412 (0.007837)

Table-5.5: Parameters estimation of GARCH-EVT Model for Non-financial data. Models are calibrated over 1990-2005 for GE, IBM and COCA-COLA and over 1996-2005 for BMW (as data prior to 1996 are not available for BMW). The numbers in parentheses represent standard errors and LL gives the Log-likelihood values of the models.

Non-Financial Assets	Upper Tail			Lower Tail		
	Threshold (u)	ξ	β	Threshold (u)	ξ	β
GE	1.1992	0.14443 (0.058346)	0.46927 (0.035881)	1.2124	-0.077657 (0.047344)	0.55007 (0.037996)
IBM	1.1347	0.2763 (0.066758)	0.46168 (0.03836)	1.126	0.10977 (0.054492)	0.56862 (0.042908)
COCA-COLA	1.1425	0.1584 (0.057123)	0.52043 (0.040109)	1.1882	0.003766 (0.050376)	0.55965 (0.040072)
BMW	1.1916	-0.061446 (0.069536)	0.66836 (0.067354)	1.2437	0.041952 (0.066971)	0.49512 (0.048401)

Table-5.6: Shows threshold values and maximum likelihood GPD parameters values for both positive and negative tails of the distribution of the non-financial data, the numbers in parentheses represent standard errors.

5.3 Backtesting Tests for Value-at-Risk and Expected Shortfall

We provide a brief overview of the tests employed for **backtesting** Value-at-Risk (VaR) and Expected Shortfall (ES); detailed descriptions are available in Chapter 4, Section 4.4**.**

5.3.1 Tests for Value-at-Risk Backtest

For Value-at-Risk (VaR**)** backtesting, we employ several statistical tests, including the Binomial (BIN) test, Traffic Light (TL) test, Proportion of Failures (POF) test, Time Until First Failure (TUFF) test, Conditional Coverage (CC) test, and Conditional Coverage Independence (CCI) test. The binomial test assesses whether the number of VaR violations conforms with VaR confidence level; a test that is based on the normal approximation of the binomial distribution. For 'N' and 'p' as number of observations and complement of

the VaR coverage level, respectively, then under the null hypothesis[16] the number of violations 'x' is statistically inferred by the test statistic[17]:

$$Zscore_{bin}=\frac{x-Np}{\sqrt{Np(1-P)}} \tag{5.9}$$

This test statistic approximately follows the normal distribution and is not reliable for either small values of 'N' or (and) small values of 'p'[18]. The test being two-sided it's $p-value$ is defined as $p\ value_{bin}=2\big(1-F(|\ Zscore_{bin}\ |)\big)<(1-\alpha)$; where 'α' is the test confidence level (often 95%) and F is the standard normal CDF.

The binomial distribution characterizes the Traffic Light test of VaR back-testing too. With the same notation of 'N', 'p' and 'x' as above, when the Binomial CDF is defined as $F(X\leq x|N,p)$, the three so called traffic light zones are characterized as , Green : $F(x|N,p)\leq 0.95$, Yellow : $0.95<F(x|N,p)\leq 0.9999$, and Red : $0.9999<F(x|N,p)$. For Basel-III with 97.5% VaR level this means that in a year with 250 days the number of VaR violations must be less or equal to 10 for the model to fall in 'Green' zone; must be between 11 and 16, inclusive, for the model to fall in 'Yellow' zone; and must be 17 or more for the model to fall in 'Red' zone.

Kupiec's PoF test rejects a model over a specified time interval for both too many and too few VaR violations over the interval; a feature that is consistent with regulatory Basel Committee's concentration too. With the notation of $p_{VaR}=1-VaR\ level=p$; the likelihood ratio test for 'PoF' test (i.e. the test statistic) is given by:

$$\begin{aligned} LR_{PoF} &= -2log\left(\frac{(1-p_{VaR})^{N-x}\ {p_{VaR}}^{x}}{\left(1-\frac{x}{N}\right)^{N-x}\left(\frac{x}{N}\right)^{x}}\right) \\ &= -2\left[(N-x)log\left(\frac{N(1-p_{VaR})}{N-x}\right)+xlog\left(\frac{Np_{VaR}}{x}\right)\right] \end{aligned} \tag{5.10}$$

[16] i.e. under the assumption that the model is correct.

[17] Commonly known as Z-score

[18] For typical uses in VaR backtesting analyses with $N = 250$ or much larger and p in the range of 1%–10% the approximation, however, gives results in line with other tests.

where 'N' and 'x' are as defined earlier. LR_{PoF} asymptotically follows Chi-square with 1-degree of freedom. So, the $p\ value_{PoF}$ is the probability that a Chi-Square variate with 1-degree of freedom, F_{Chi} ,exceeds the value LR_{PoF}. That is $p\ value_{PoF} = 1 - F_{Chi}(LR_{PoF})$. Hence the test accepts a model with LR_{PoF} if $F_{Chi}(LR_{PoF}) < F_{Chi}(\alpha)$ where 'α' is the test confidence level (often 95%). TUFF, is kind of a particular case of PoF test above corresponding to x=1, (Kupiec, TECHNIQUES FOR VERIFYING THE ACCURACY OF RISK MEASUREMENT MODELS, 1995). It measures the time the model takes for first VaR violation to occur. Thus, the test statistic for TUFF test is:

$$LR_{TUFF} = -2log\left(\frac{(1-p_{VaR})^{N-1}\, p_{VaR}{}^{1}}{\left(1-\frac{1}{N}\right)^{N-1}\left(\frac{1}{N}\right)^{1}}\right) \tag{5.11}$$

LR_{TUFF} falling below critical value from F_{Chi} , as of PoF, leads to an acceptance, otherwise a rejection. However, this test is not much efficient in identifying poor VaR models.

The conditional coverage independence test is also a likelihood ratio test first proposed by (Christoffersen P. F., Evaluating Interval Forecasts, 1988) . Over consecutive time steps the CCI test assess whether the VaR violations are independent or not. The test statistic is defined as, (Christoffersen P. F., Evaluating Interval Forecasts, 1988):

$$LR_{CCI} = -2log\left(\frac{(1-p_{uc})^{(N00+N10)}\, p_{uc}{}^{(N01+N11)}}{(1-p^{01})^{N00}p^{01}{}^{N01}\,(1-P^{11})^{N10}P^{11}{}^{N11}}\right). \tag{5.12}$$

Here $N00$=number of periods with no VaR violation followed by VaR violations; $N10$= number of periods with VaR violations followed by no VaR violations; $N01$= number of periods with no VaR violations followed by VaR violations; $N11$= number of periods with VaR violations followed by VaR violations. With these quantities the conditional probability of seeing a VaR violation at any time period given there was no VaR violation at immediately previous time period is $p^{01} = \frac{N01}{N01+N00}$; and of seeing a VaR violation at any time period given there was a VaR violation at immediately previous time period too is $p^{11} = \frac{N11}{N10+N11}$. Besides these, the unconditional probability of observing a VaR violation at any time period (i.e. probability of seeing a VaR violation irrespective of

immediately previous period being a violation or not) is $p_{uc} = \frac{N01+N11}{N01+N00+N11+N10}$. As before LR_{CCI} asymptotically follows the F_{Chi} with 1-degree of freedom. Therefore it's $p\ value_{cci} = 1 - F_{Chi}(LR_{CCI})$. Hence the test accepts a model with LR_{CCI} if $F_{Chi}(LR_{CCI}) < F_{Chi}(\alpha)$ where 'α' is the test confidence level (often 95%).

A test devoted to warn the clustering of VaR violations is the conditional coverage test. The idea is not only to examine the frequency of VaR violations but also to see if the violations occur consecutively[19]. The test statistic of conditional coverage (cc) test is given by (Christoffersen & Pelletier, 2004) :

$$LR_{CC} = LR_{PoF} + LR_{CCI}$$

which is asymptotically distributed as Chi-square with 2-degrees of freedom. So based on the details of PoF and CCI, the *p*-value of the cc test is the probability that a chi-square distribution with 2 degrees of freedom exceeds the likelihood ratio LR_{CC}

$$p\ value_{cc} = 1 - F^2_{chi}(LR_{CC})$$

Thus, a VaR model gets accepted by cc test if $F^2_{chi}(LR_{CC}) < F^2_{chi}(\alpha)$; '$\alpha'$ being the test confidence level and F^2_{chi} being the CDF of a chi-square variate with 2 degrees of freedom.

5.3.2 Tests for Expected Shortfall Backtest

For ES we use the test based on unconditional test statistic proposed by (Acerbi & Szekely, Back-testing Expected Shortfall, 2014).

$$Z_{unconditional} = \frac{1}{N(1-\alpha)} \sum_{t=1}^{N} \frac{X_t I_t}{ES_t} + 1 \qquad (5.13)$$

ES backtest uses unconditional test statistic that is sensitive to both the severity of the VaR failures relative to the ES estimate, as well as to the frequency of VaR failures. As a consequence, a rare, or very few such rare, but very large VaR failure's relative to the ES

[19] Which must be much more worrying compare to few violations occurring over a long period with good spreads. Since consecutive violations do not allow banks (companies) to overcome certain shocks by redistributing assets and/or taking putative measures.

may result in the rejection of a model over a particular timeframe. On the other hand, on a violation day when the ES estimate is also large (i.e. severity ratio is close to one, so the contribution to drive the test statistic negative is not big[20]) may not impact the test results as much as a large loss would have if a smaller ES would be encountered. Similarly, a model can also be rejected due to many VaR failures, even if all the VaR violations are just slightly higher than the VaR, because each such failures contributes in driving the test statistics negative. So, glimpses of asymmetry in number of VaR violations among different models as well as asymmetry in 'expected' and 'observed' severity (the so-called severity ratio) are both important for the ES backtest in eq.5.13.

5.4 Data

This chapter analyzes two panels of data: financial and non-financial assets. The financial panel includes four institutions (mainly Bank) — JP Morgan (JPM), Bank of America (BAC), American International Group (AIG), and Prudential Financial (PRU). For these assets, we focus on a relatively stable market period from 2010 to 2018, using 2010–2015 for in-sample calibration and 2016–2018 for out-of-sample backtesting. The non-financial panel consists of General Electric (GE), IBM, Coca-Cola, and BMW. The sample period for all assets spans from February 2, 1990, to February 18, 2022, except for BMW, which starts from November 11, 1996, due to data unavailability before that date. Each series is divided into two segments: 1990–2005 (15 years) is used for in-sample parameter estimation, and 2006–2022 (18 years) is reserved for out-of-sample forecasting evaluation. Tables 5.7 and 5.8 summarize the descriptive statistics of daily return series that demonstrate the characteristic of stylized facts: positive average return, skewness, excess kurtosis, volatility behaviour, and distributional features. These stylized facts indicate

[20] This ES backtest starts rejecting models after the test statistic falls below a certain negative value depending on test level. So only models underestimating risk can get rejected. This ES backtest doesn't penalize for overestimation; e.g. if there is no VaR violation at all the test statistic gets a value of 'one' and the model readily passes the ES test. This is a big concern for GARCH-EVT model which has a tendency of overestimating the risk.

deviations of normal distributions and the existence of time-varying risk, facilitating the application of volatility models such as GARCH, Conditional GARCH model.

Financial Assets	Observations	mean	St. dev.	Skewness	Kurtosis	Minimum	Maximum
JPM	2263	0.0375	1.6500	-0.2799	7.6546	-10.4472	8.5607
BAC	2263	0.0211	2.1866	0.2200	17.4605	-19.2406	26.5022
AIG	2263	0.0105	2.0444	0.1962	11.9822	-15.8791	17.7378
PRU	2263	0.0212	1.6973	-0.5414	6.2434	-10.9882	7.2143

Table-5.7: shows the Summary statistics of the financial assets, mainly banks JP Morgan (JPM), Bank of America (BAC), Prudential Capital (PRU) and American Investment Group (AIG) returns series during the period January 4, 2010 to December 31, 2018.

Non-financial Assets	Observations	mean	St. dev	Skewness	Kurtosis	Minimum	Maximum
GE	8097	0.0107	2.0062	0.2172	13.6458	-17.5846	20.4264
IBM	8097	0.0211	1.7273	-0.1369	10.1652	-15.5302	15.0145
COCA -COLA	8097	0.0316	1.3554	-0.2990	10.9934	-15.8807	11.0091
BMW	6479	0.0257	2.2628	-0.2943	8.9597	-18.9828	16.5485

Table-5.8: shows the Summary statistics of the non-financial assets returns series during the period January 2, 1990 to February18, 2022 except BMW whose sample period is November 11, 1996 to February 18, 2022 as data prior to 1996 are not available.

5.5 Empirical Results with Discussion

The main empirical analysis is conducted for financial sector assets only, mainly banks. For this part we select relatively crisis free period of the market i.e. we use 2010 to 2018 data; using six years (2010-15) for in-sample calibrations and three years (2016-18) for out-sample back-testing. The banks we choose are JP Morgan (JPM), Bank of America (BAC), Prudential Capital (PRU) and American Investment Group (AIG). Though analytic implementation is the key, simulation studies can also be carried out to get ideas of

robustness and back-testing statistics for Conditional Analytic Markov (CAM) approach; for both single and multi-period risk forecasts. Square root rule for multi-period risk forecasts[21] are compared with simulation based multi-period forecasts from GARCH-EVT model and with analytic multi-period forecasts from Conditional Analytic Markov (CAM) approach.

For our robustness analysis we, however, consider a more extended sample and consider different assets, this time from non-financial sector. We select General Electronics (GE), Cola-Cola, IBM and BMW data over the period of 1990 to 2022; using 15 years for in-sample calibration and rest of the 18 years for out-sample back-testing, both VaR and ES and for both single and multi-period cases. The robustness study includes carrying out similar investigations as in the main calibrations, but over different periods and for different assets. Our robustness study includes an out-sample period that includes both 2008 subprime crisis as well as 2019 COVID crisis for out-sample back-testing; whereas our primary study includes periods free of big swings in the markets for both in sample calibrations as well as out sample back-testing.

The primary calibrations and back-testing; the robustness calibrations and back-testing, as well as simulations studies, clearly prefer proposed Conditional Analytic Markov (CAM) approach for risk forecast over GARCH-EVT and BS model[22]; for both single and multi-period cases. So, the attraction of implementing a model without any optimization or numerical search required, which offers analytic estimation of VaR and ES on varying background of assets current level that can revert back to any level[23], will certainly appeal to investors at grassroots level in practicing risk management on their own.

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[21] 2 days, 5 days and 10 days cases.

[22] Surprisingly over a long 18 years of back-testing, BS model occasionally yields forecasts that are better than the GARCH-EVT forecasts.

[23] Nonetheless when there is no discernible mean reversion in assets evolution the relative out performance of CAM approach could be suppressed by methods as is presented at the end of the paper using long Microsoft data.

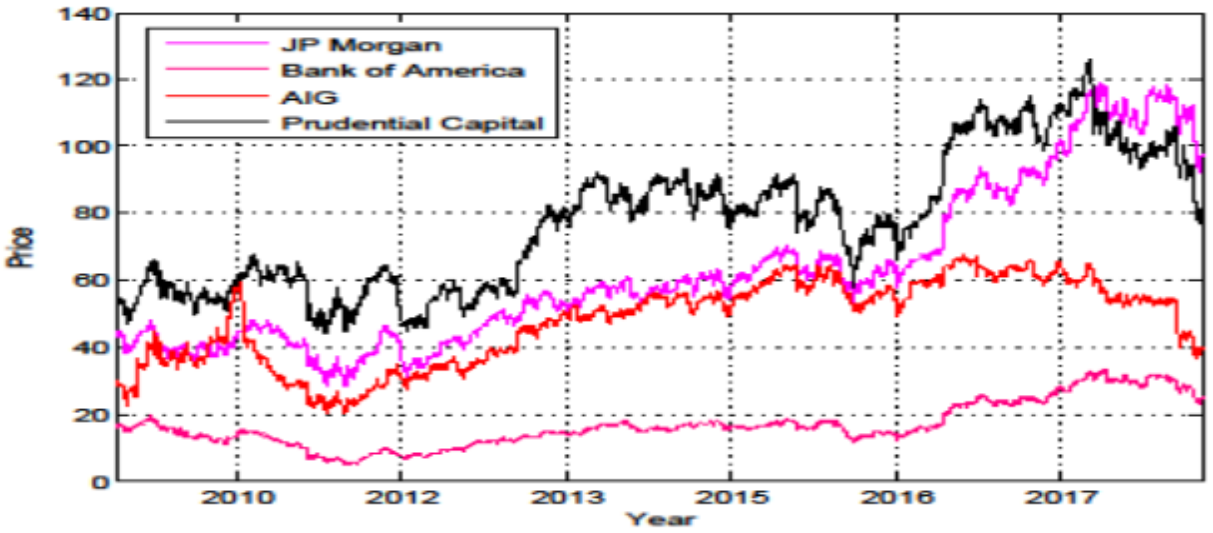


Figure-5.1: The graph shows the price levels of financial assets of JP Morgan, Bank of America, American Investment Group (AIG) and Prudential Capital used in main empirical analysis over 2010-18.



Figure-5.2: The graph shows the price levels of non-financial assets of BMW, IBM, General Electronics (GE) and Coca-Cola used in robustness study over 1990-2022.

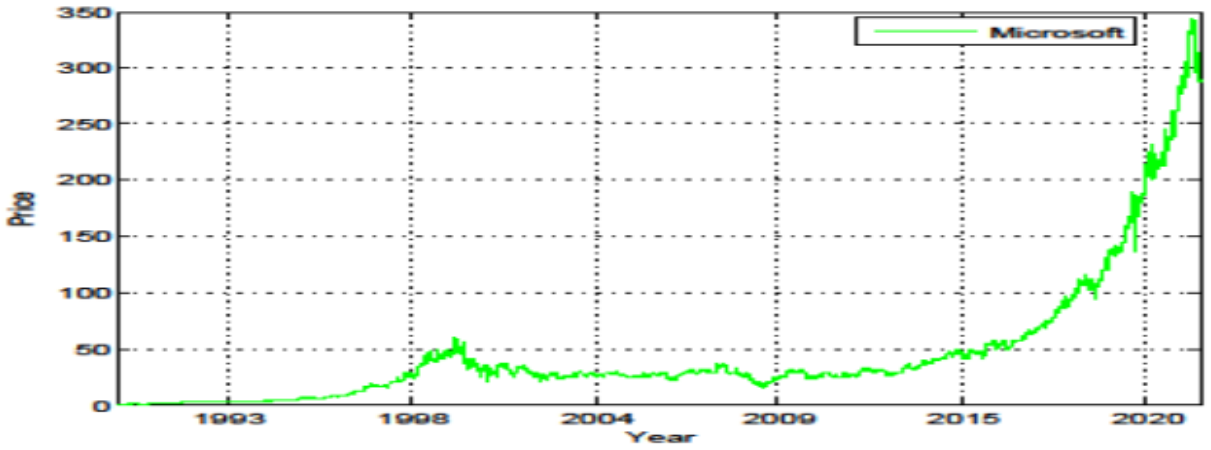


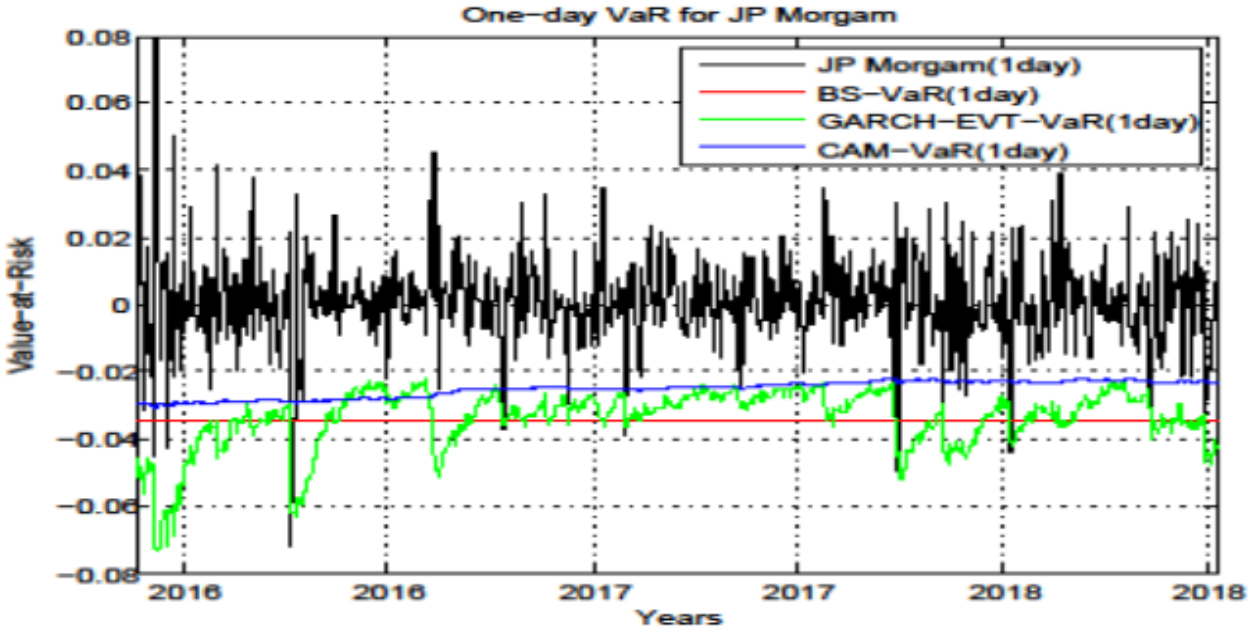
Figure-5.3: The graph shows the price level of Microsoft over 1990-2022 that has been used to study the performance of CAM method when there is no discernible mean revision in the data.

In Figure-5.1 the graph shows the price levels of financial assets of JP Morgan, Bank of America, American Investment Group (AIG) and Prudential Capital used in main empirical analysis over 2010-18, whereas Figure-5.2 shows the price levels of non-financial assets of BMW, IBM, General Electronics (GE) and Coca-Cola used in robustness study over 1990-2022. Figure-5.3 shows the downside of CAM approach when there is clearly no evidence of mean reversion in the data, here the Microsoft share prices. As also this figure shows over 2012-2022 Microsoft share prices are predominantly increasing, i.e. it lacks evidence of mean reversion. That means for a Markovian model with volatility $\sigma_i^{cam} = \frac{\hat{\tilde{\sigma}}}{\sqrt{S_i}}$ the increasing series of $\{S_i\}$ leads to decreasing volatilities to be used in VaR and ES forecasts over 2012-2022. Moreover, since CAM approach is not based on volatility background, historical or near past volatilities has nothing to do with future volatility forecasts. As a result, Markovian CAM-VaR and CAM-ES are not affected (even when the non-Markovian estimates of return volatilities vary considerably), as long as the current asset levels predominantly keep increasing[24] resulting future Markovian volatilities keep

[24] Clearly exhibiting the evidence of lack of mean reversion.

decreasing[25]. Figures 5.4–5.11 illustrate that the combination of time-varying volatility and heavy-tailed distributions significantly enhances the precision of both VaR and ES predictions. The findings demonstrate that the GARCH–EVT model delivers superior protection against extreme losses by more effectively capturing tail risk, while the CAM model presents a reliable alternative with competitive forecasting capabilities. The findings underscore the shortcomings of the Black–Scholes model and advocate for the implementation of dynamic volatility models to achieve more accurate market risk. Figures **5.12–5.39** present the out-of-sample performance of three competing risk models whereas the dynamic volatility models outperform the BS model. The BS model generally produces the largest number of VaR exceedances and higher ES severity ratios because its assumption of constant volatility prevents it from adapting to changes in market conditions. Consequently, it tends to underestimate downside risk, particularly during periods of elevated market volatility. The GARCH–EVT model consistently records the fewest VaR failures and the lowest average ES severity ratios across almost all financial assets and forecast horizons. By combining time-varying conditional volatility with Extreme Value Theory, the model captures both volatility clustering and heavy-tailed return distributions, leading to more accurate estimation of tail losses. The CAM model also demonstrates strong forecasting performance. Although it occasionally exhibits slightly more VaR violations than the GARCH–EVT model, it consistently outperforms the Black–Scholes approach across all assets. The results also demonstrate that parametric multi-period risk forecasting is superior to the square-root- rule, particularly at longer investment horizons. These findings support the use of GARCH–EVT as the preferred model for tail-risk estimation, while CAM serves as a reliable alternative that balances forecast stability and responsiveness, making both models more suitable than the traditional Black–Scholes framework for financial risk management and regulatory capital assessment.

[25] Falling volatilities yielding rise to price levels, so called leverage effects, seems to be secondary when evidence of mean reversions lacks significantly.



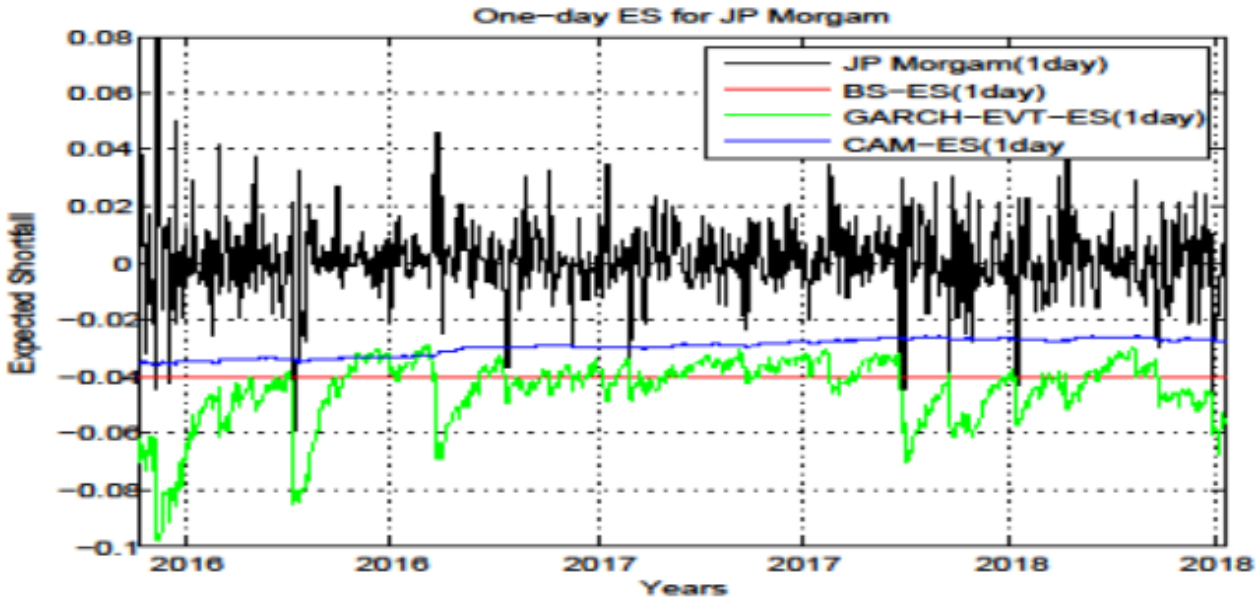


Figure-5.4 Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) back-testing outcomes of the financial assets **JPM** with a **1-day horizon** for the **years 2016–2018** using the different models. Models are calibrated over 2010-15.

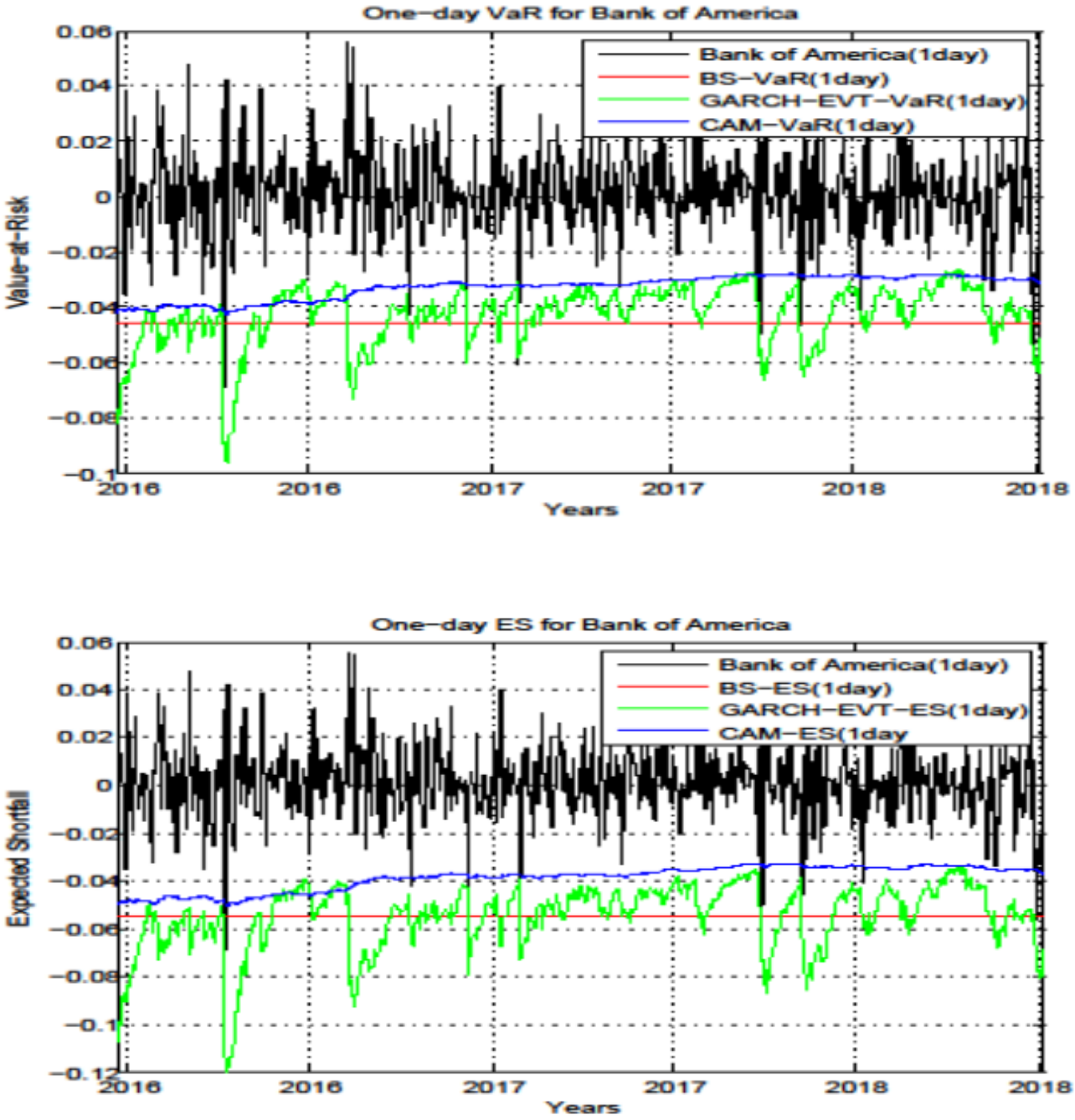


Figure-5.5: Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) back-testing outcomes of the financial assets **Bank of America (BAC)** with a **1-day horizon** for the year 2016-18 using three different models. Models are calibrated over 2010-15.

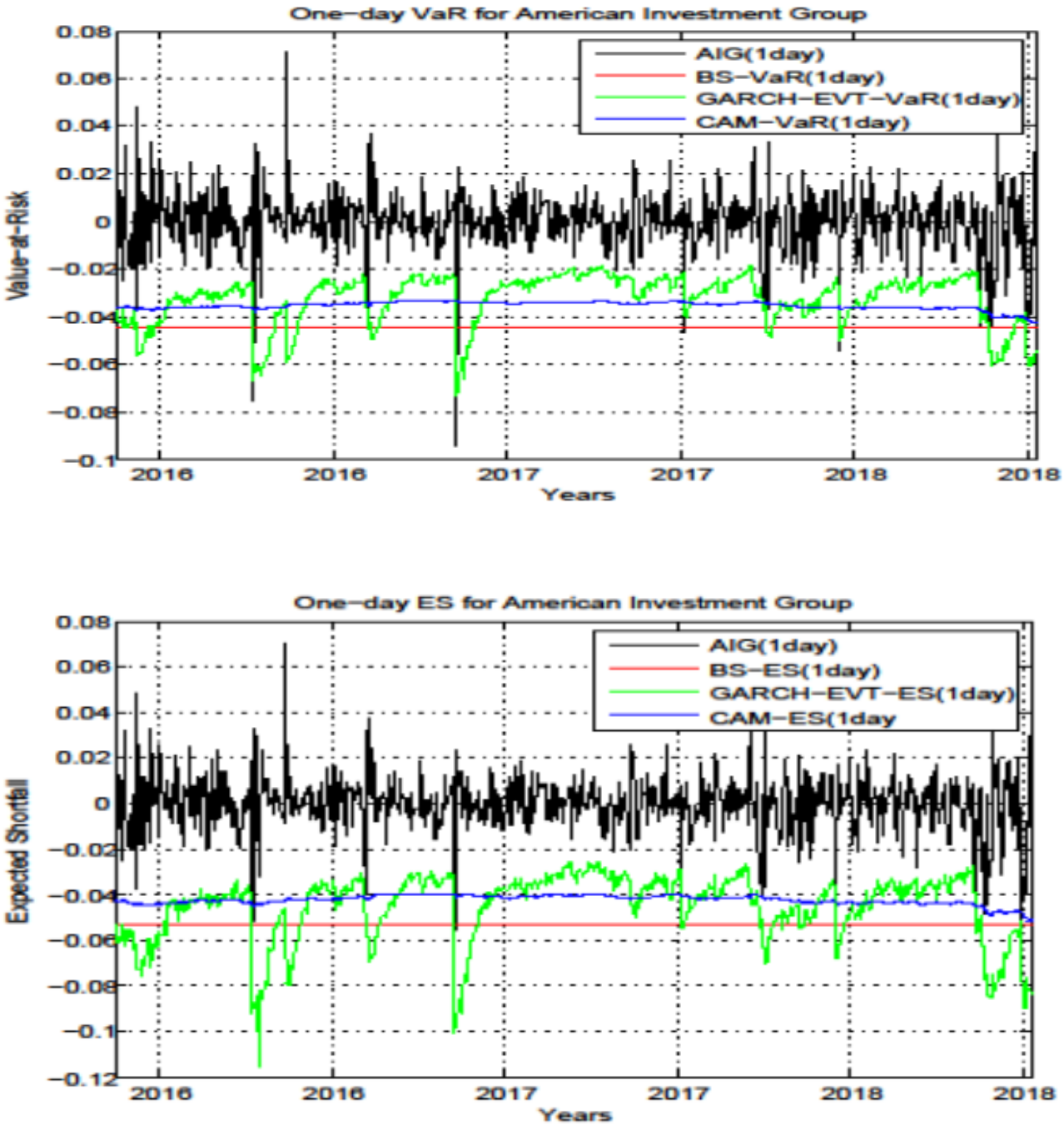


Figure-5.6: Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) back-testing outcomes of the financial assets **American Investment Group (AIG)** with a **1-day horizon** for the year 2016-18 using three different models. Models are calibrated over 2010-15.

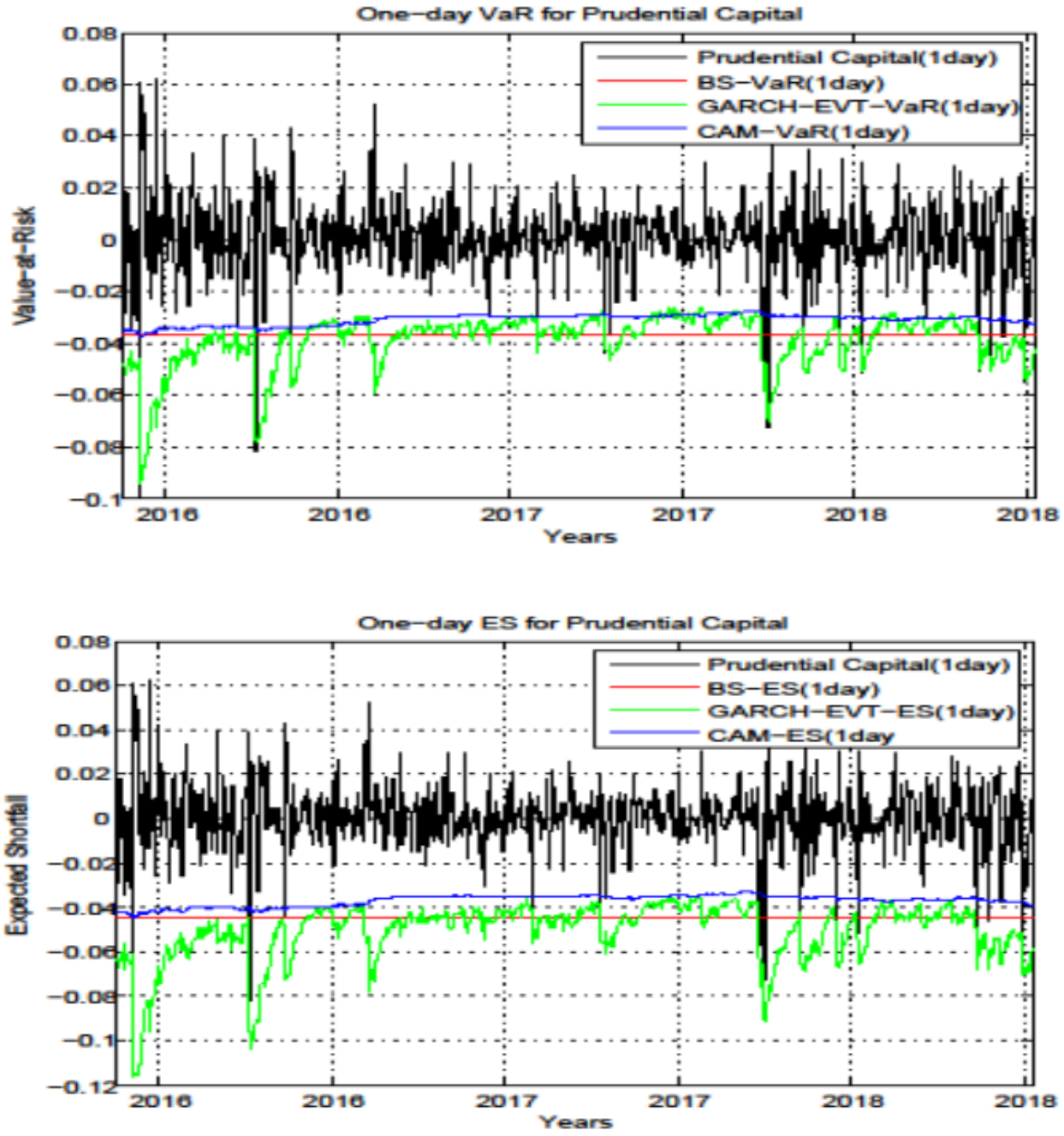


Figure-5.7: Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) back-testing outcomes of the financial assets **Prudential Capital (PU)** with a **1-day horizon** for the year 2016-18 using three different models. Models are calibrated over 2010-15.

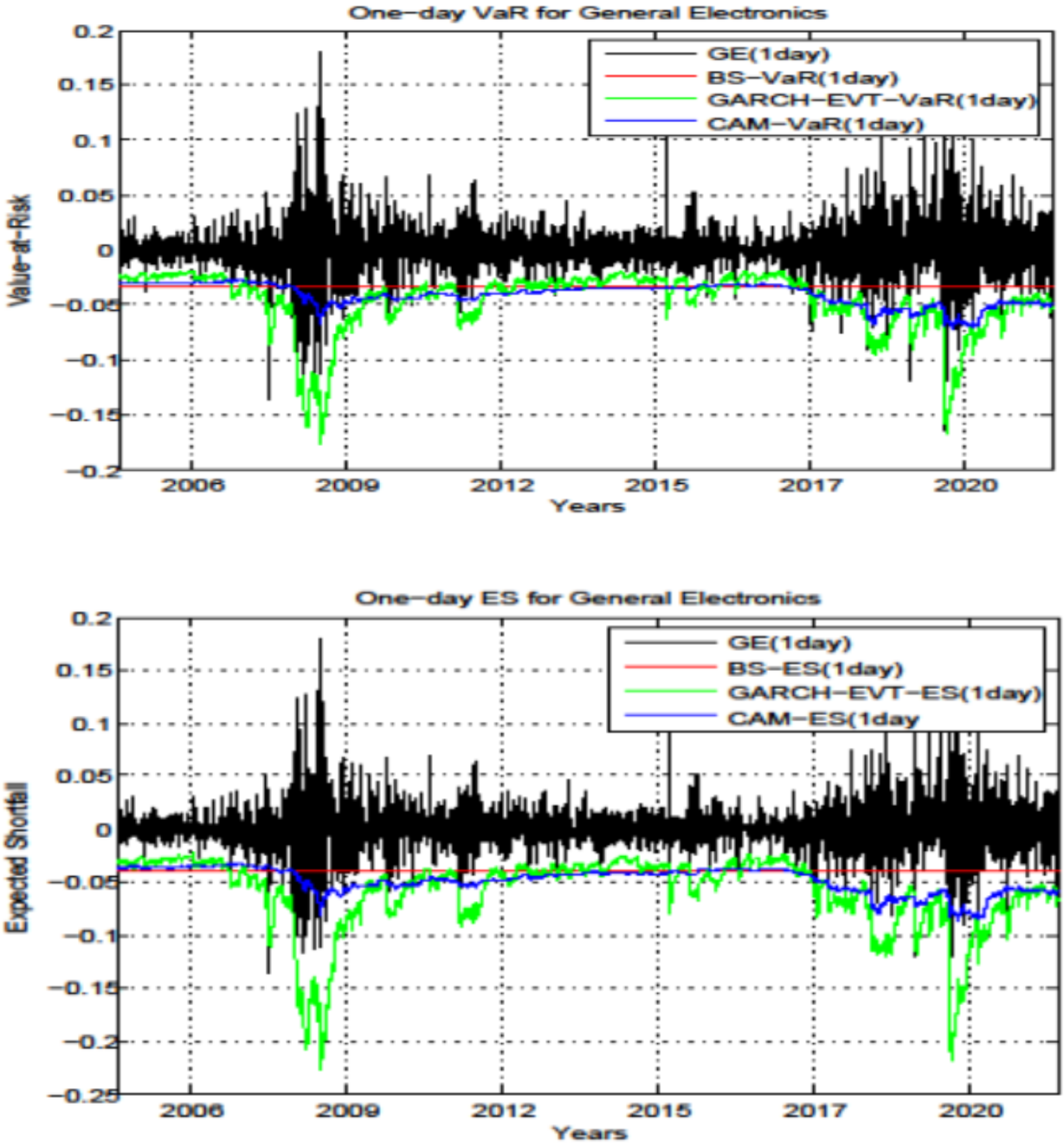


Figure-5.8: Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) used in robustness studies for backtesting of the assets **General Electronic (GE)** with **1-day horizon** for the year 2006-2022 using three different models. Models are calibrated over 1990-2005.

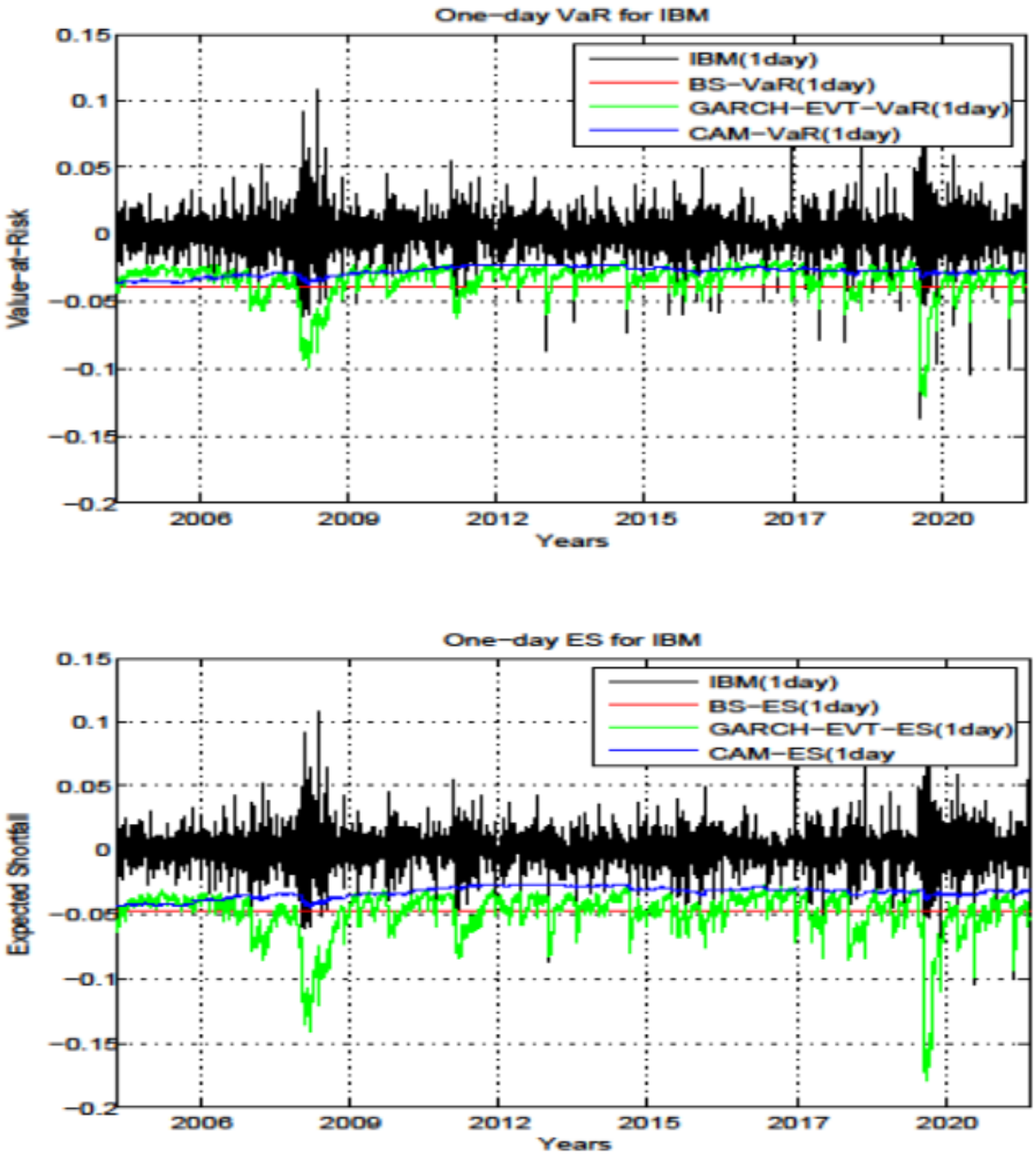


Figure-5.9: Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) used in robustness studies for backtesting of the assets **IBM** with **1-day horizon** for the year 2006-2022 using three different models. Models are calibrated over 1990-2005.

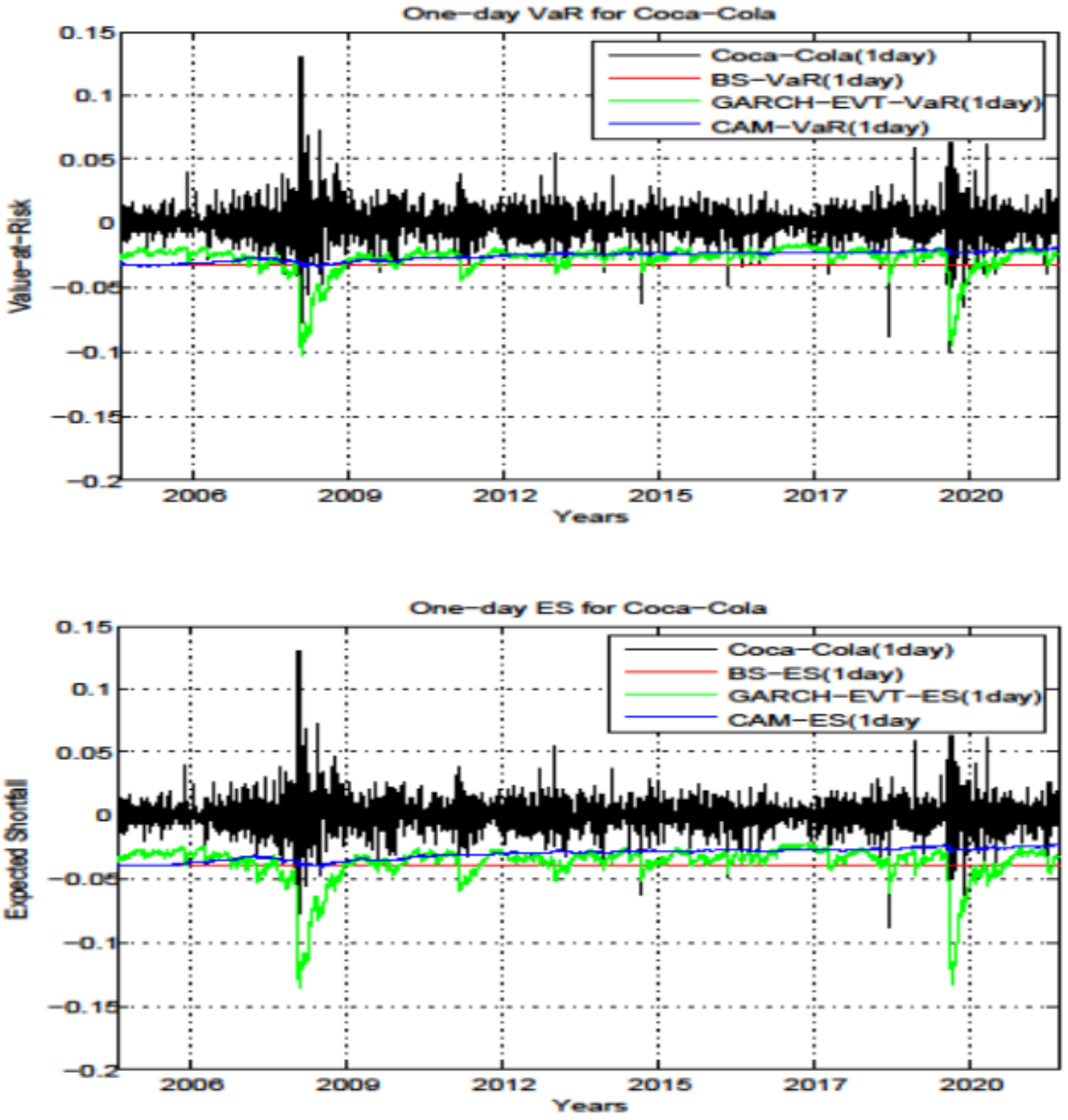


Figure-5.10: Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) used in robustness studies for backtesting of the assets **Coca-Cola** with **1-day horizon** for the year 2006-2022 using three different models. Models are calibrated over 1990-2005.

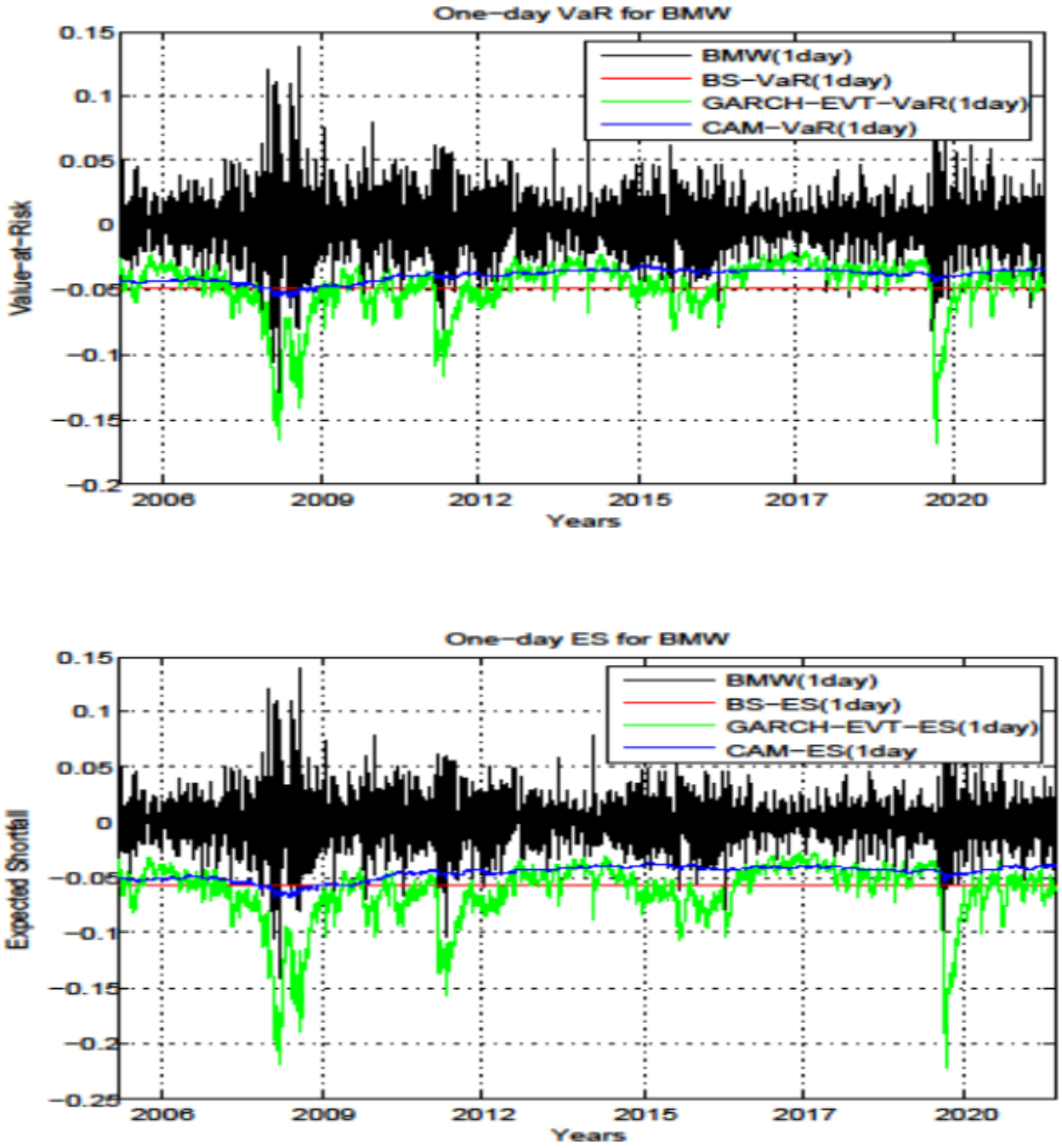


Figure-5.11: Out-of-sample 97.5% VaR (top) and 97.5% ES (bottom) used in robustness studies for backtesting of the assets **BMW** with **1-day horizon** for the year 2006-2022 using three different models. Models are calibrated over 1990-2005.

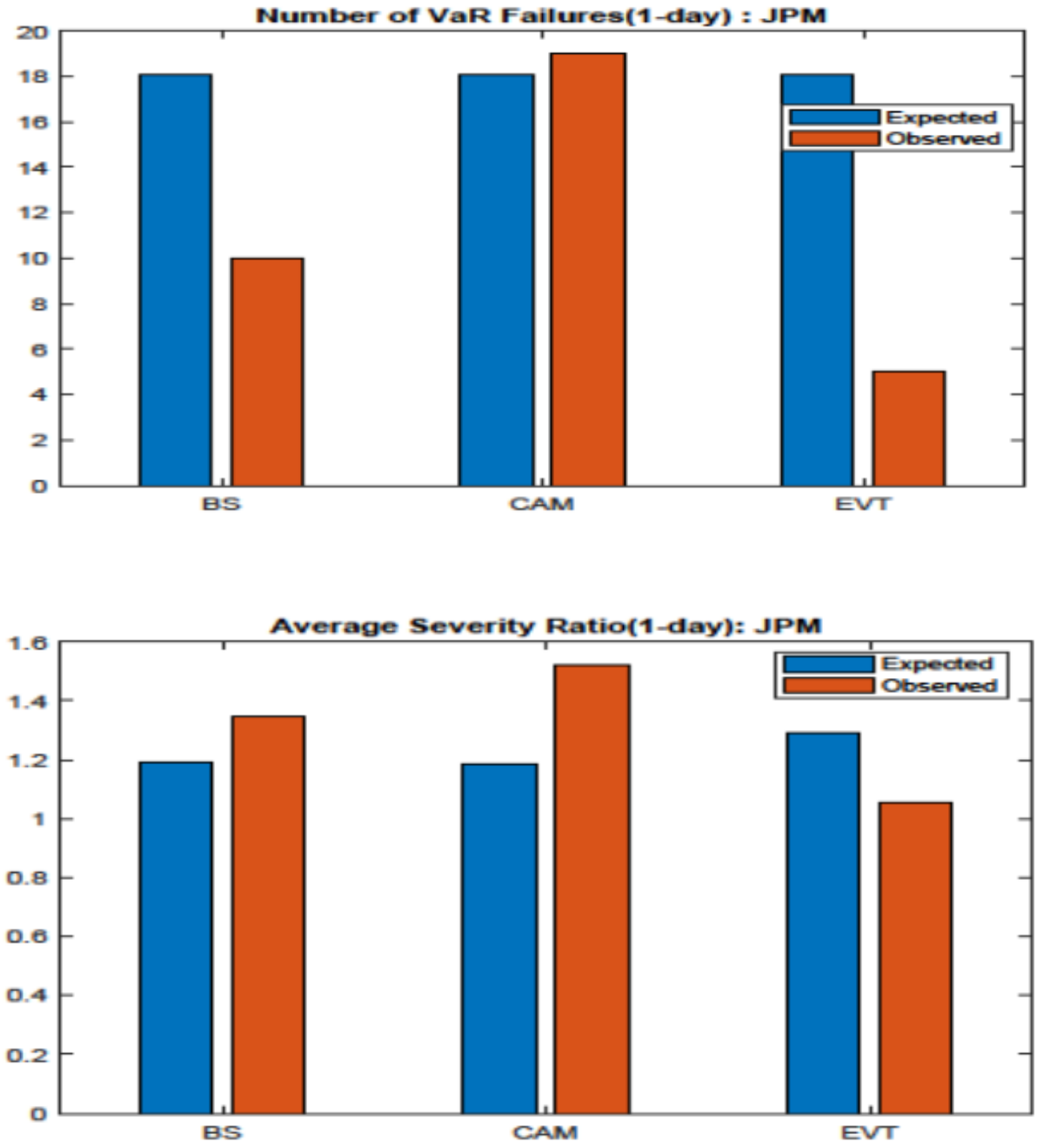


Figure-5.12: Number of **VaR (97.5%) failures (top)** and **average severity ratio of ES (bottom)** of financial assets **JP Morgan (JPM)**, 1-day horizon for the year (2016-2018) using three different Models.

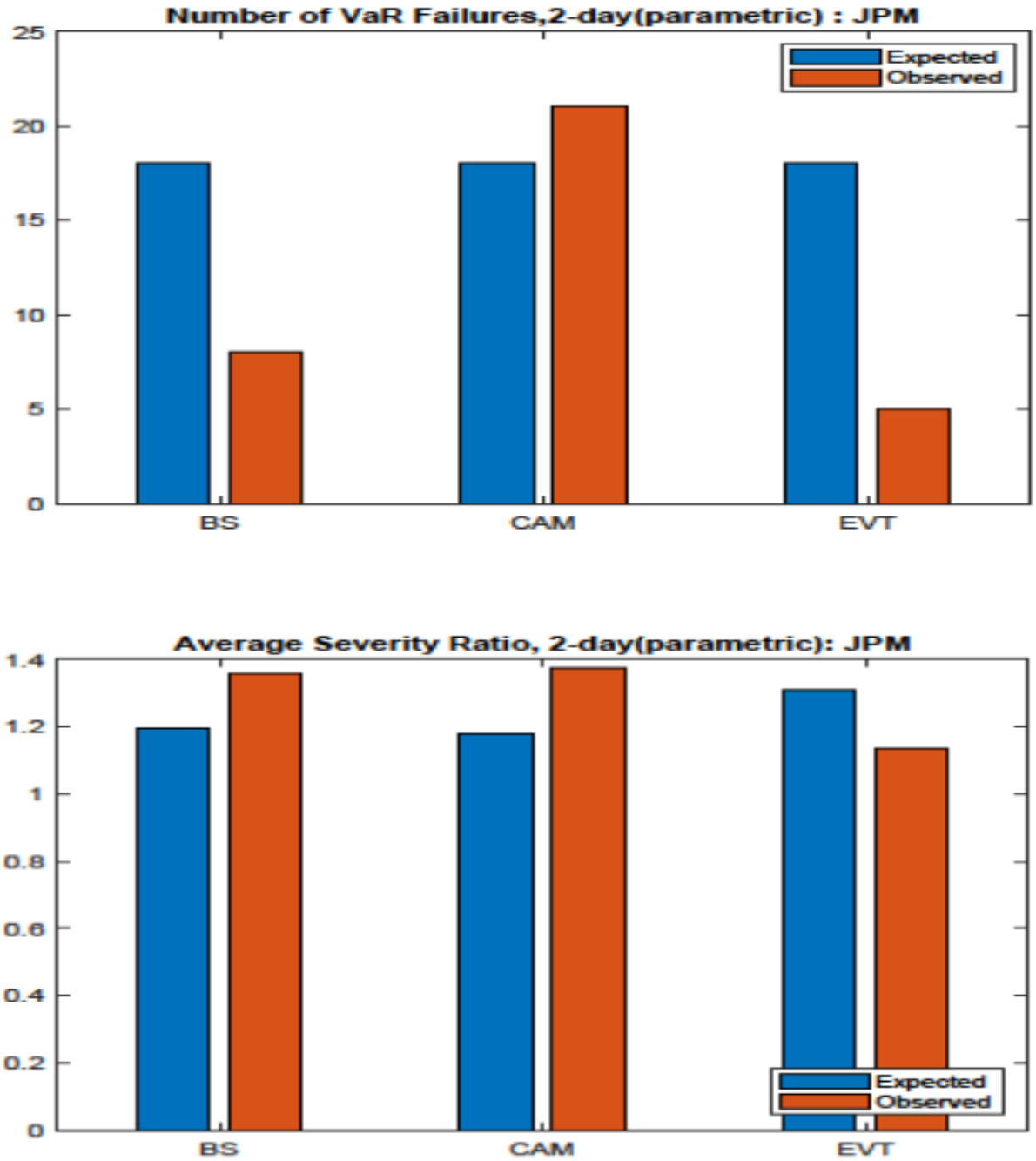


Figure-5.13: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets **JP Morgan (JPM), 2-days horizon** for the year (2016-2018) under **parametric multi-period characterization** using three different Models.

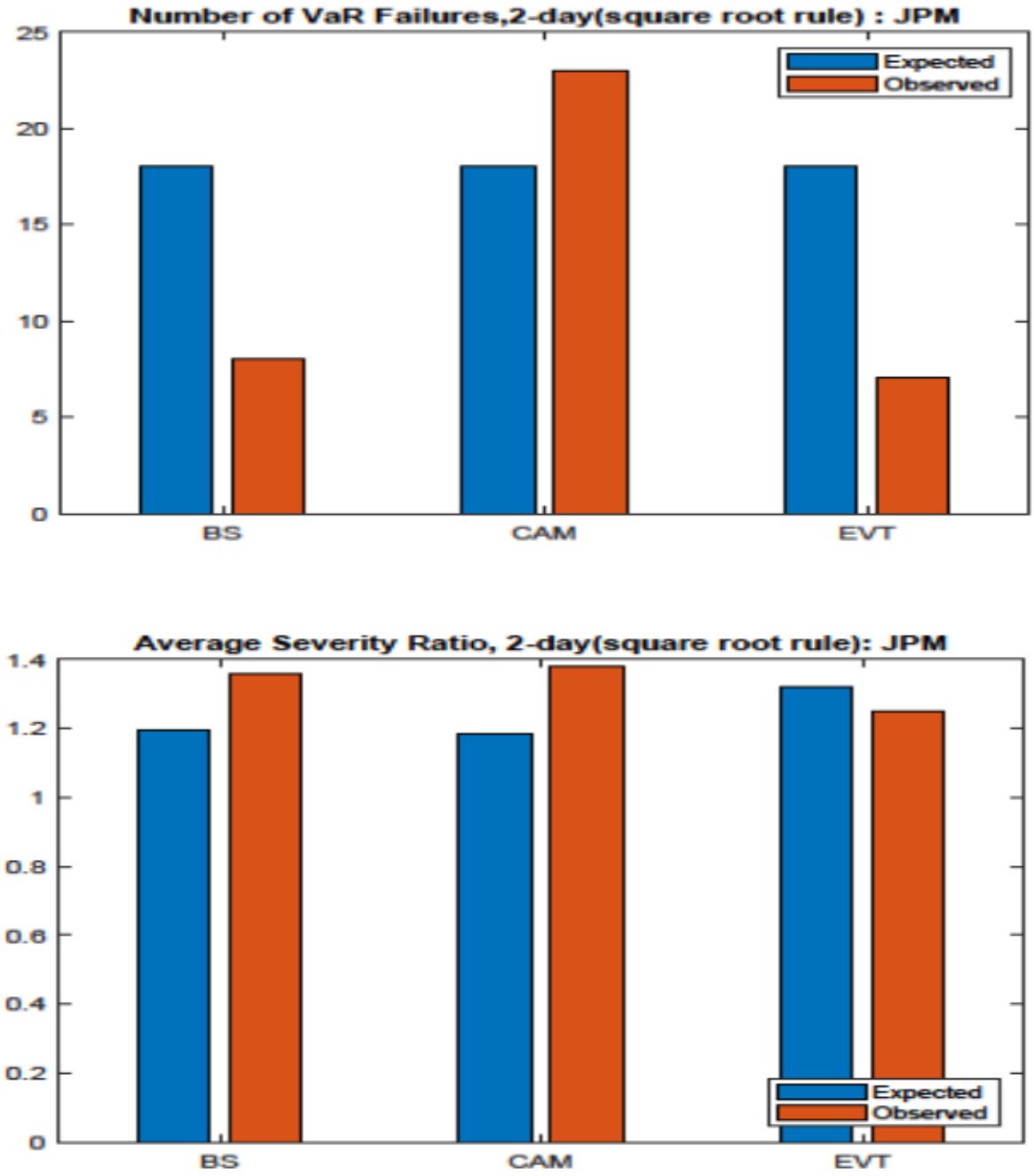


Figure-5.14: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets JP Morgan (JPM), **2-days horizon** for the year **(2016-2018)** under **square-root-rule** multi-period characterization using three different Models.

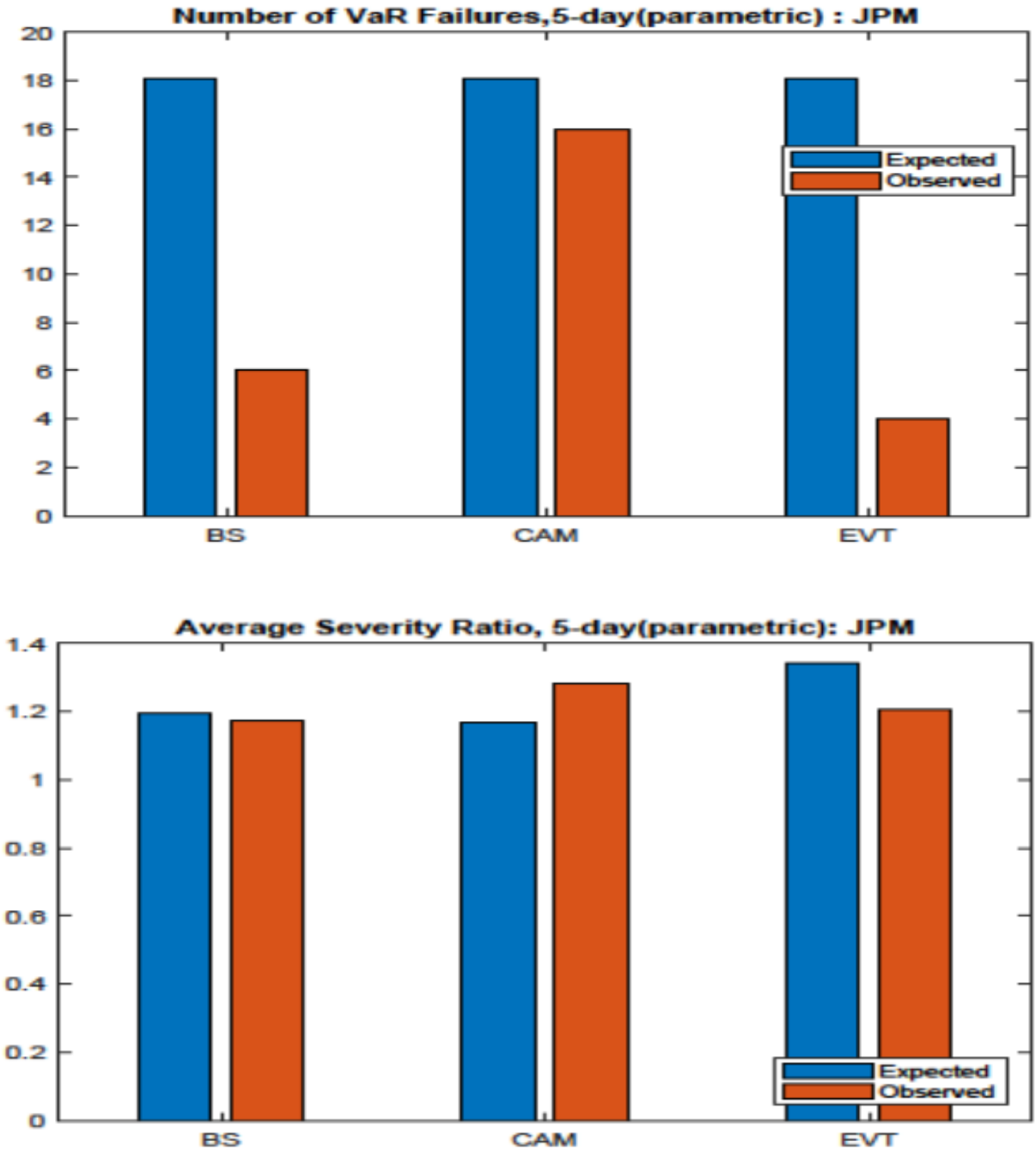


Figure-5.15: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets JP Morgan (JPM), **5-days horizon** for the year **(2016-2018)** under **parametric** multi-period characterization using three different Models.

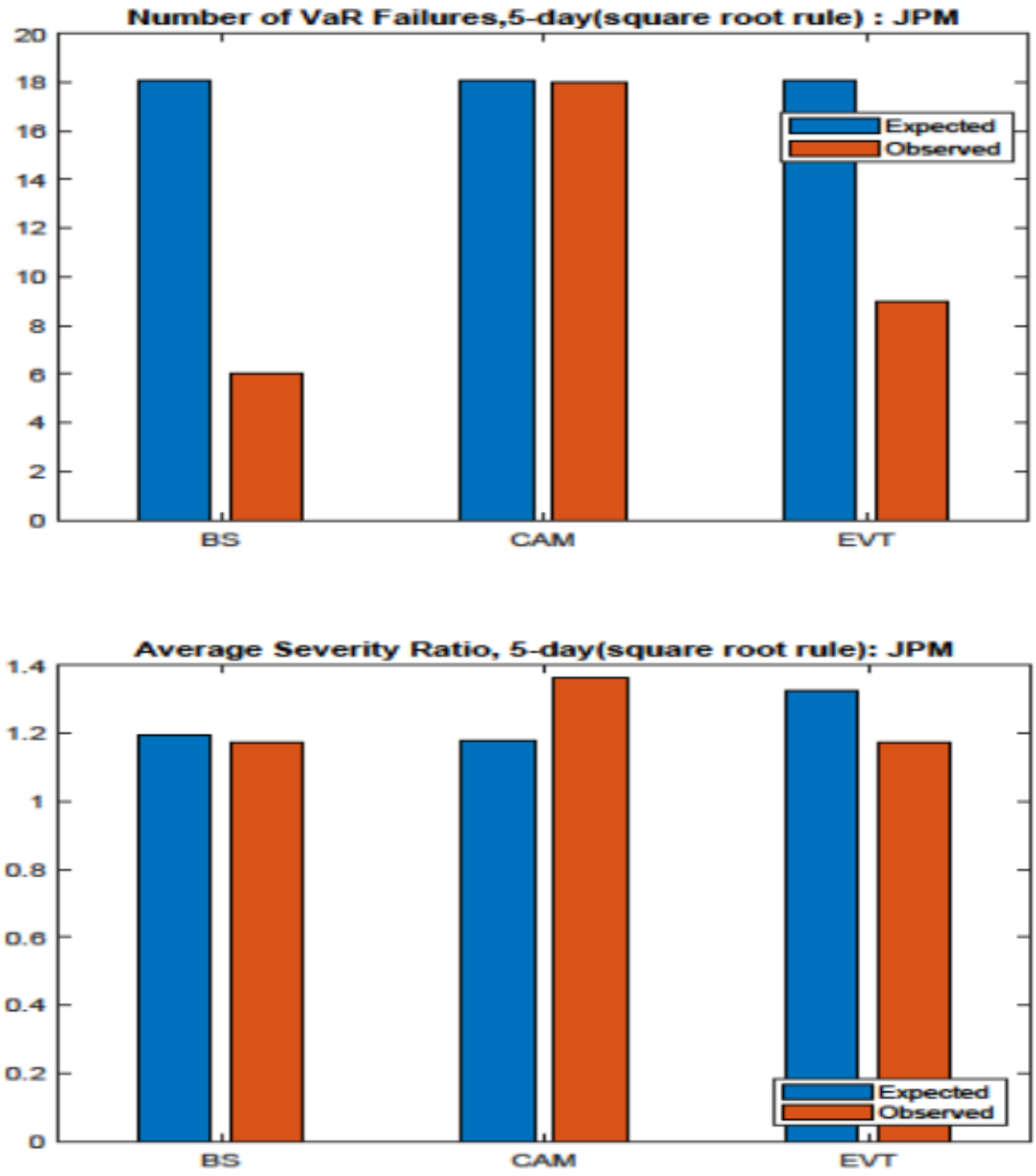


Figure-5.16: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets JP Morgan (JPM), **5-days horizon** for the year **(2016-2018)** under **square-root-rule** multi-period characterization using three different Models.

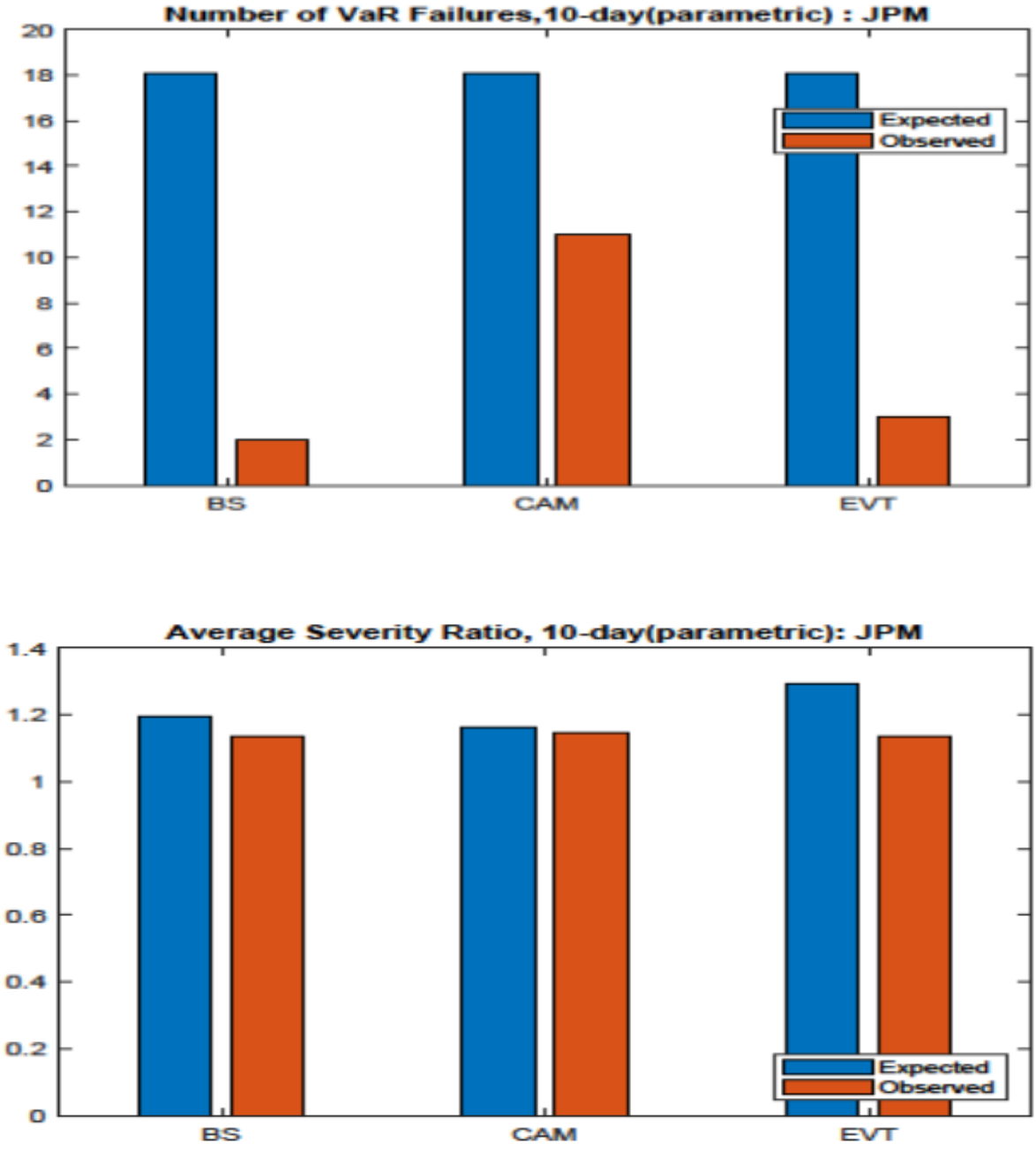


Figure-5.17: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets JP Morgan (JPM), **10-days horizon** for the year **(2016-2018)** under **parametric** multi-period characterization using three different Models.

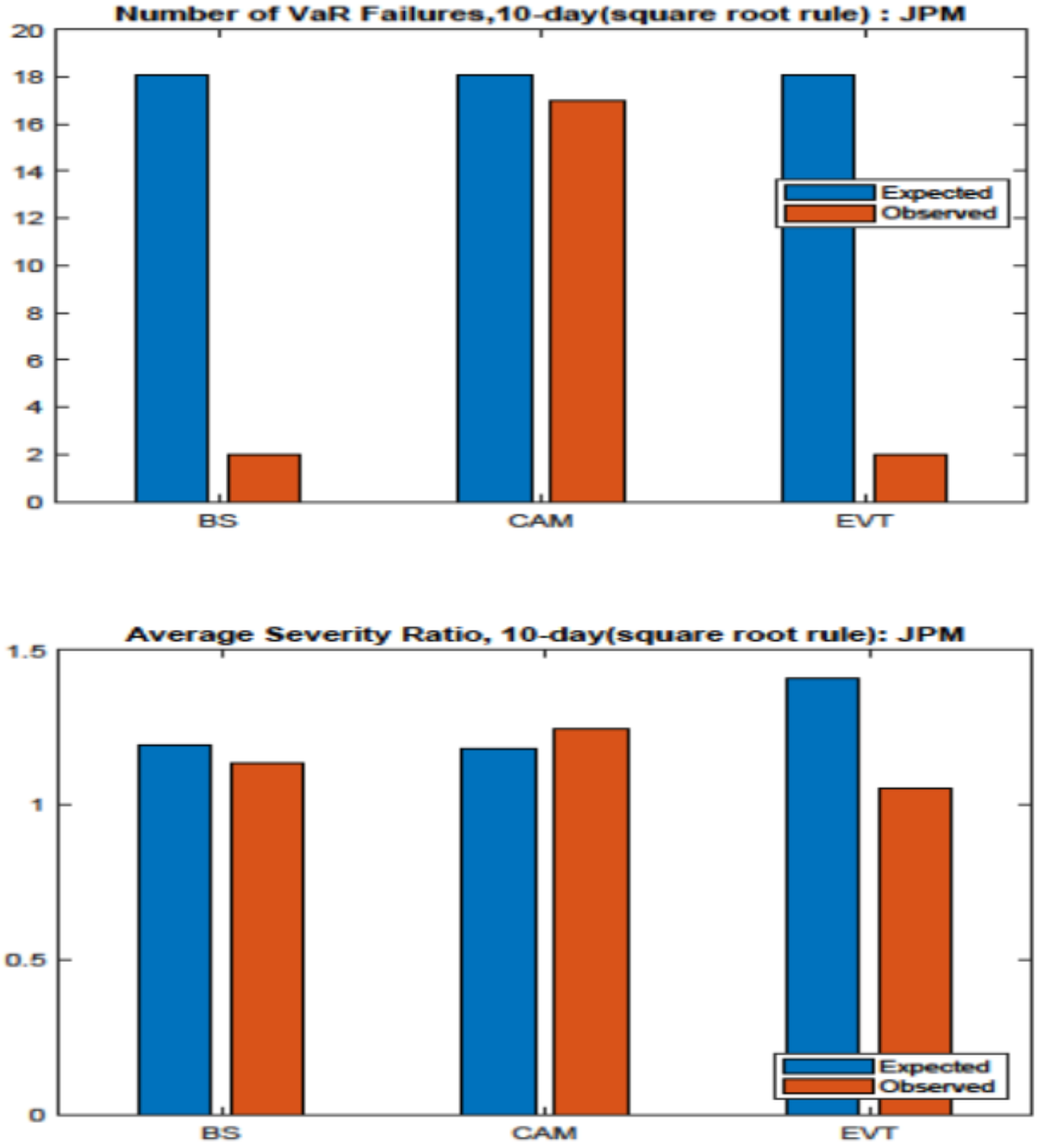


Figure-5.18: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets JP Morgan (JPM), **10-days horizon** for the year **(2016-2018**) under **square-root-rule** multi-period characterization using three different Models.

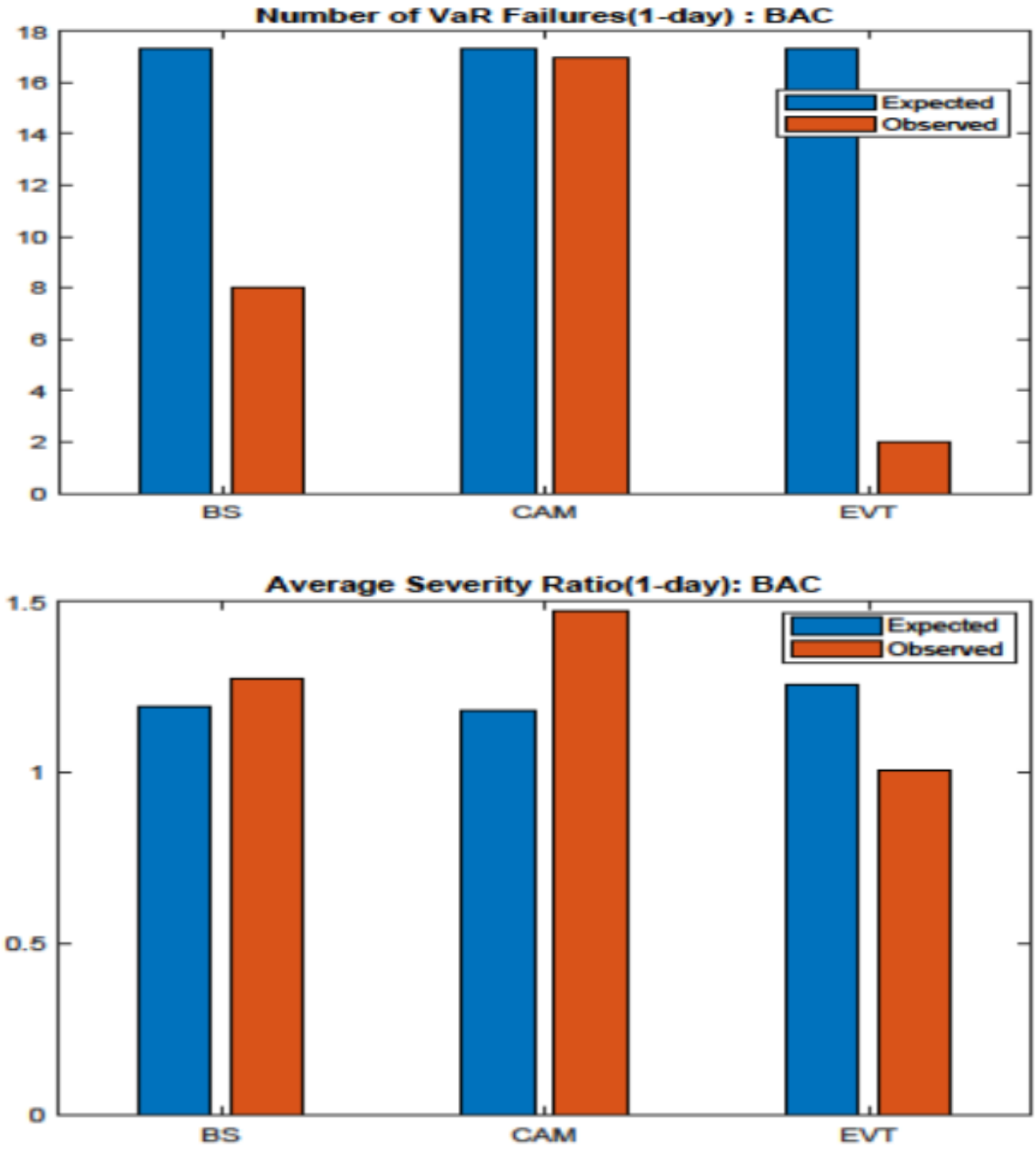


Figure-5.19: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Bank of America (BAC), **1-day horizon** for the year **(2016-2018**) using three different Models.

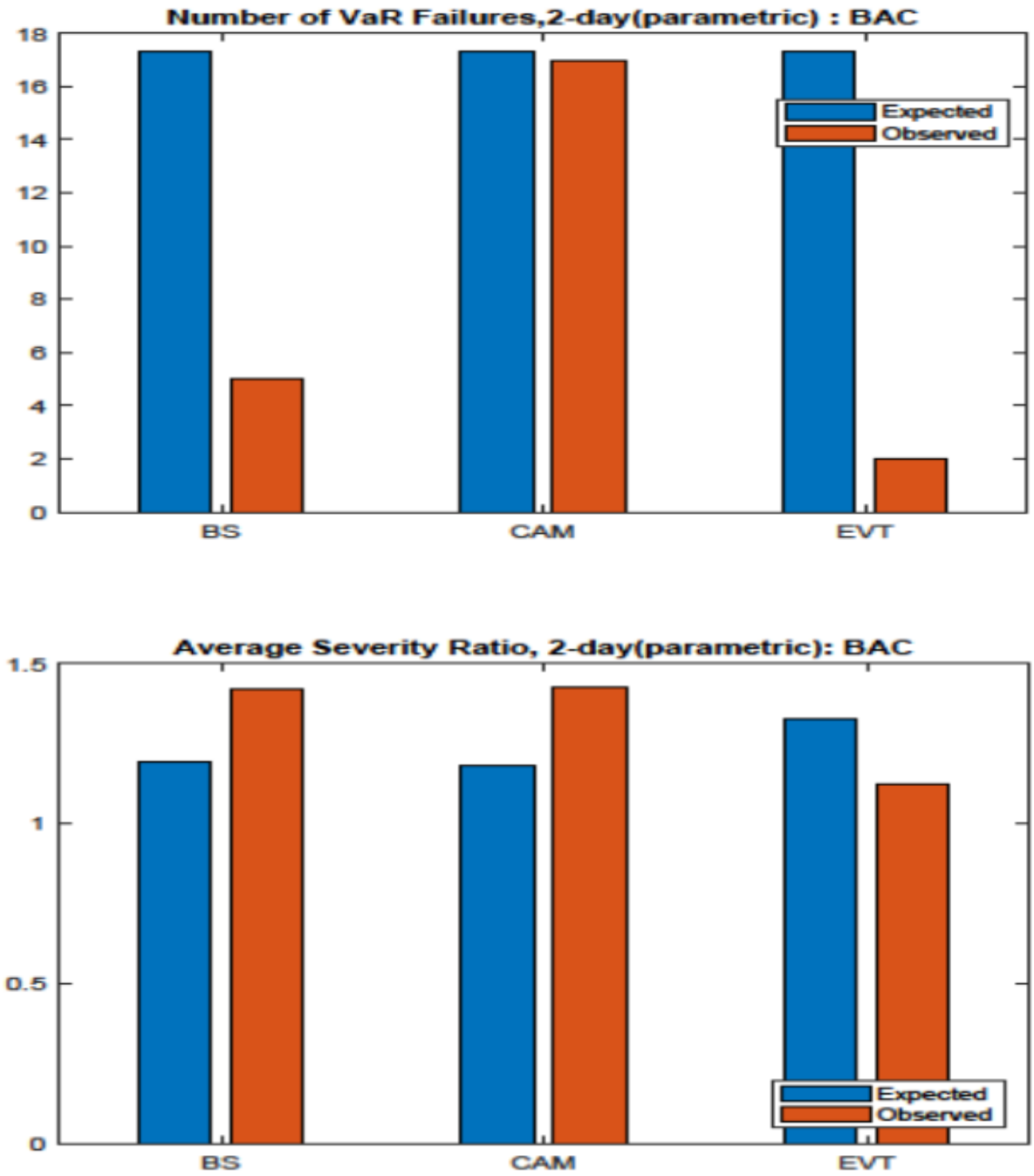


Figure-5.20: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Bank of America (BAC), **2-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

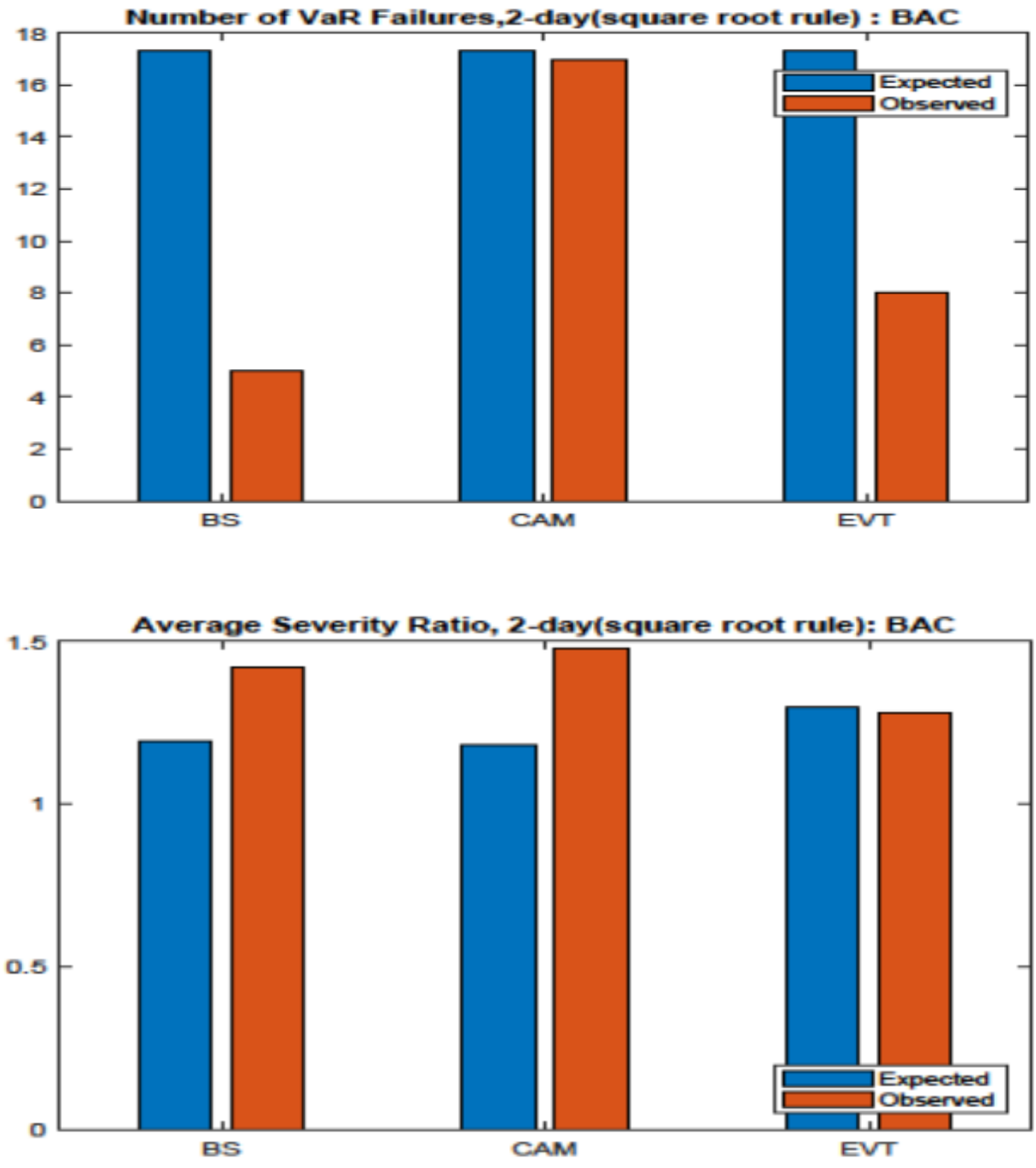


Figure-5.21: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Bank of America (BAC), **2-days horizon** for the year **(2016-2018**) under **square-root-rule** multi-period characterization using three different Models.

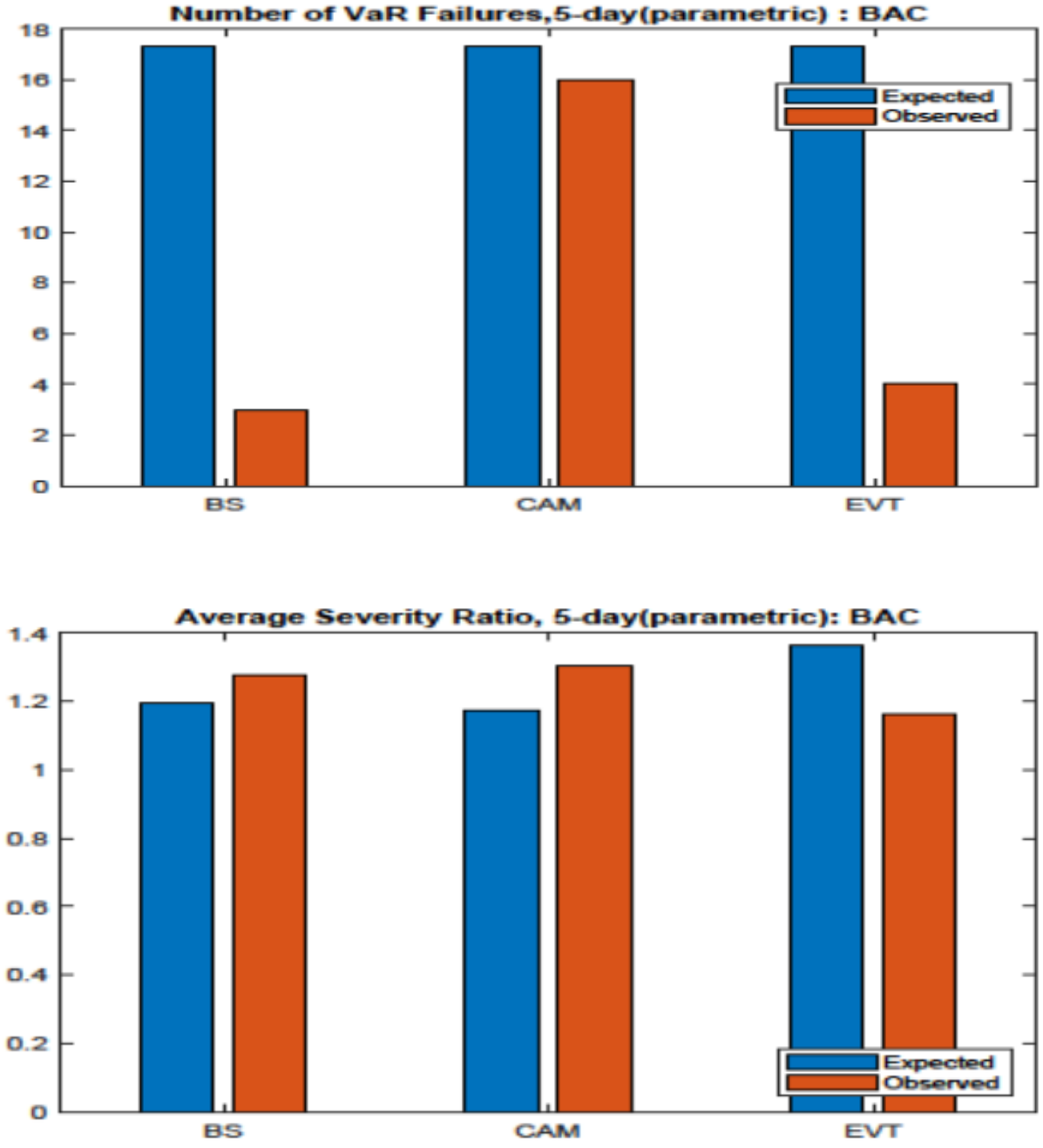


Figure-5.22: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Bank of America (BAC), **5-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

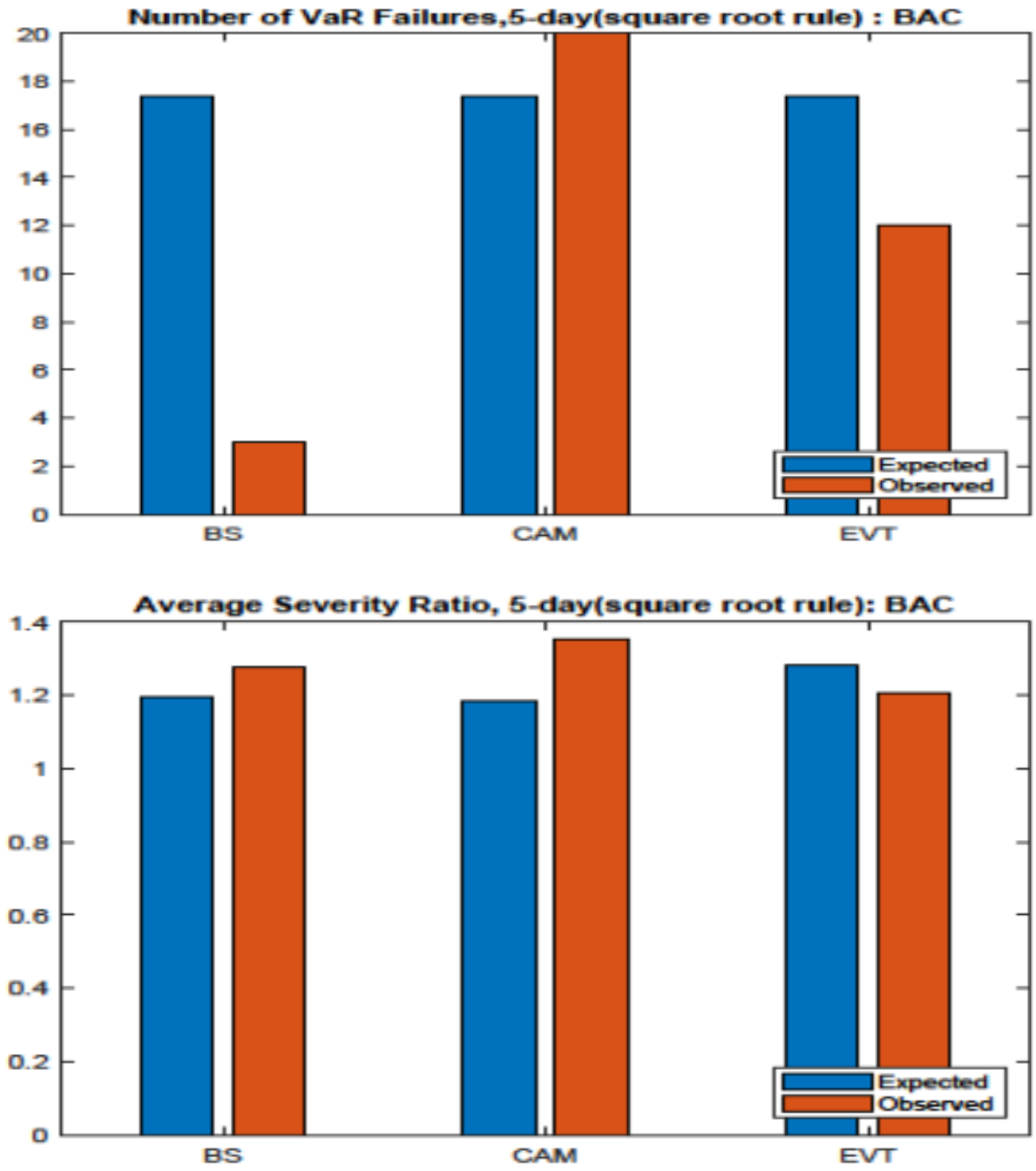


Figure-5.23: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Bank of America (BAC), **5-days horizon** for the year **(2016-2018**) under **square-root-rule** multi-period characterization using three different Models.

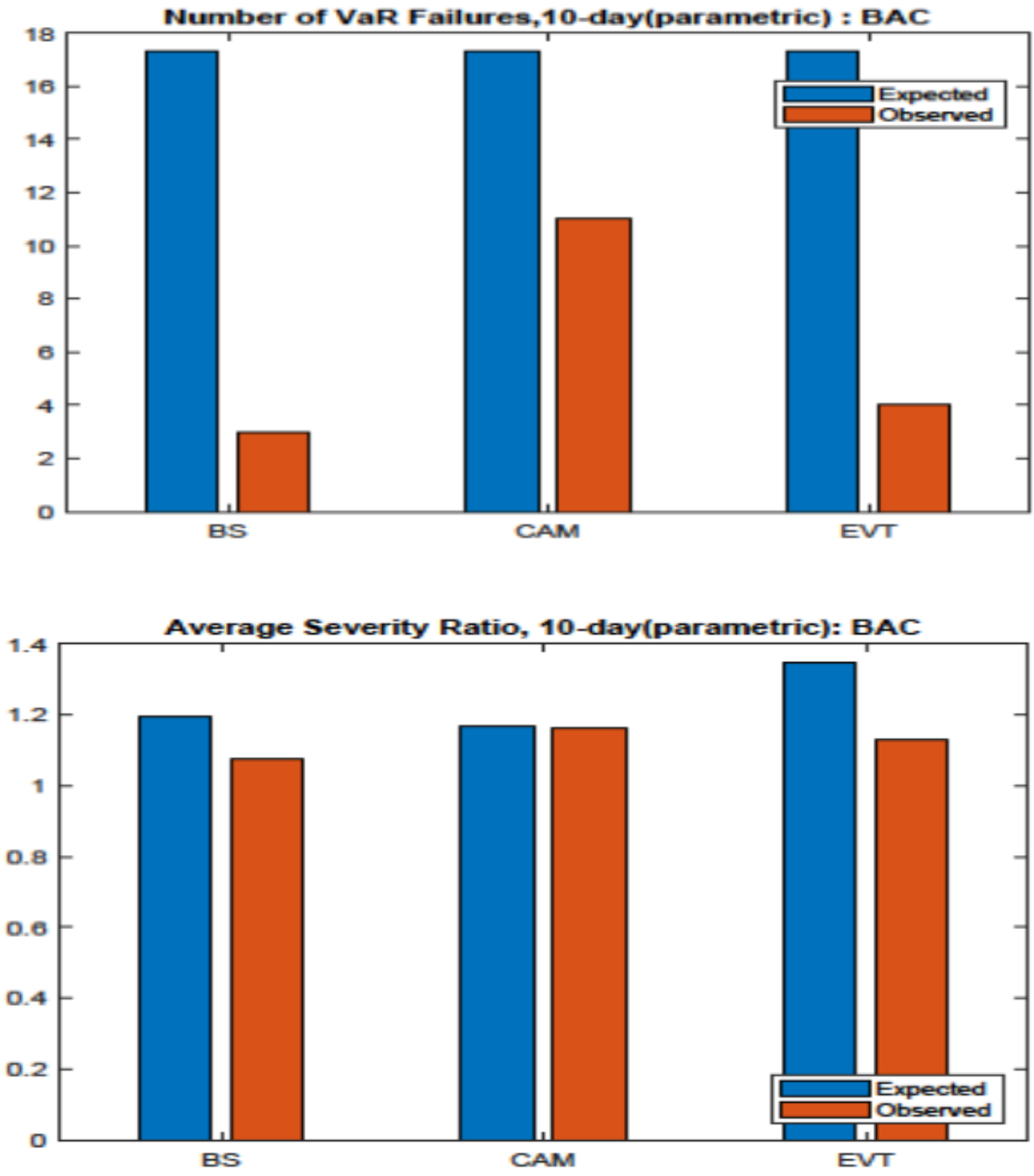


Figure-5.24: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Bank of America (BAC), **10-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

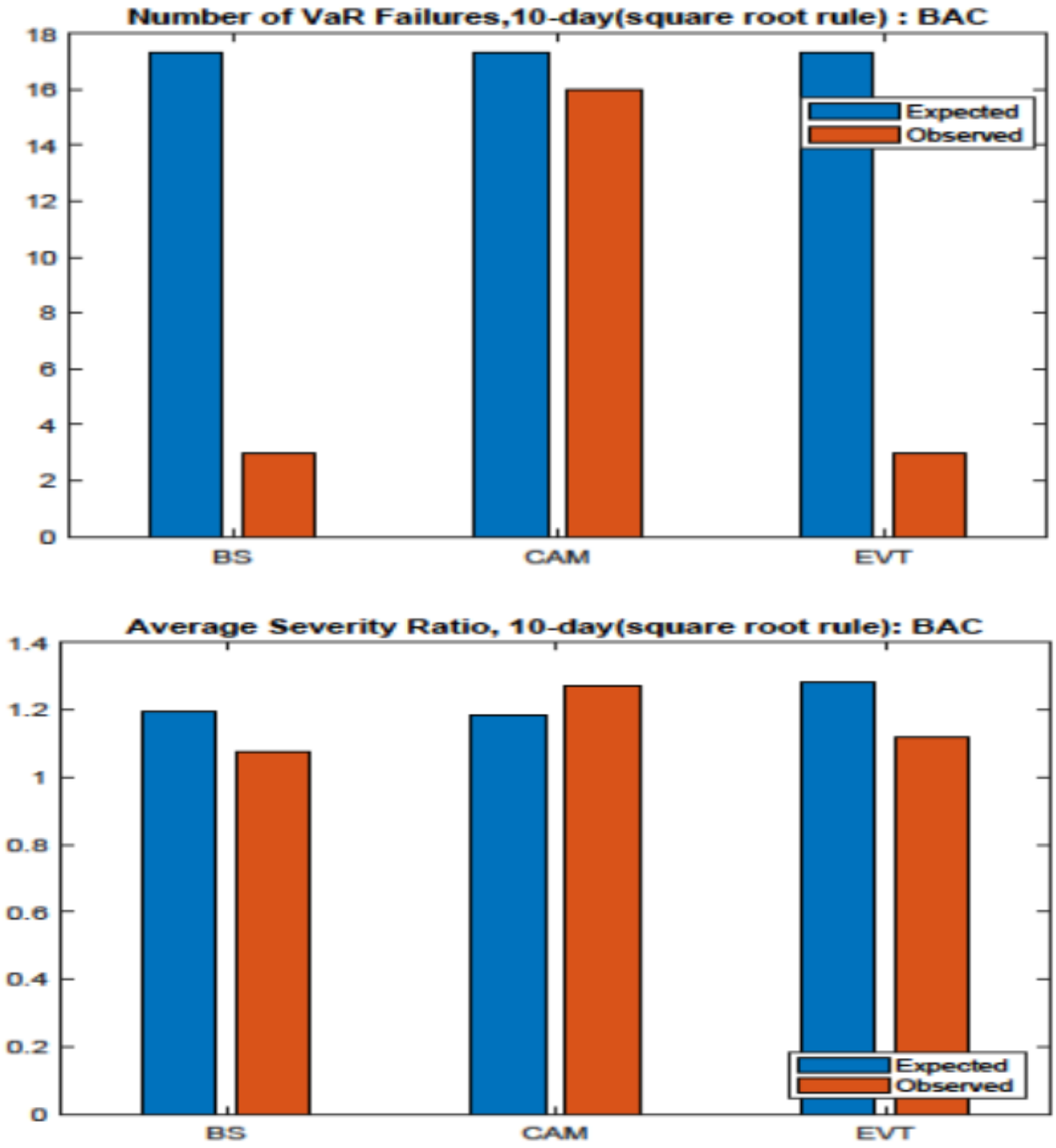


Figure-5.25: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Bank of America (BAC), **10-days horizon** for the year **(2016-2018**) under **square-root-rule** multi-period characterization using three different Models.

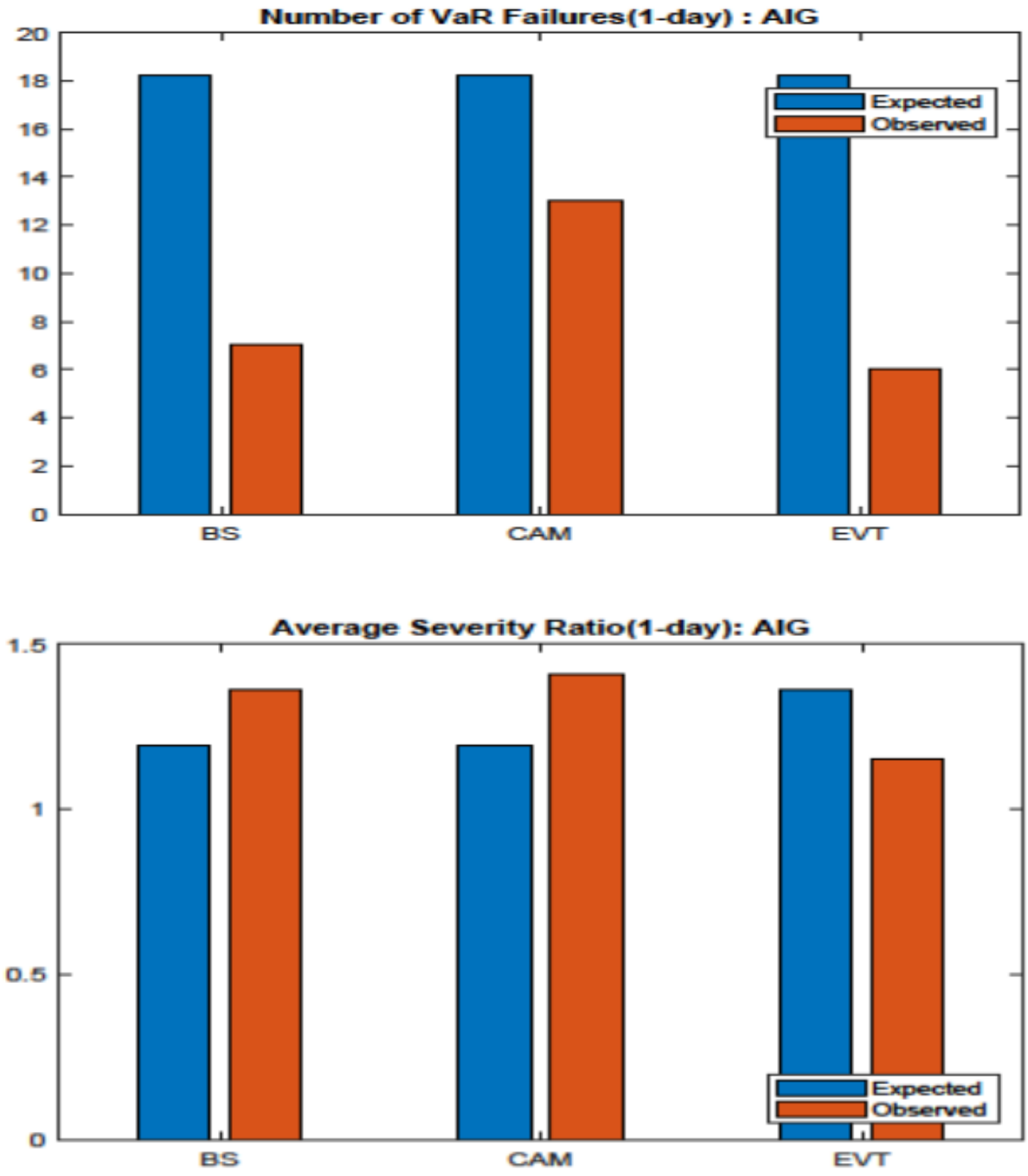


Figure-5.26: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets American Investment Group (AIG), **1-day horizon** for the year **(2016-2018**) using three different Models.

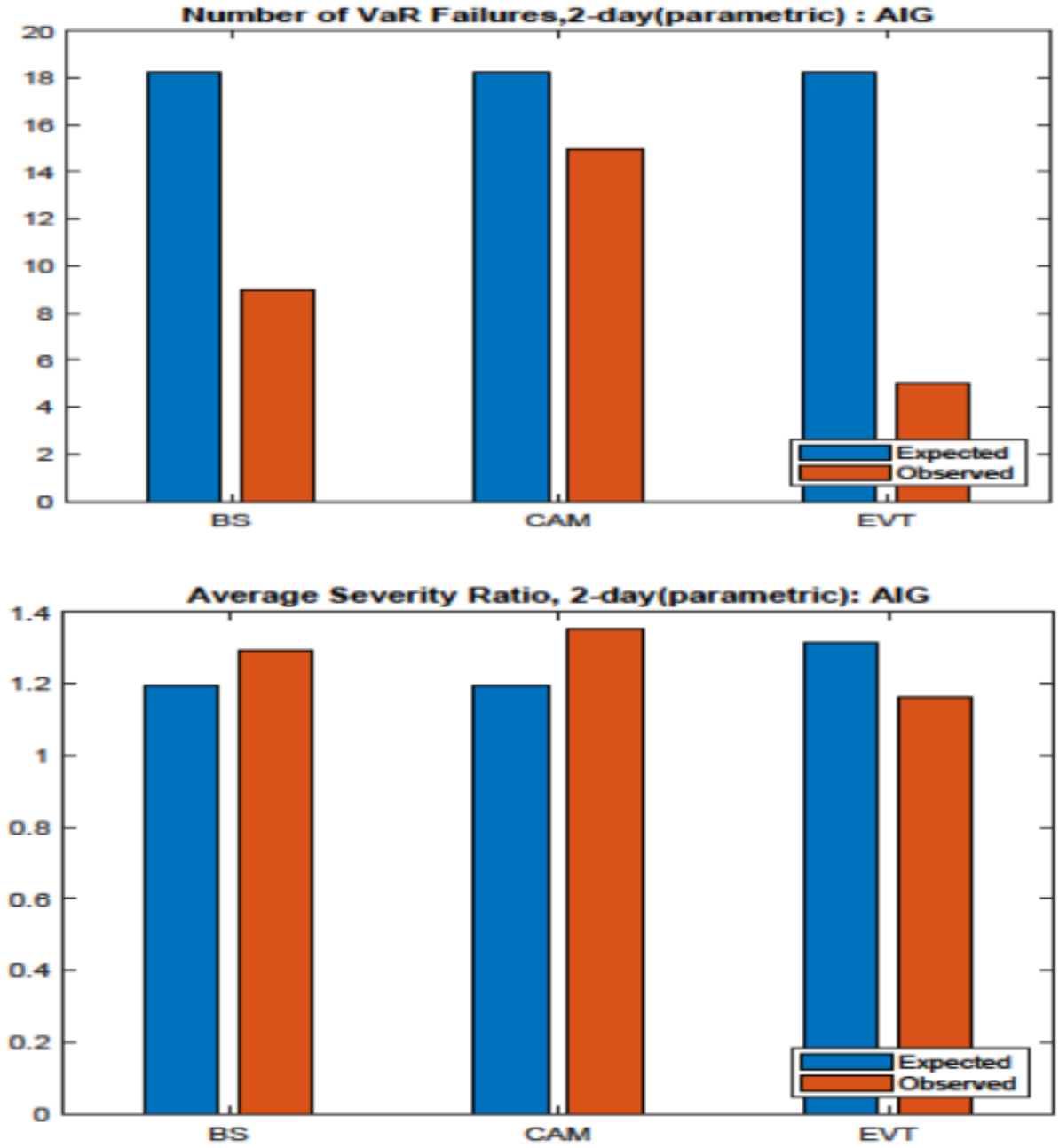


Figure-5.27: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets American Investment Group (AIG), **2-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

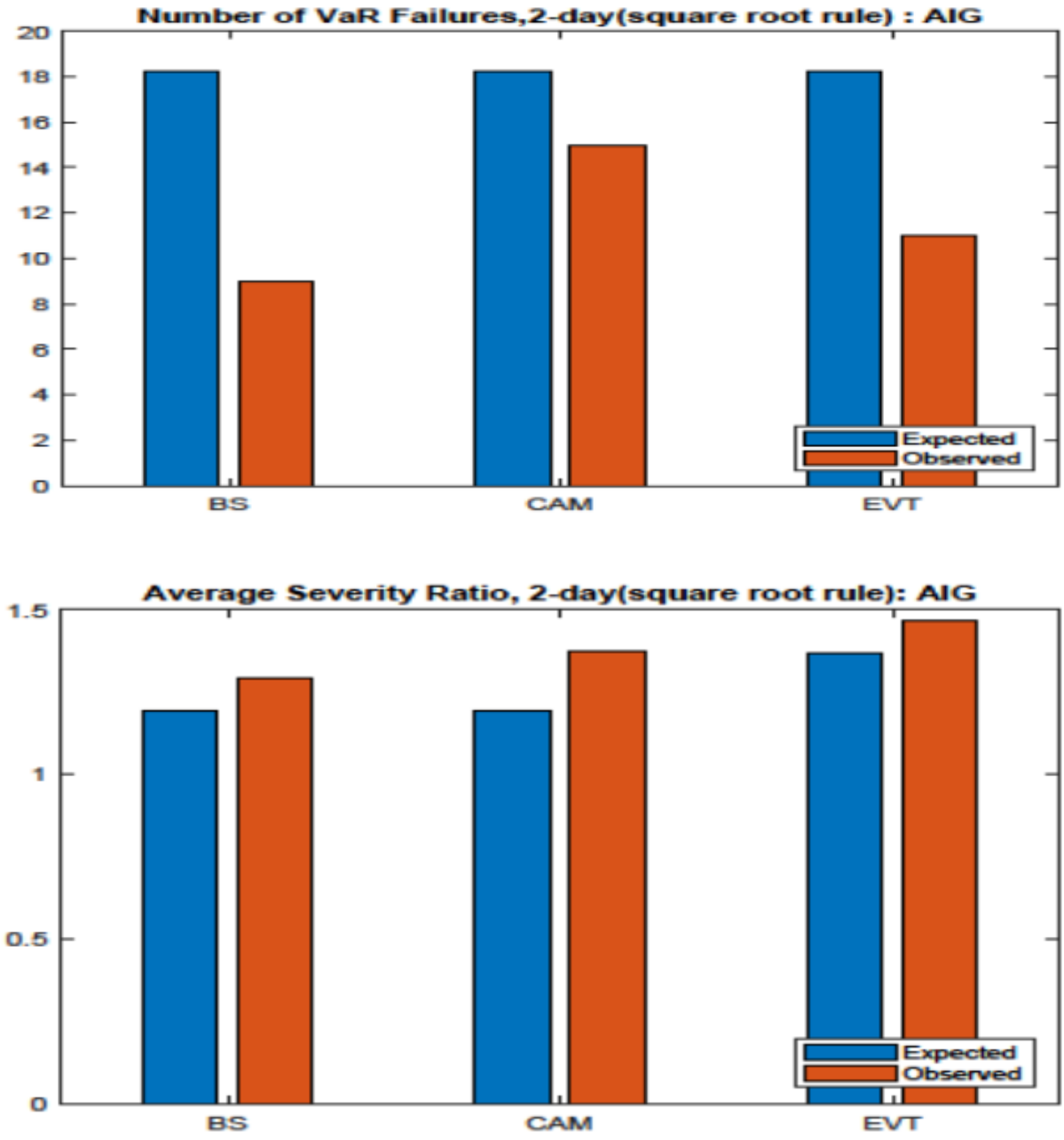


Figure-5.28: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets American Investment Group (AIG), **2-days horizon** for the year **(2016-2018)** under **square-root-rule** multi-period characterization using three different Models.

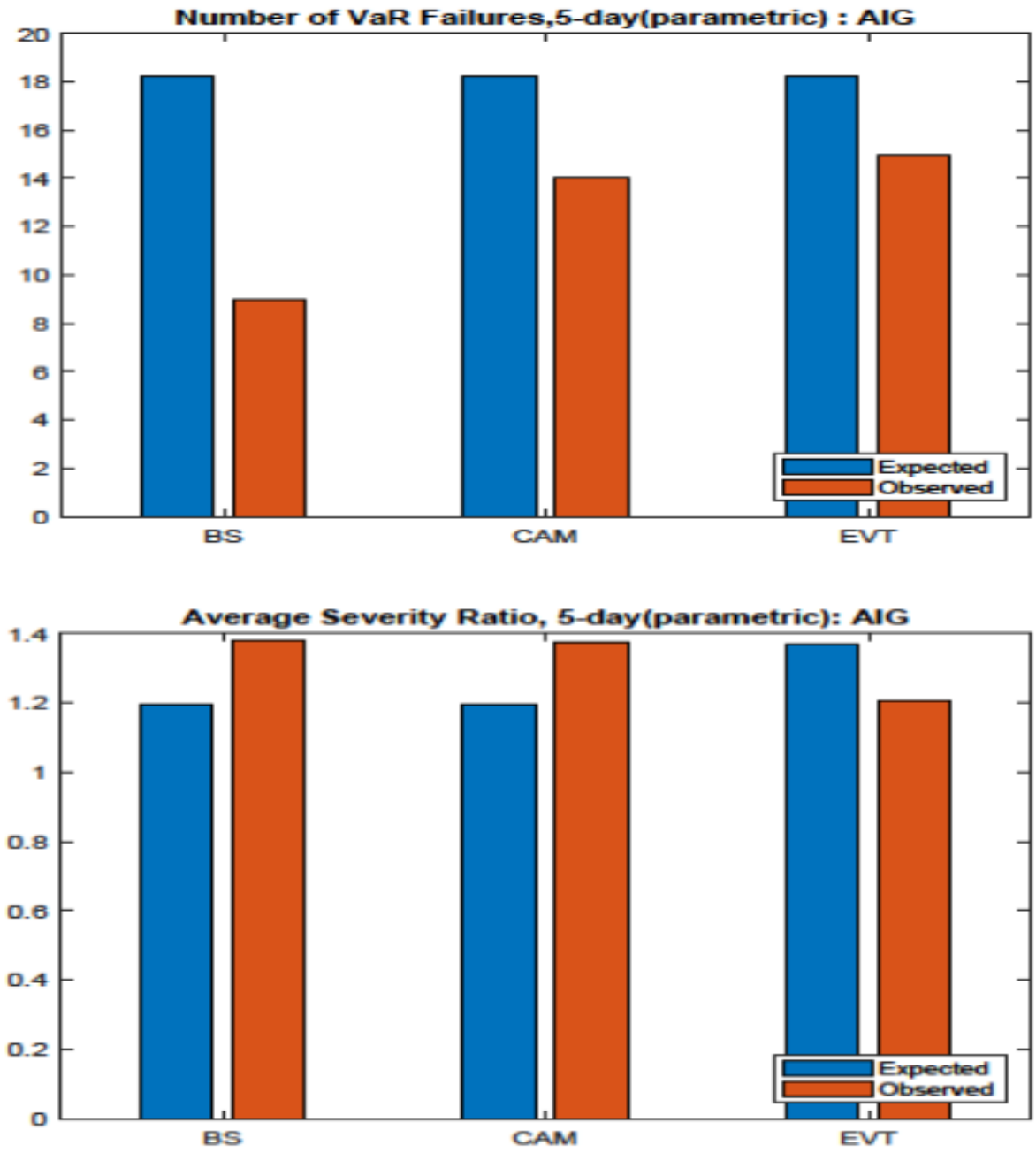


Figure-5.29: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets American Investment Group (AIG), **5-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

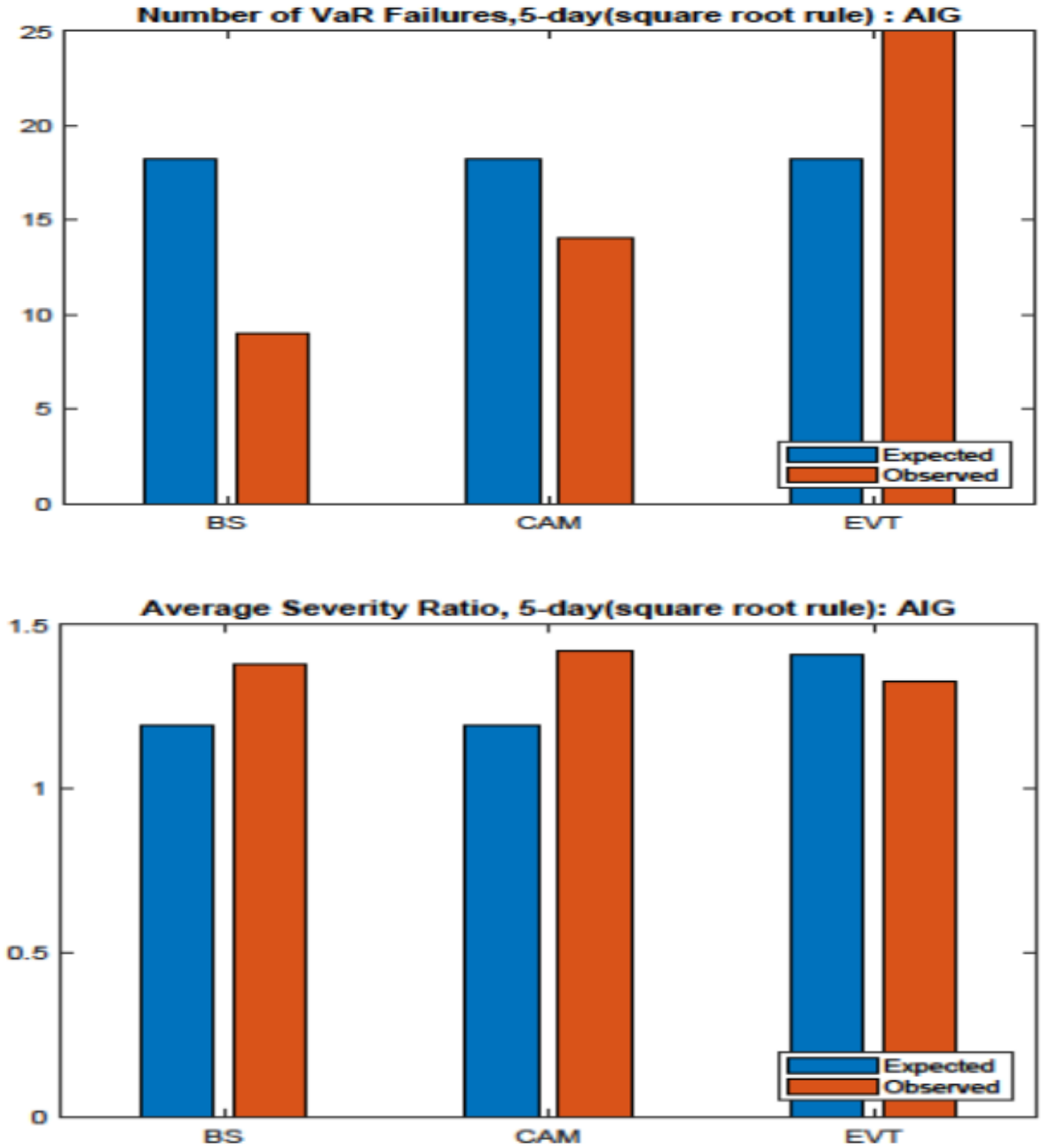


Figure-5.30: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets American Investment Group (AIG), **5-days horizon** for the year **(2016-2018)** under **square-root-rule** multi-period characterization using three different Models.

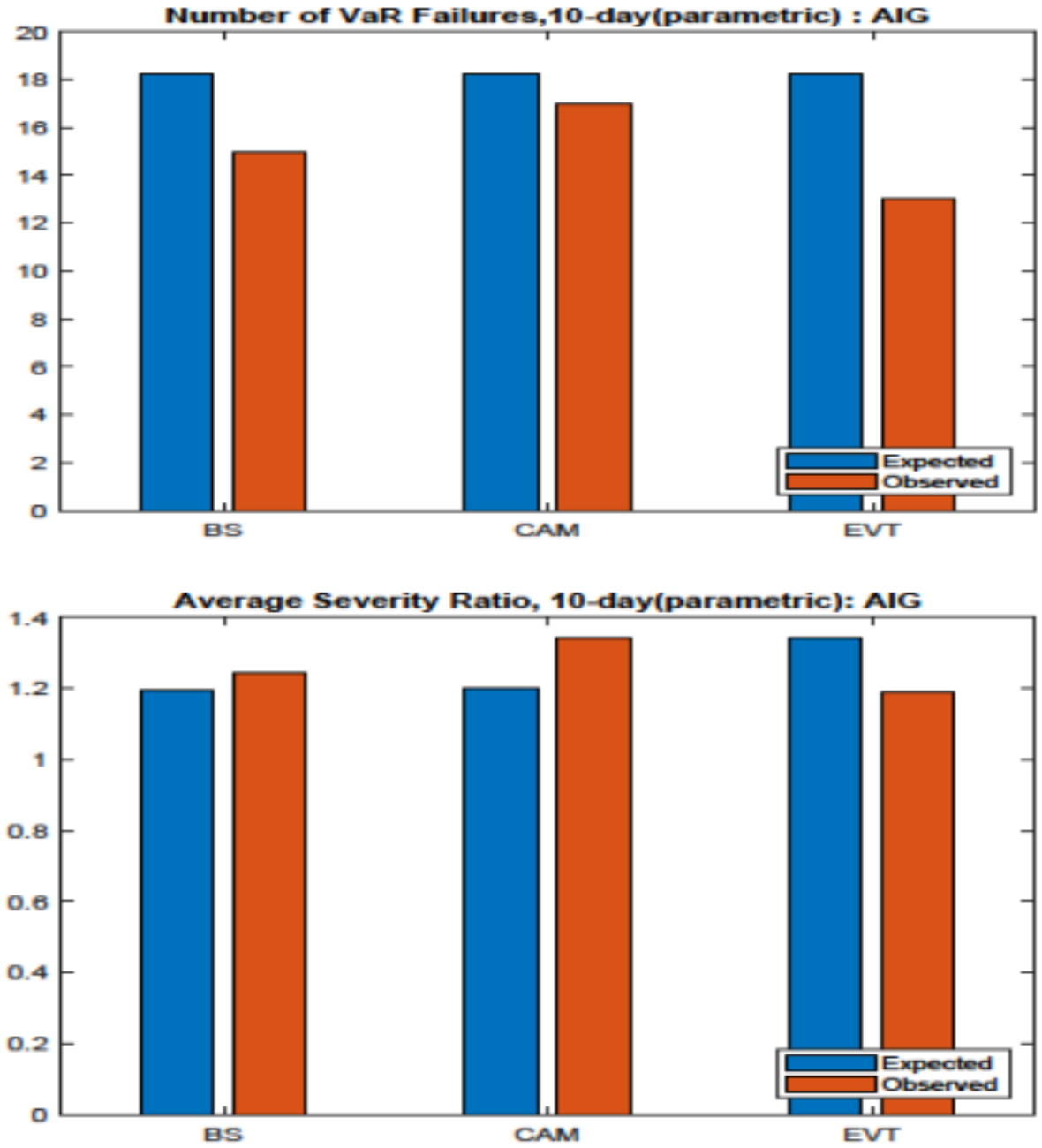


Figure-5.31: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets American Investment Group (AIG), **10-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

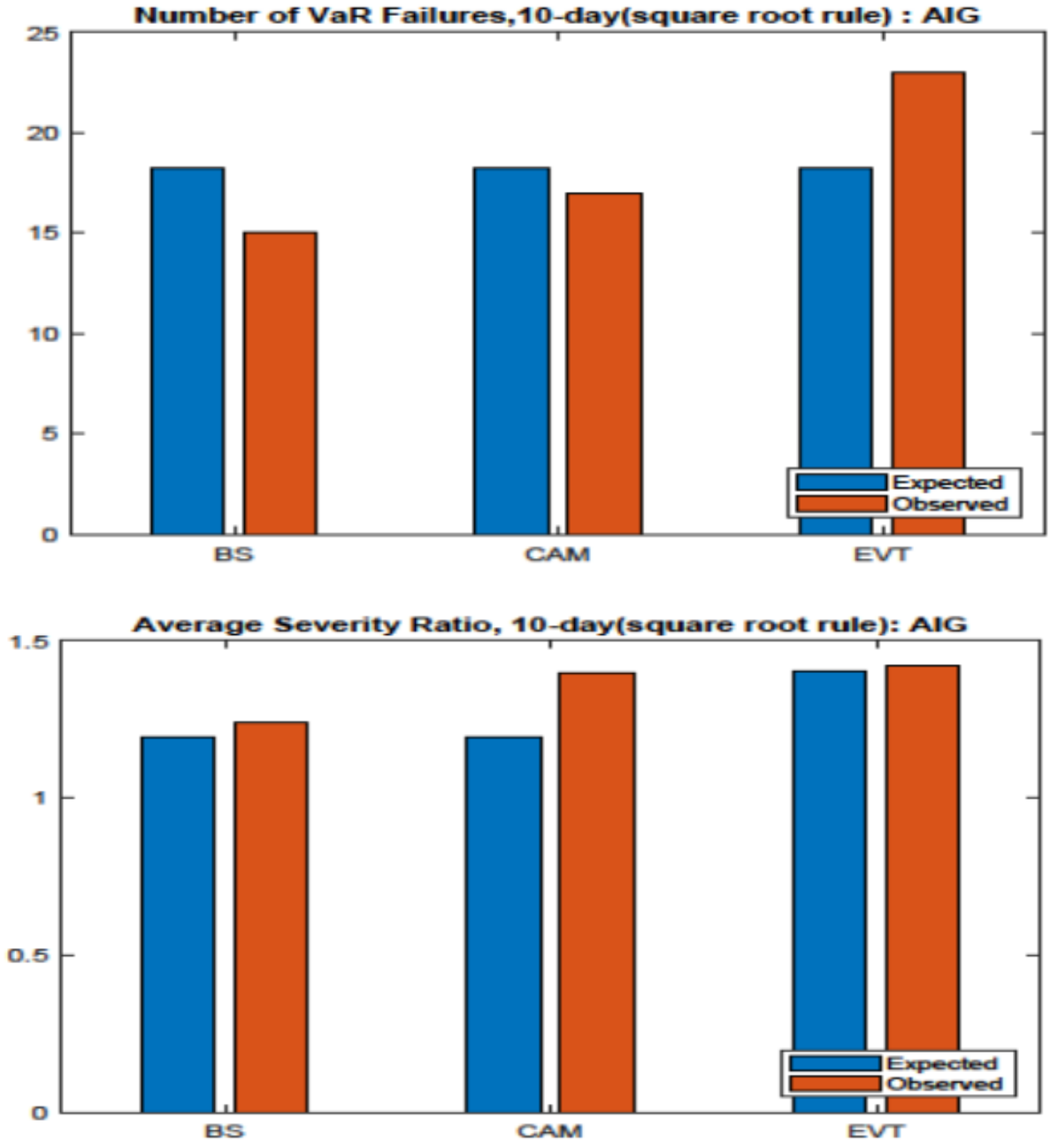


Figure-5.32: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets American Investment Group (AIG), **10-days horizon** for the year **(2016-2018)** under **square-root-rule** multi-period characterization using three different Models.

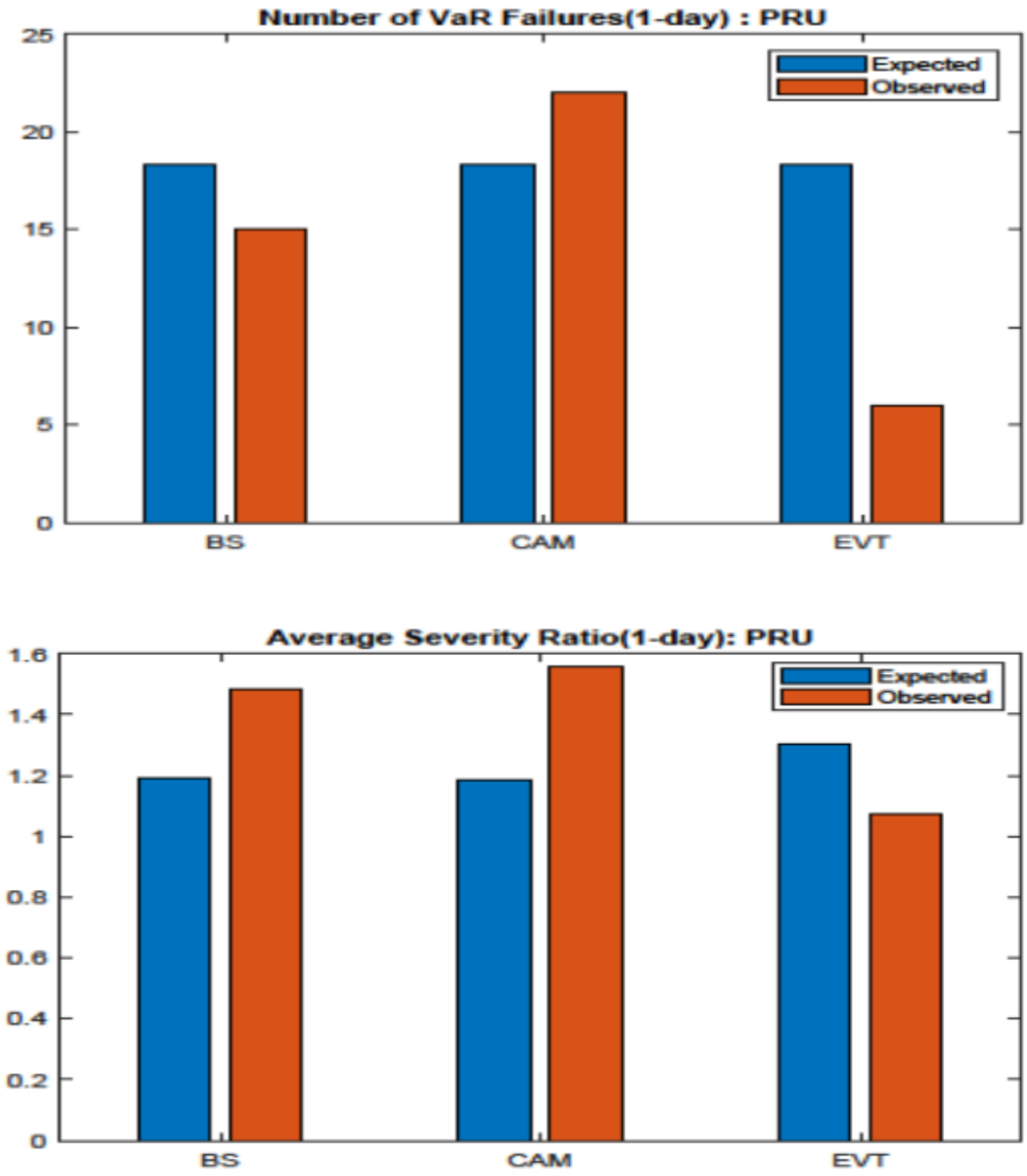


Figure-5.33: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Prudential Capital (PRU), **1-day horizon** for the year **(2016-2018**) using three different Models.

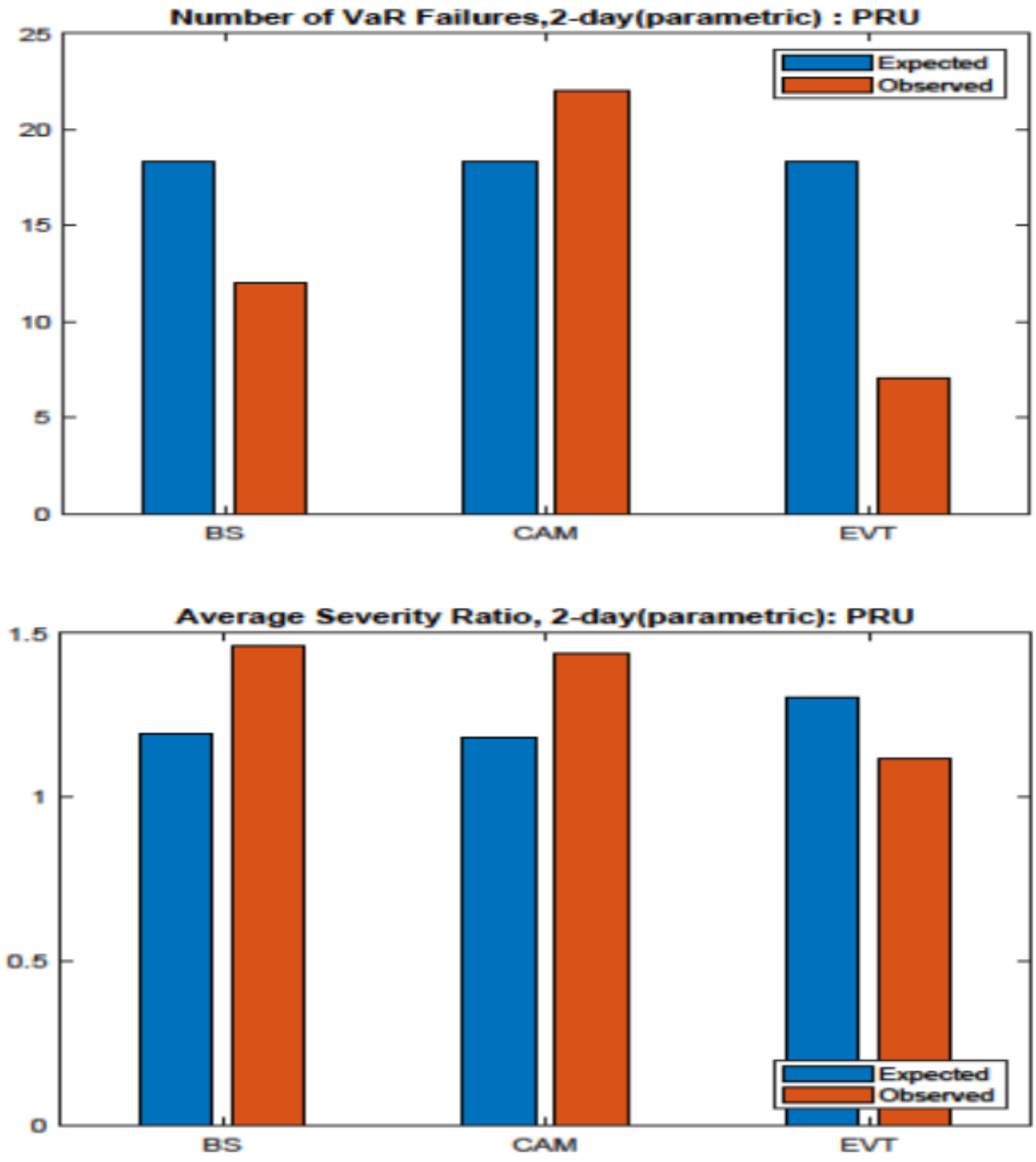


Figure-5.34: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Prudential Capital (PRU), **2-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

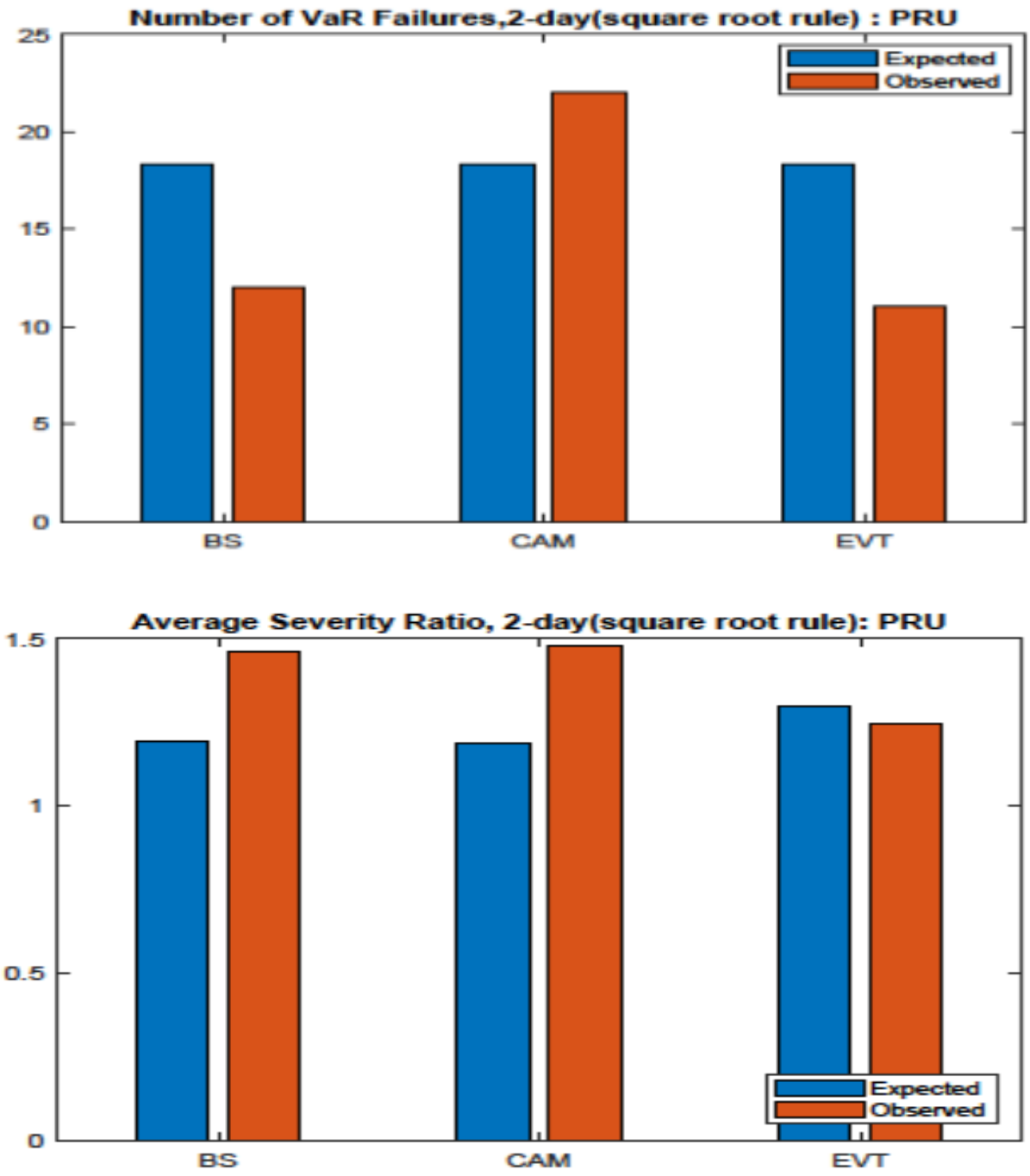


Figure-5.35: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Prudential Capital (PRU), **2-days horizon** for the year **(2016-2018**) under **square-root-rule** multi-period characterization using three different Models.

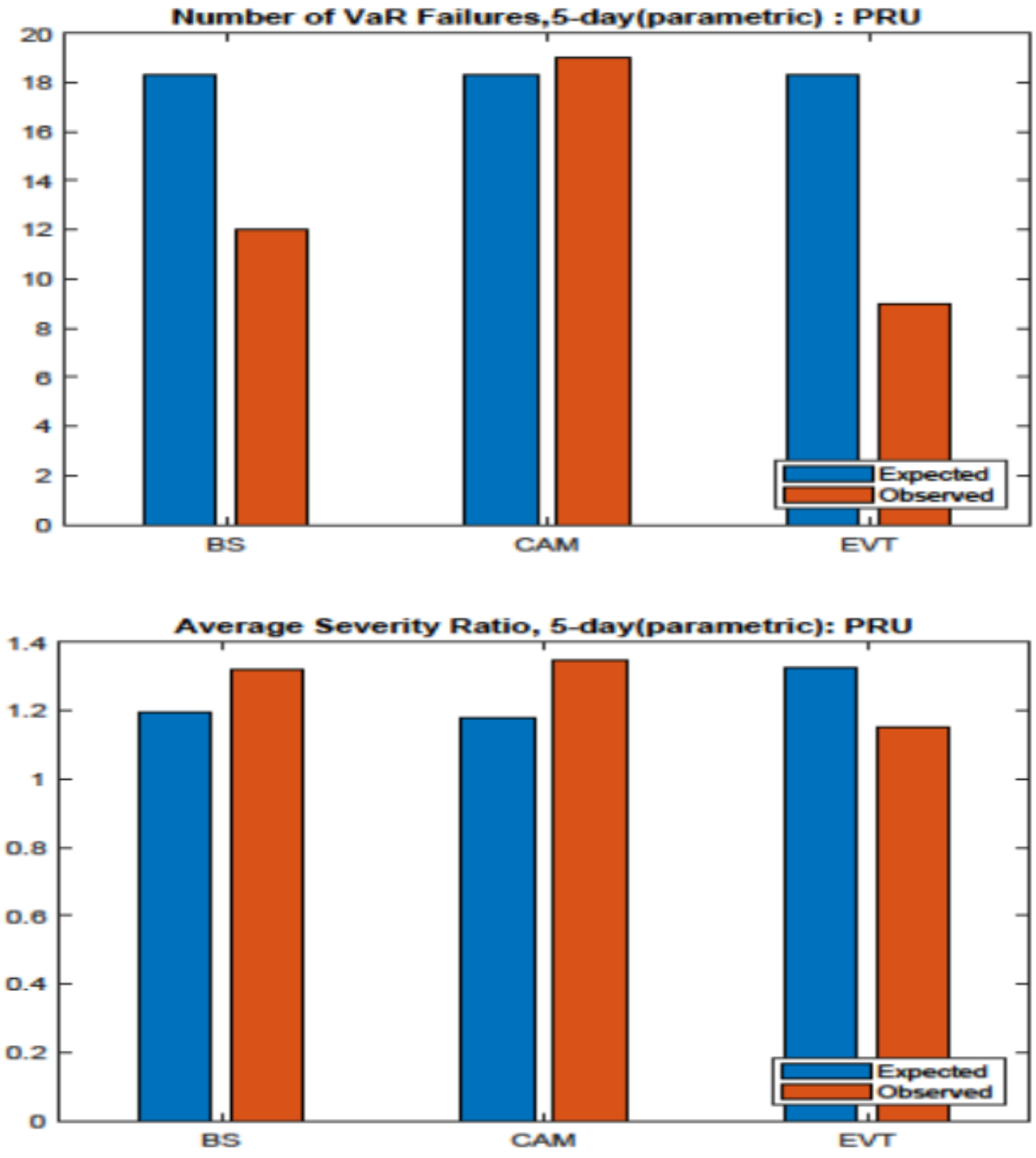


Figure-5.36: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Prudential Capital (PRU), **5-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

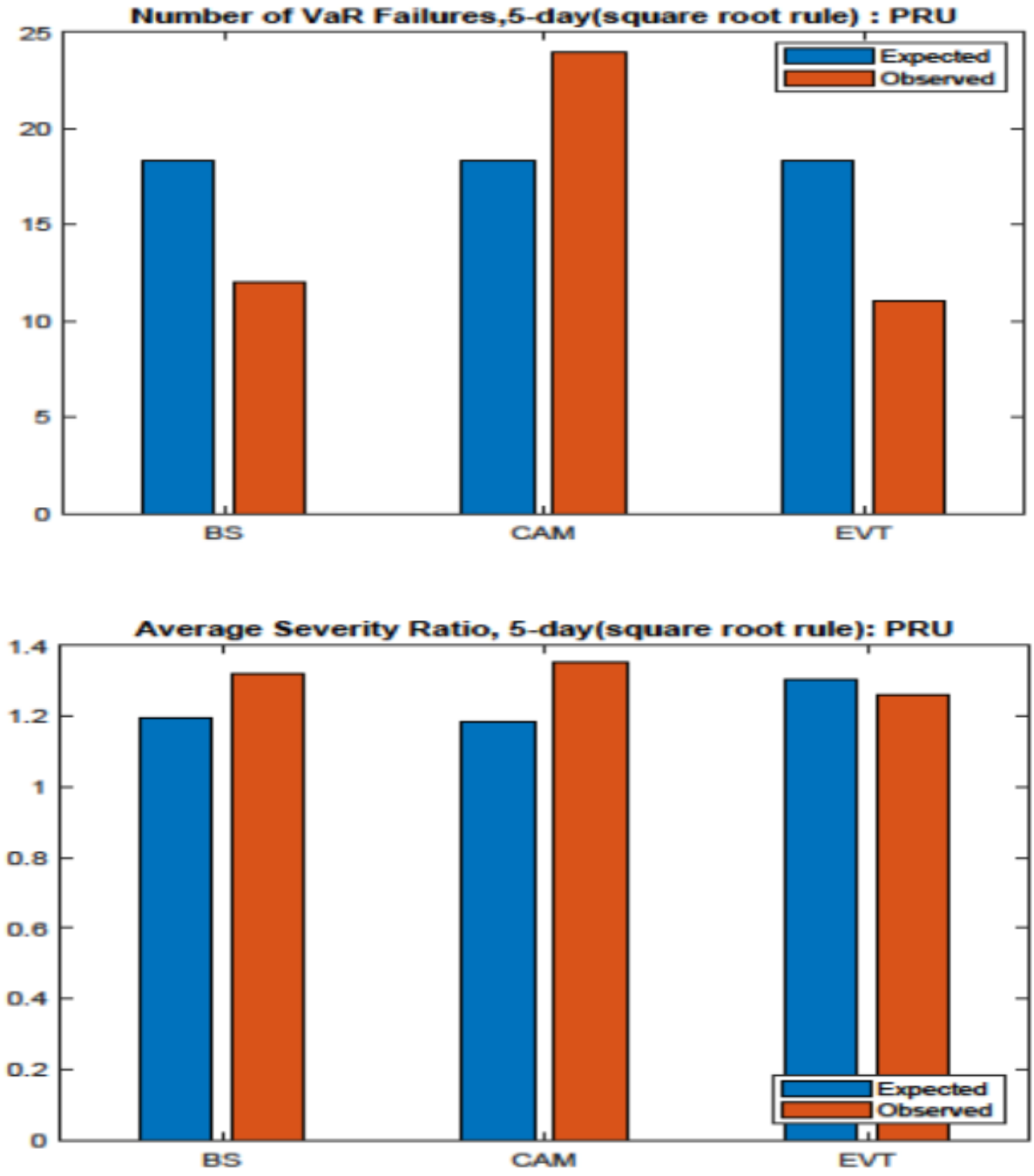


Figure-5.37: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Prudential Capital (PRU), **5-days horizon** for the year **(2016-2018**) under **square-root-rule** multi-period characterization using three different Models.

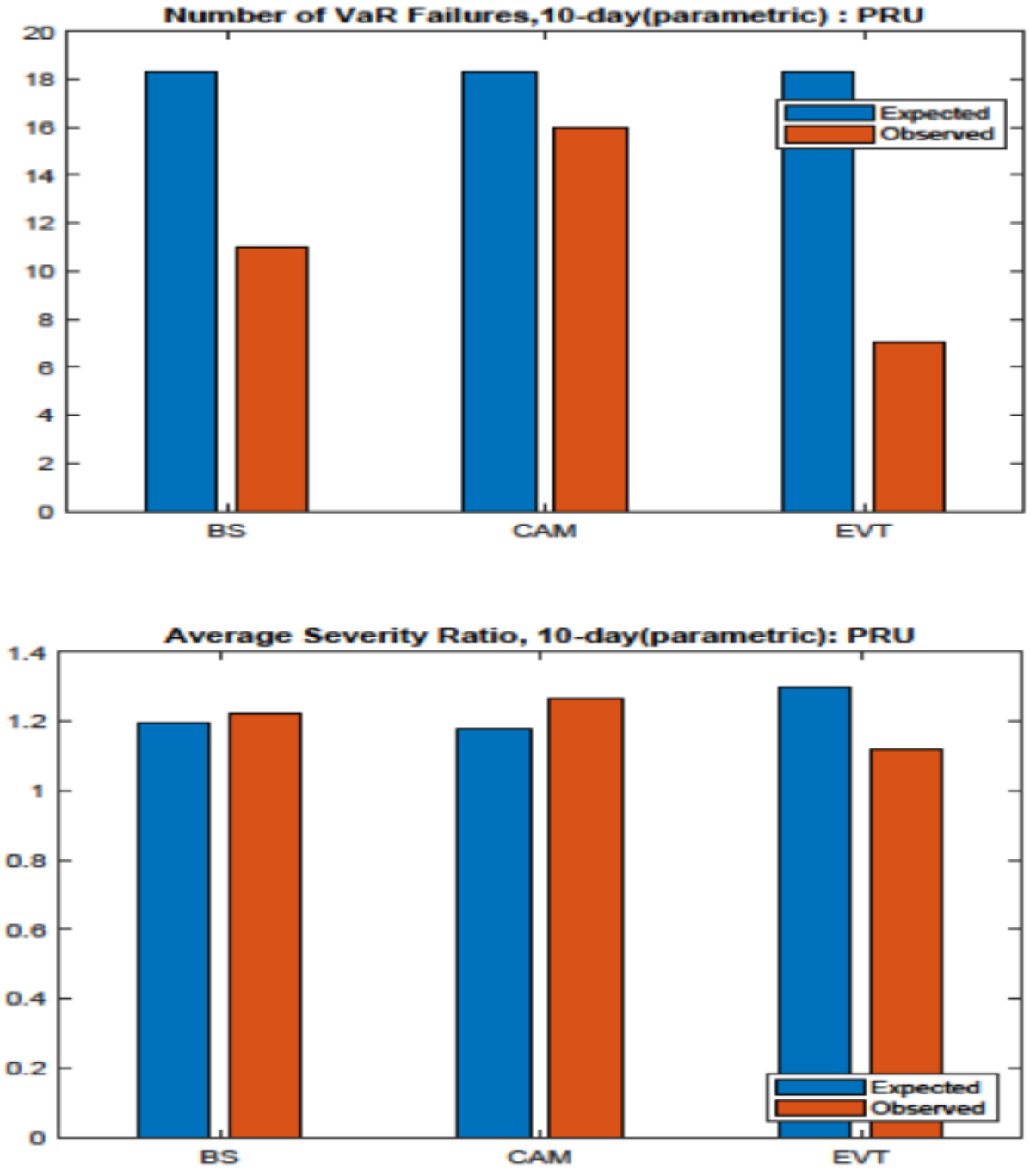


Figure-5.38: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Prudential Capital (PRU), **10-days horizon** for the year **(2016-2018**) under **parametric** multi-period characterization using three different Models.

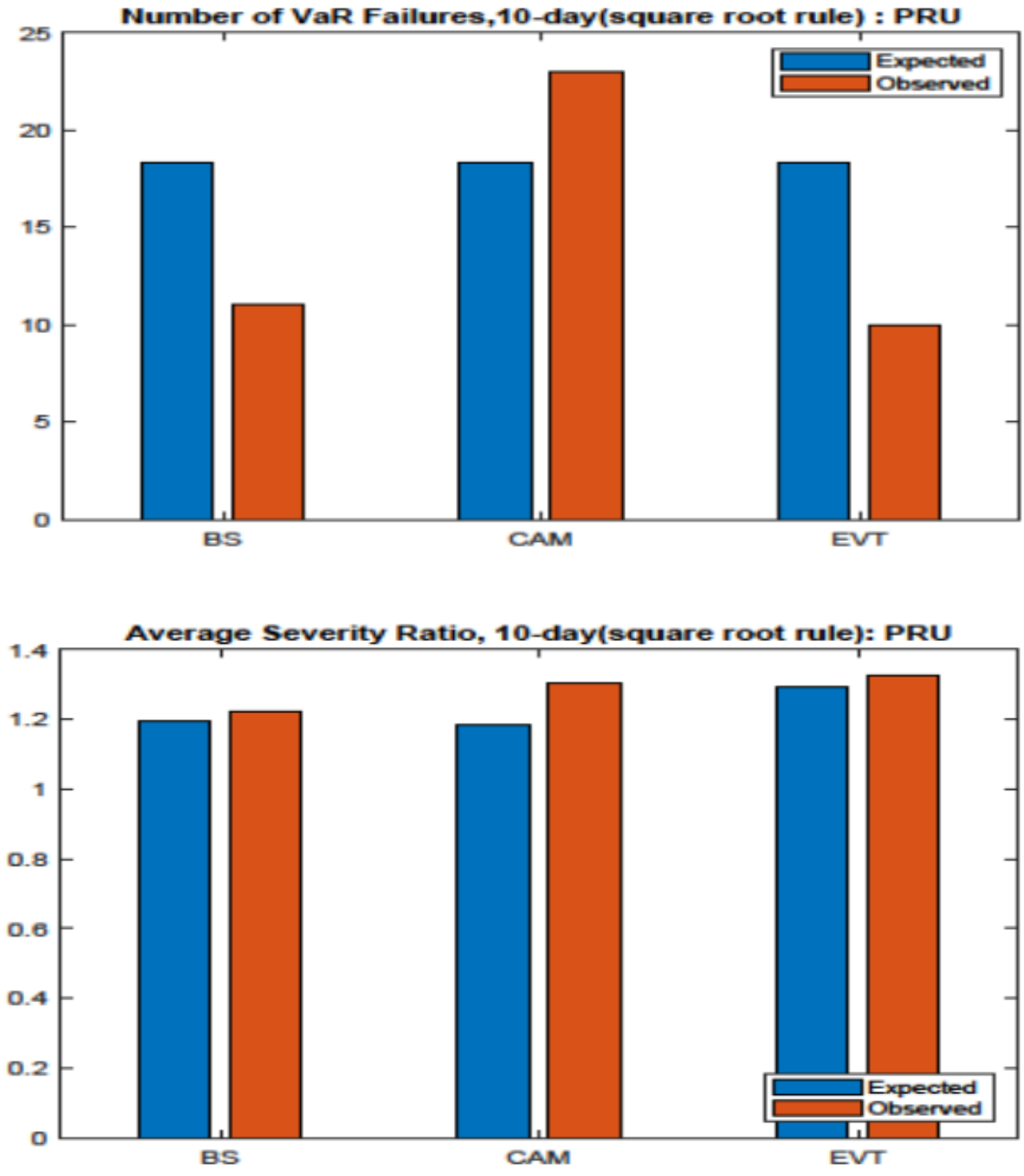


Figure-5.39: Number of VaR (97.5%) failures (top) and average severity ratio of ES (bottom) of financial assets Prudential Capital (PRU), **10-days horizon** for the year **(2016-2018**) under **square-root-rule** multi-period characterization using three different Models.

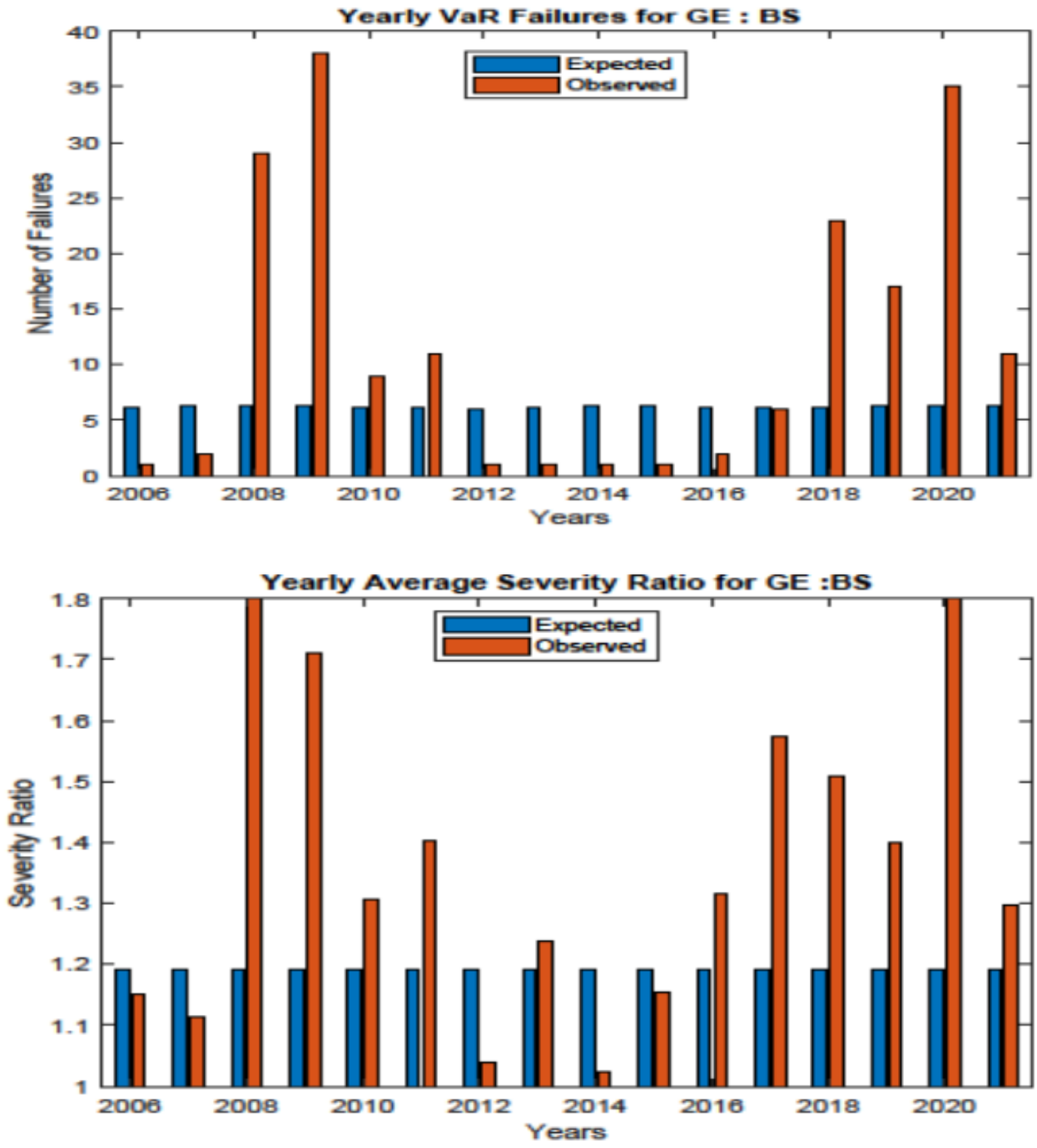


Figure-5.40: Yearly number of VaR (97.5%) failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets General Electronics (GE) using Black and Scholes (BS) model.

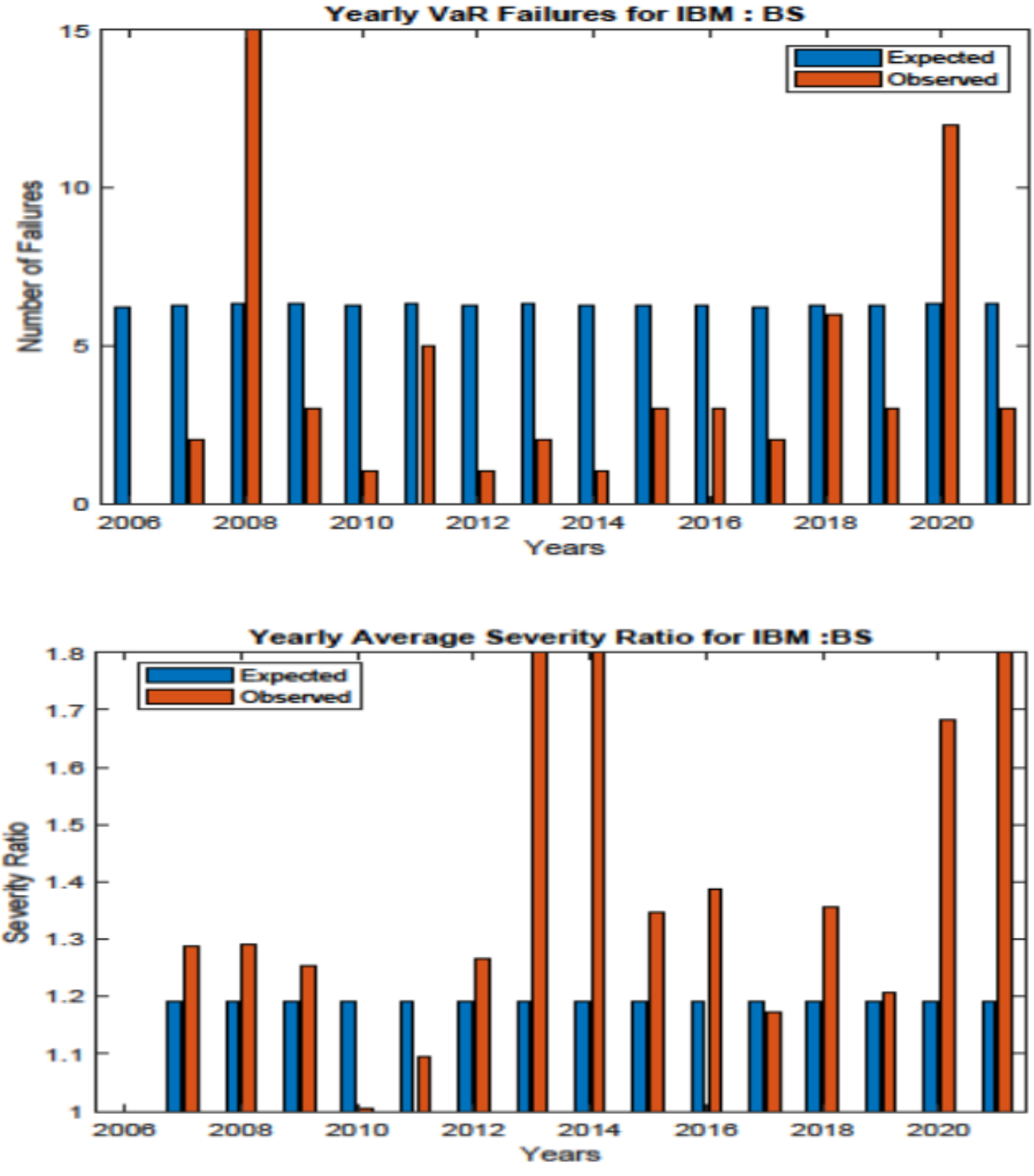


Figure-5.41: Yearly number of VaR (97.5%) failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets IBM using Black and Scholes (BS) model.

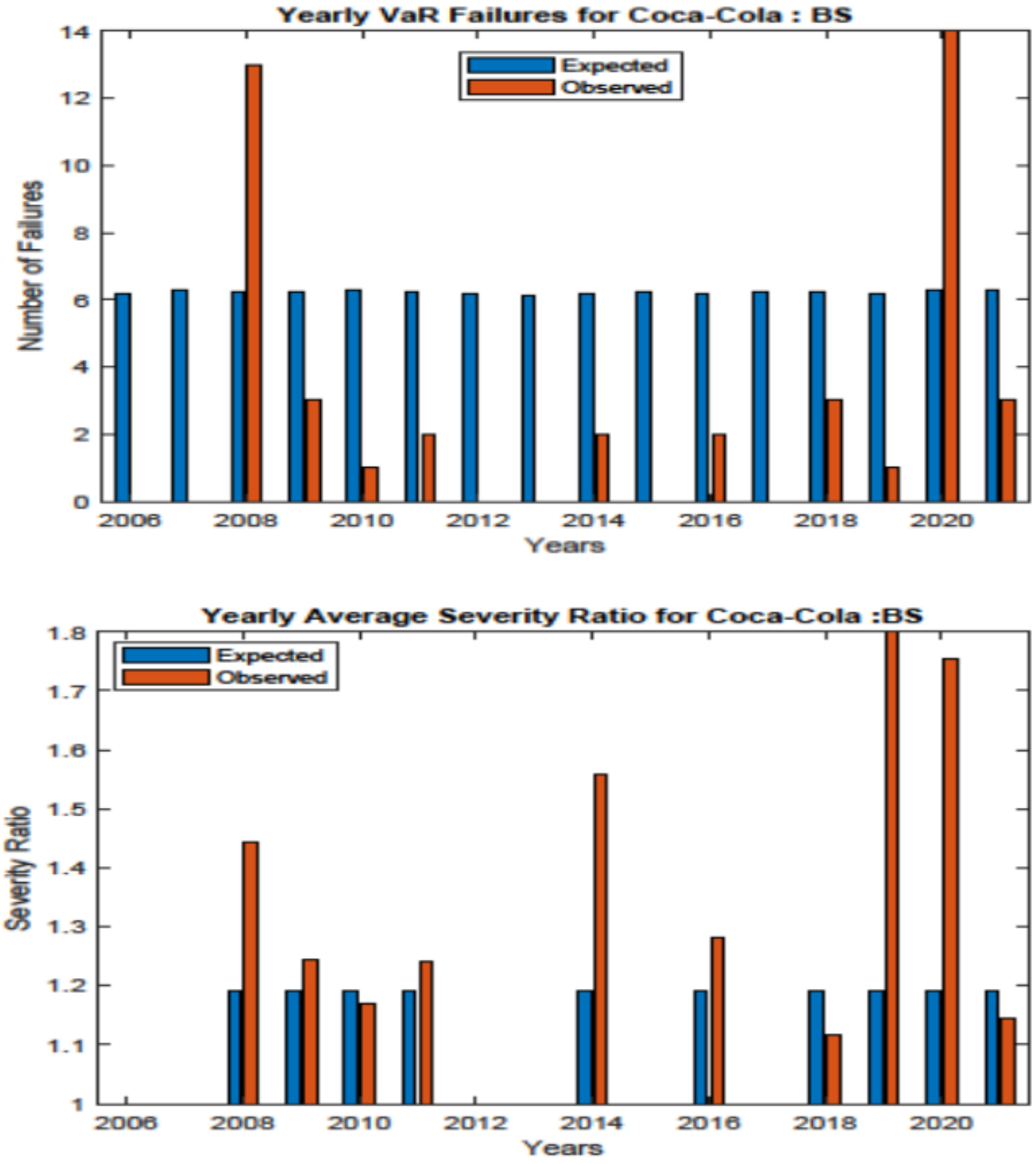


Figure-5.42: Yearly number of VaR (97.5%) failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets Coca-Cola using Black and Scholes (BS) model.

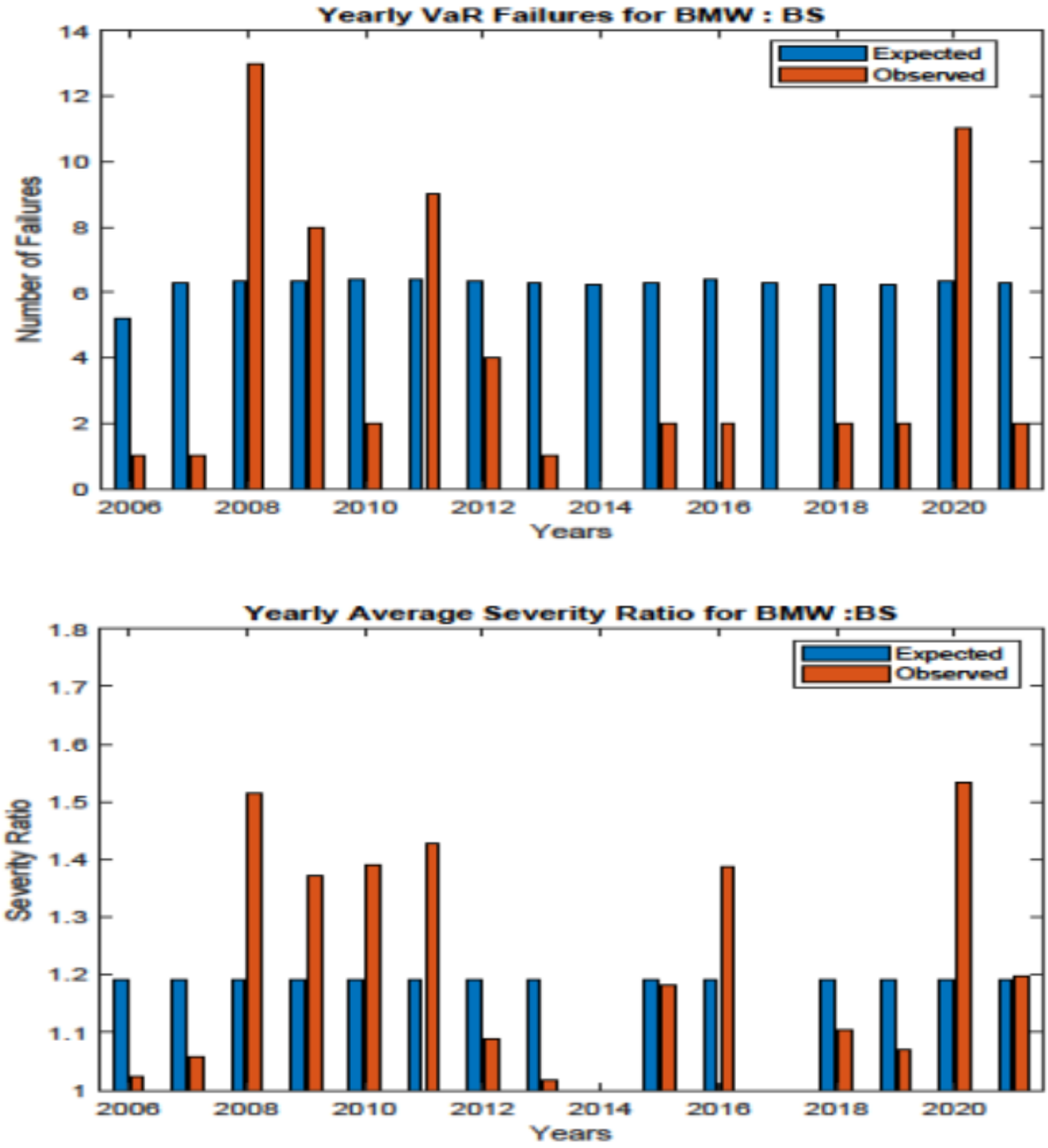


Figure-5.43: Yearly number of VaR (97.5%) failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets BMW using Black and Scholes (BS) model.

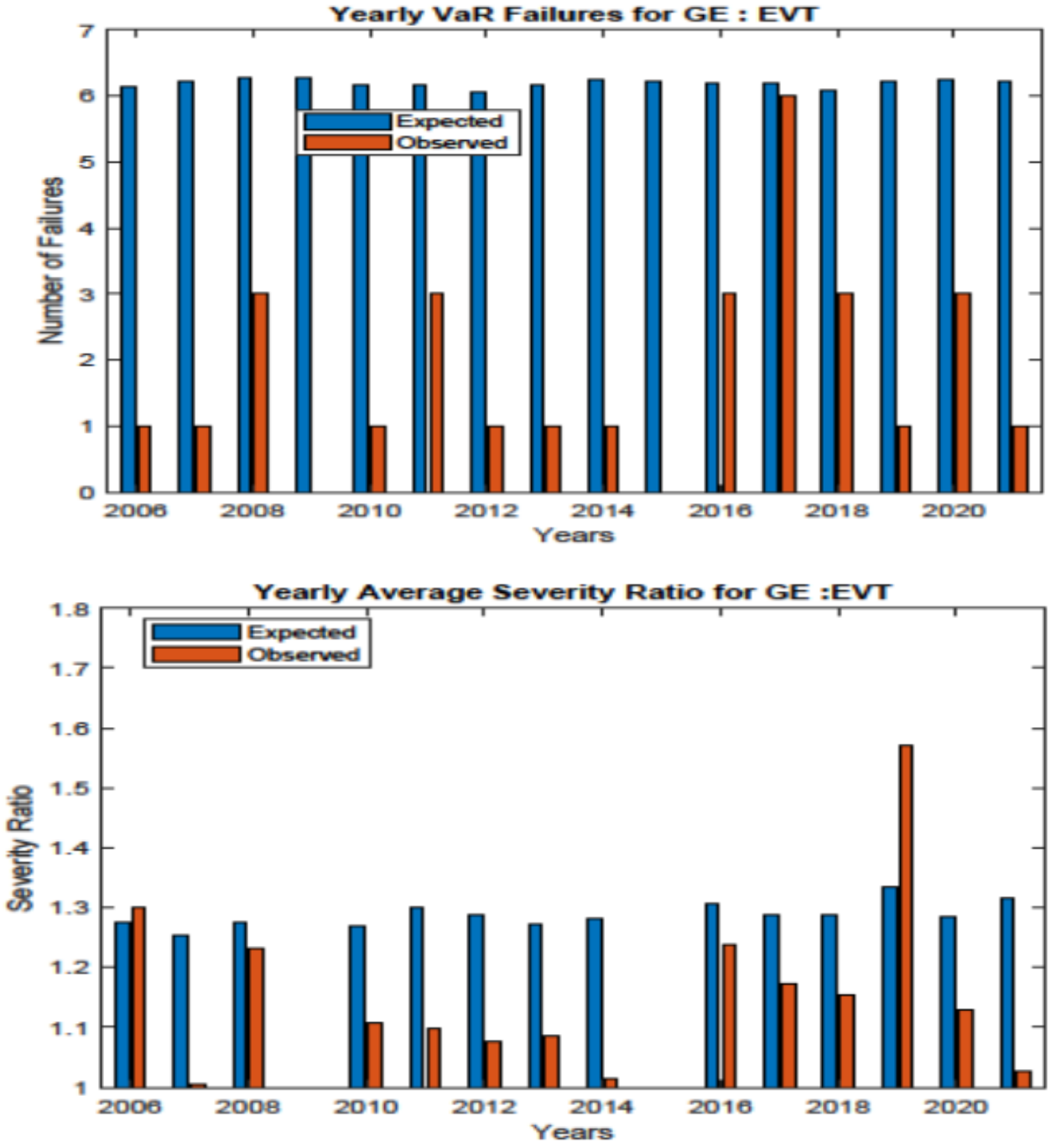


Figure-5.44: Yearly number of VaR (97.5%) failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets General Electronics (GE) using Conditional EVT model.

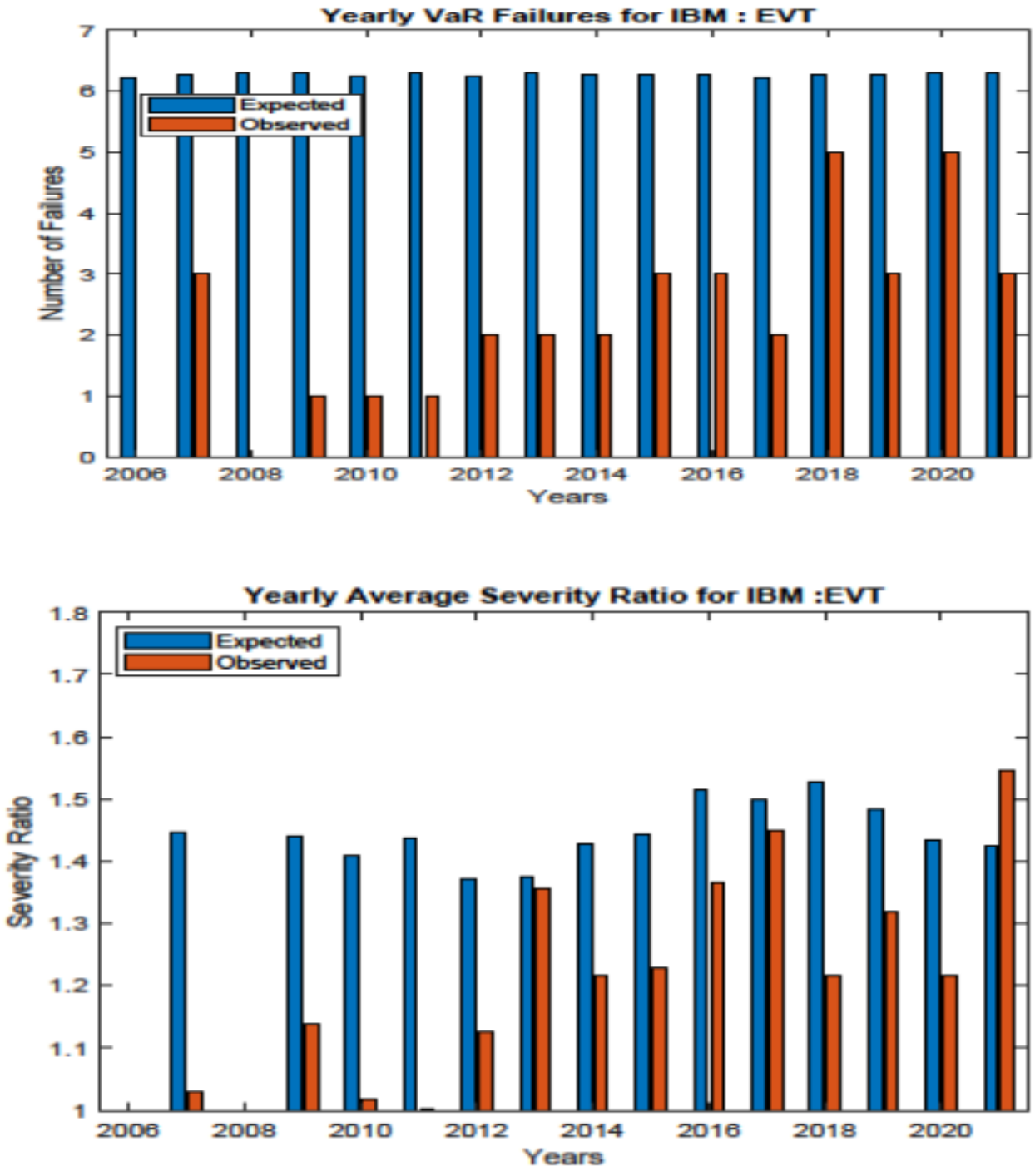


Figure-5.45: Yearly VaR failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets IBM using Conditional EVT model.

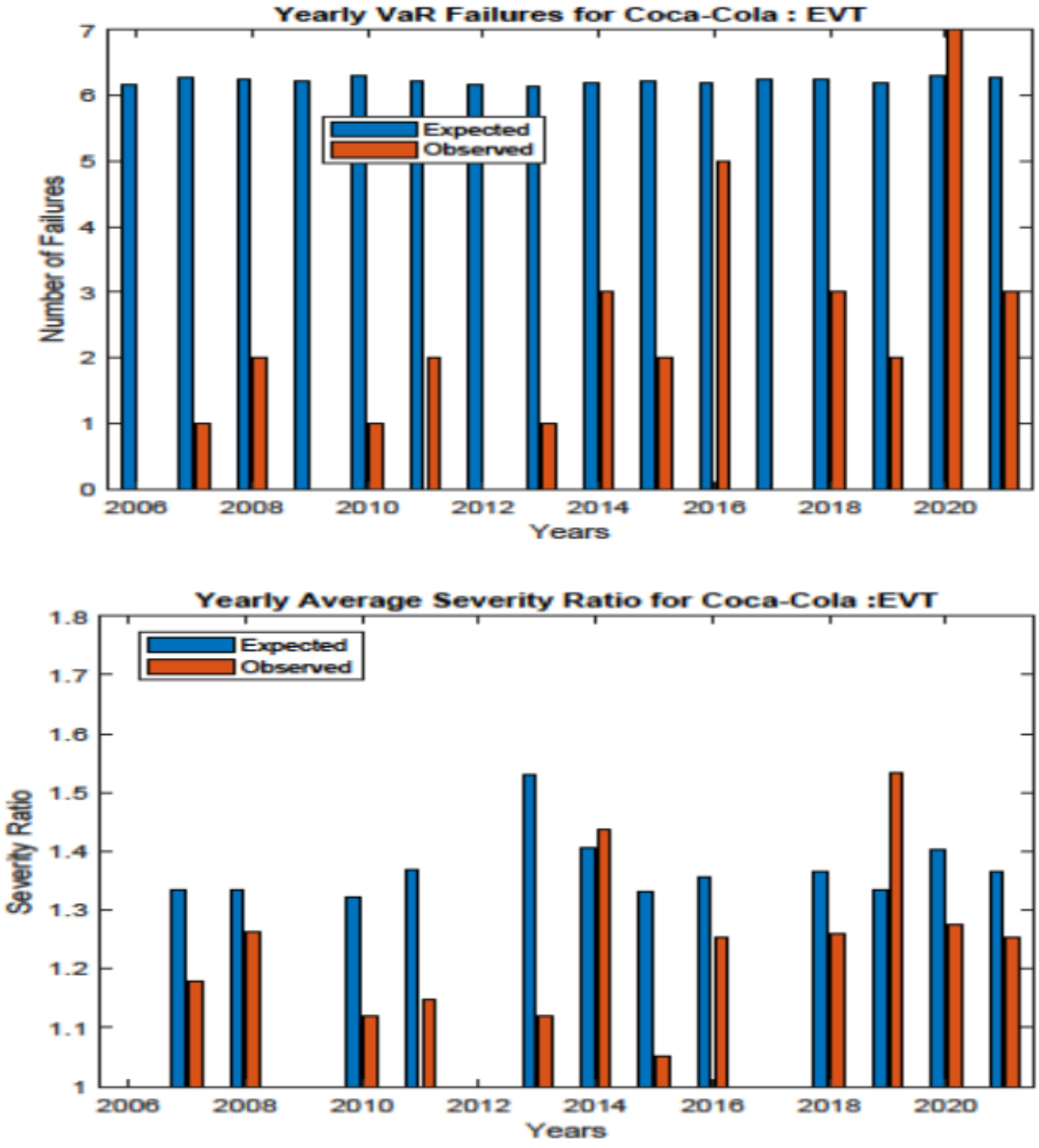


Figure-5.46: Yearly VaR failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets Coca-Cola using Conditional EVT model.

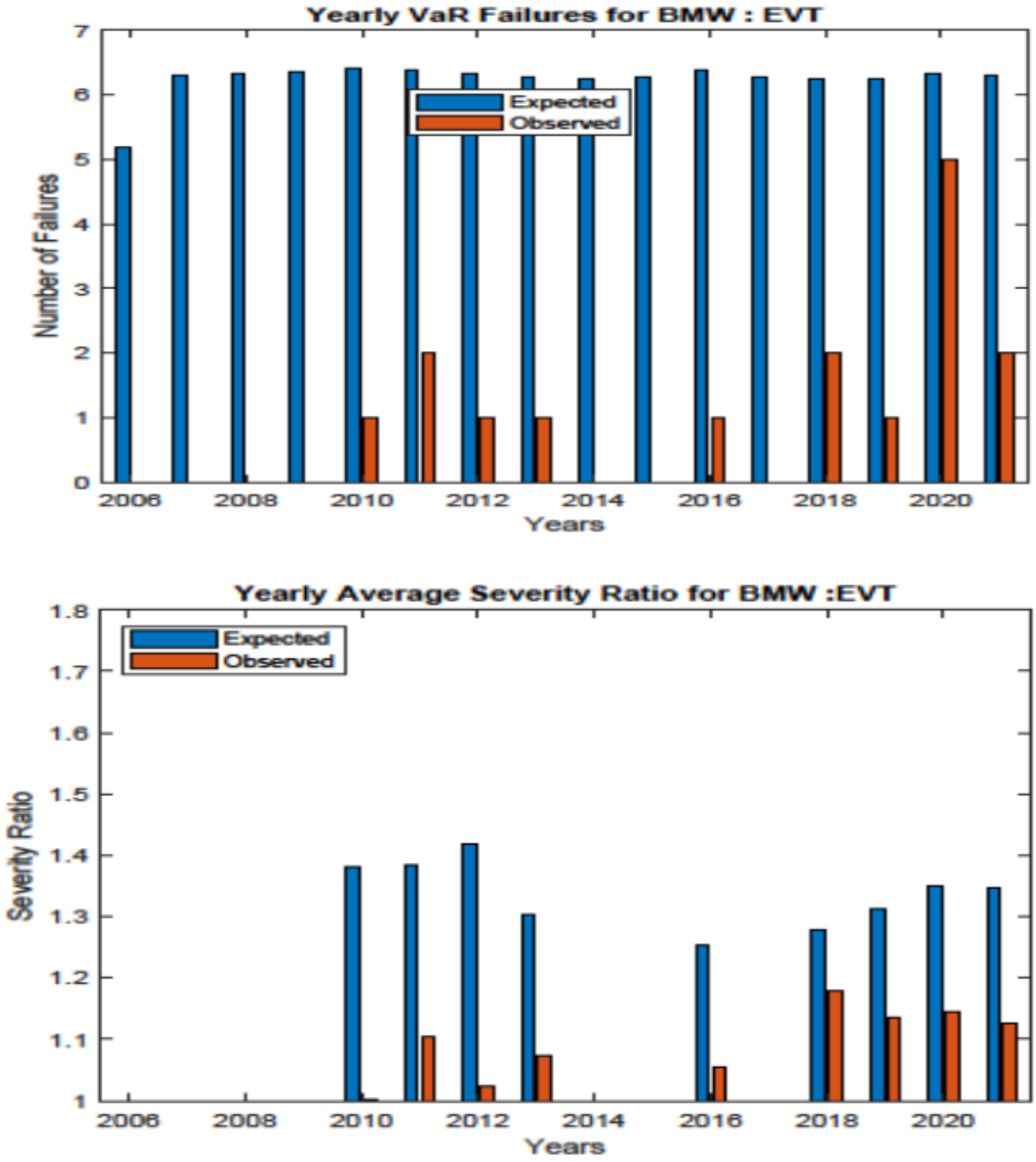


Figure-5.47: Yearly VaR failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets BMW using Conditional EVT model.

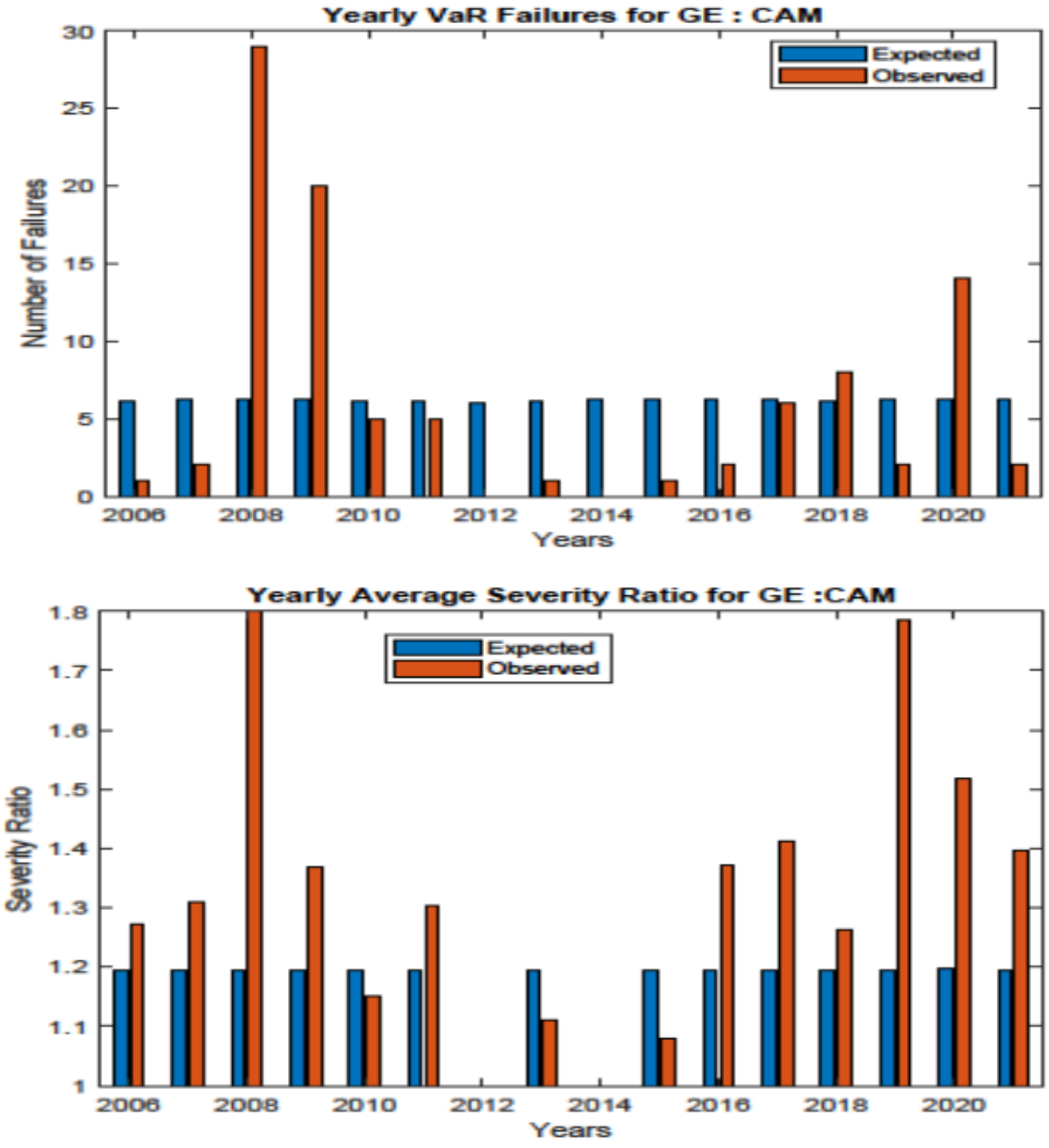


Figure-5.48: Yearly VaR failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets General Electronics (GE) using Conditional Analytic Markov (CAM) model.

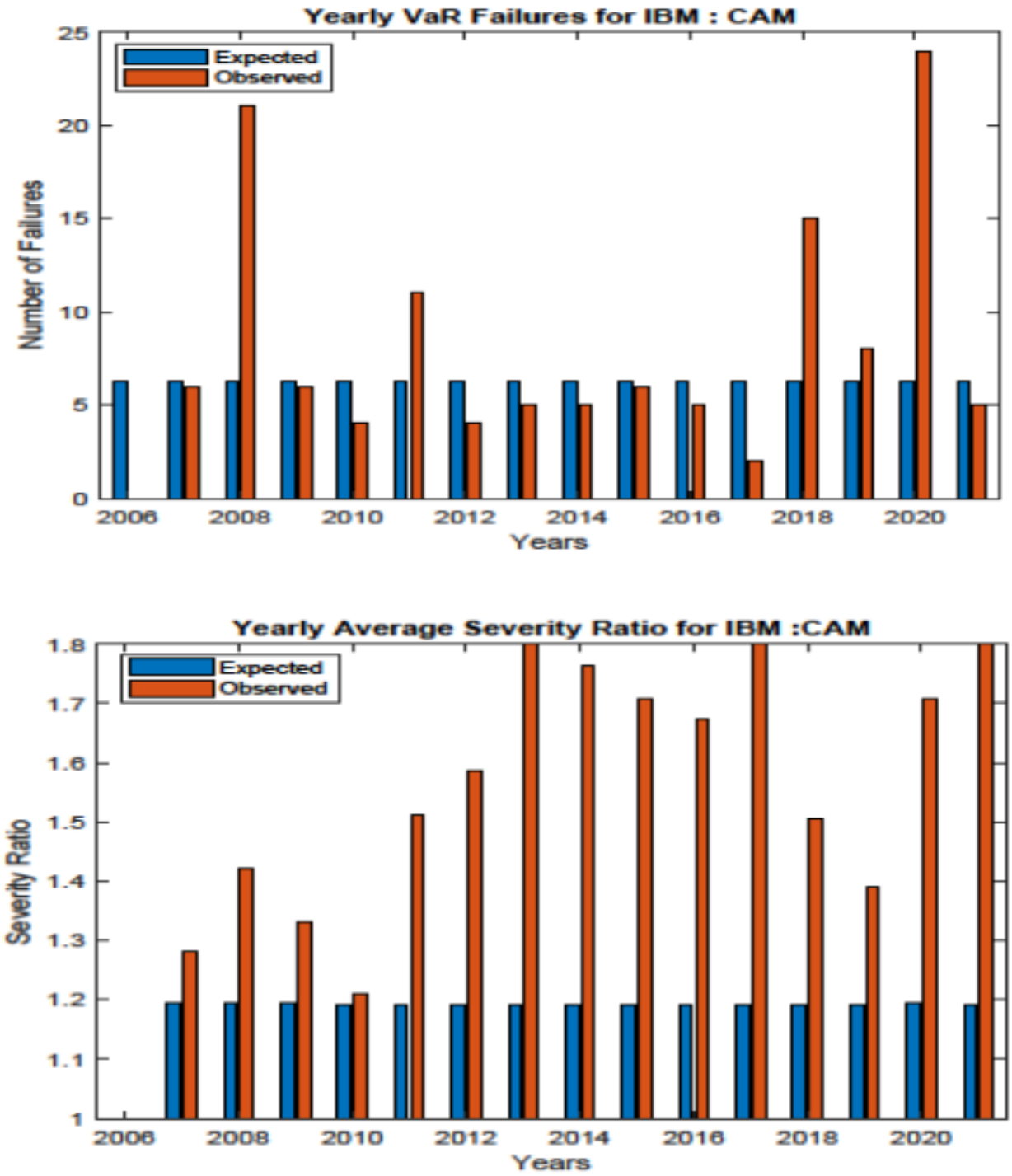


Figure-5.49: Yearly VaR failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets IBM using Conditional Analytic Markov (CAM) model.

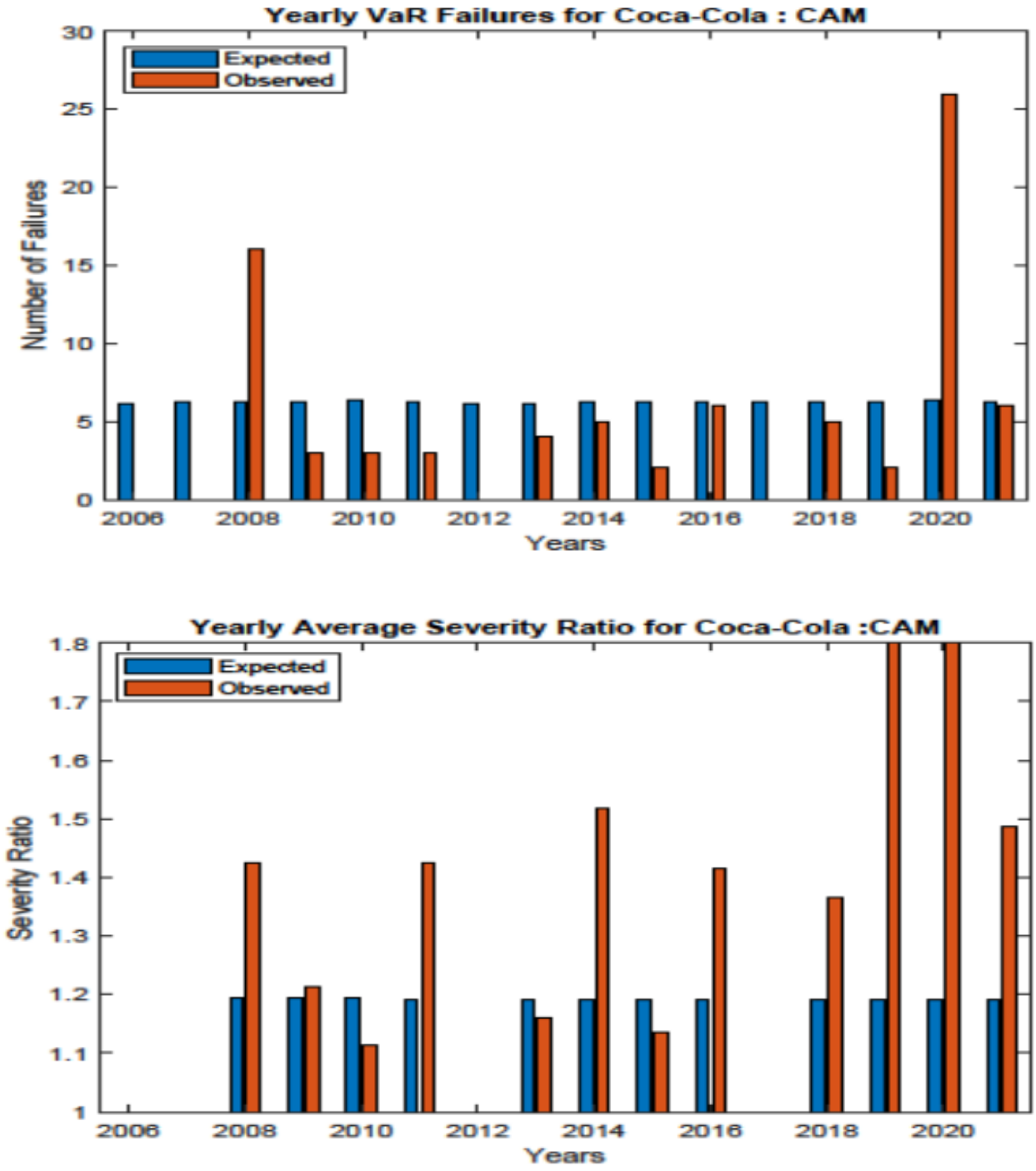


Figure-5.50: Yearly VaR failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets Coca-Cola using Conditional Analytic Markov (CAM) model.

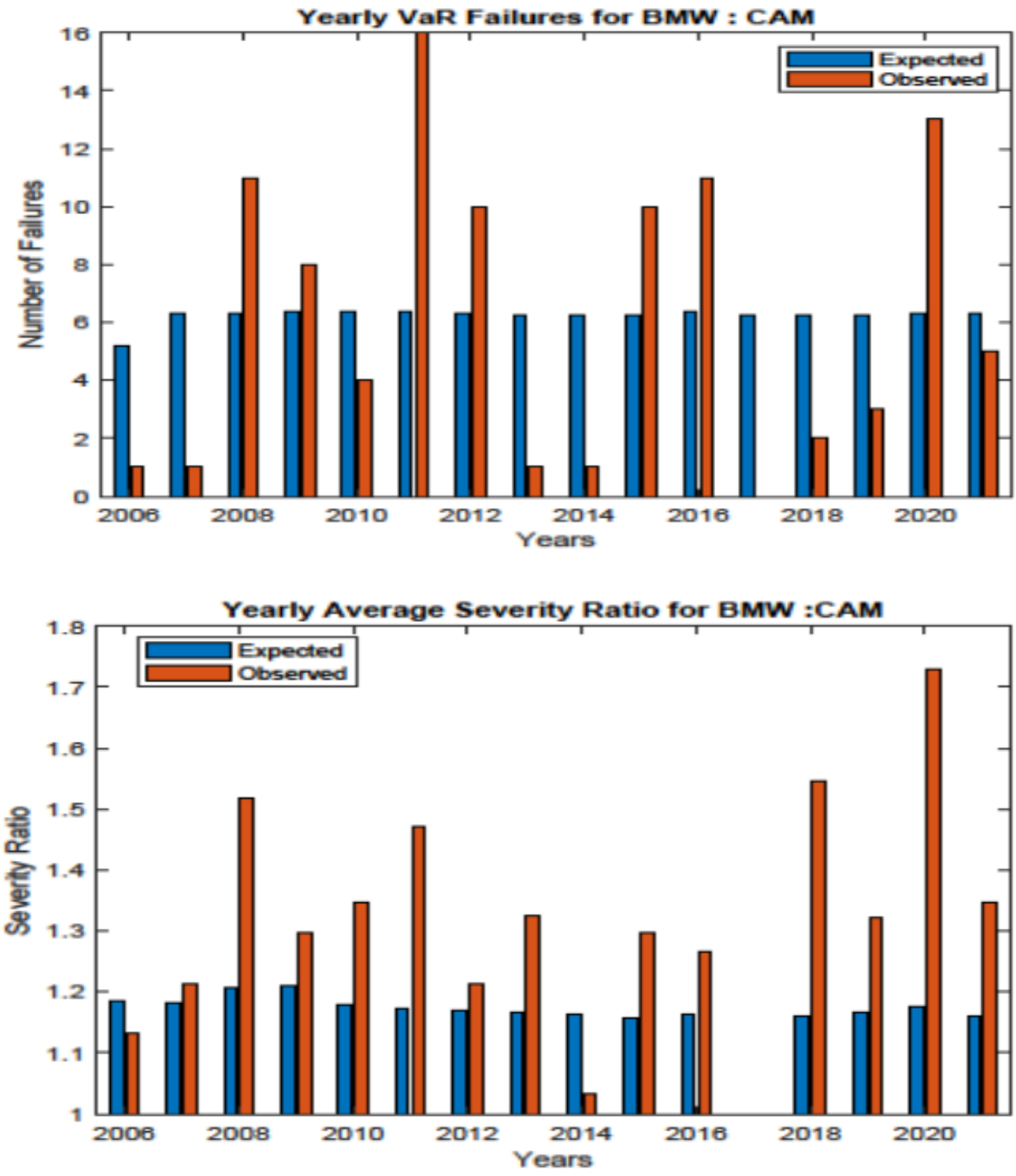


Figure-5.51: Yearly VaR failures (top) and yearly average severity ratio (bottom) over 2006-2022 for the assets BMW using Conditional Analytic Markov (CAM) model.

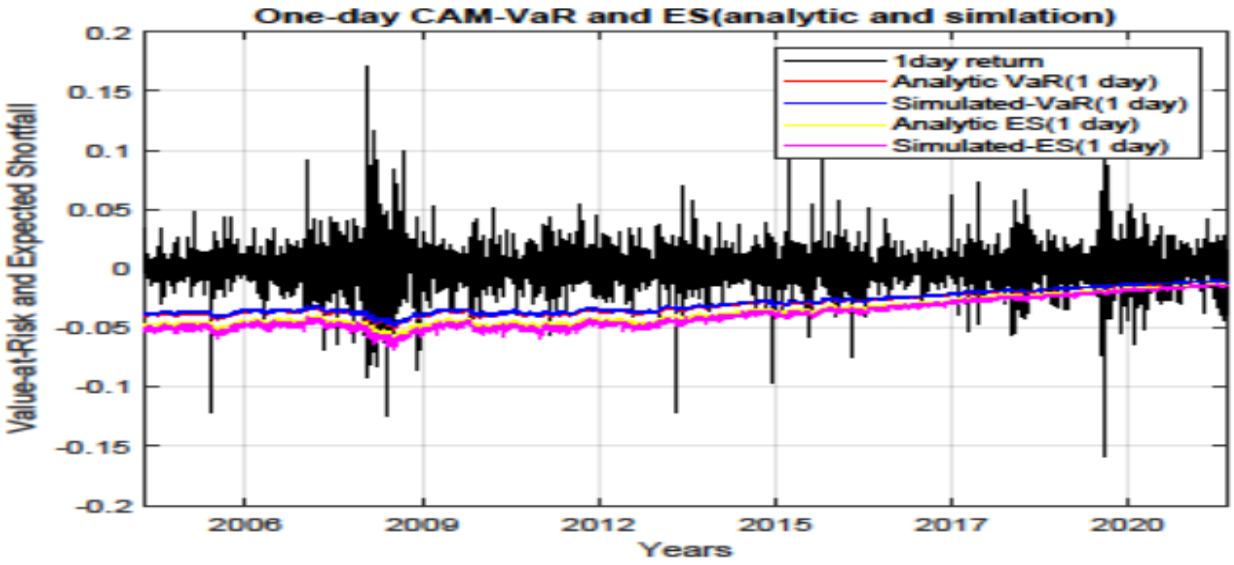


Figure-5.52: Analytic and Simulated VaR and ES forecasts for 1- days ahead under Conditional Analytic Markovian (CAM) model apparently in the absence of mean reversion for Microsoft returns. Moreover, in figure-5.3 shows that Microsoft index is clearly increasing over 2006-2022 and lacks evidence of mean reversion.

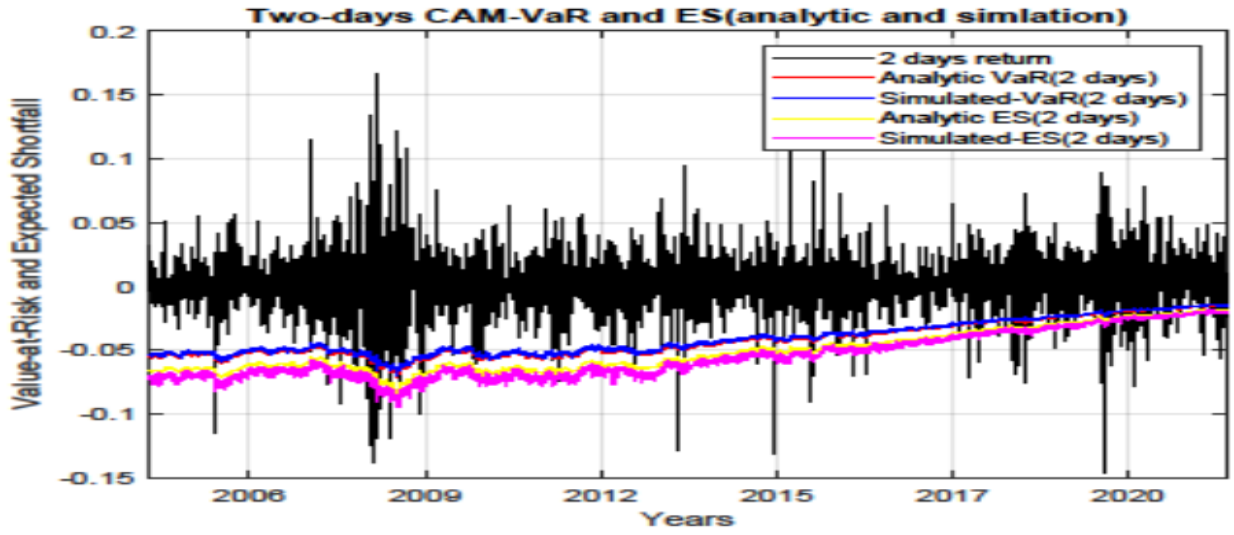


Figure-5.53: Analytic and Simulated VaR and ES forecasts for 2- days ahead under Conditional Analytic Markovian (CAM) model apparently in the absence of mean reversion for Microsoft returns. Moreover, in figure-5.3 shows that Microsoft index is clearly increasing over 2006-2022 and lacks evidence of mean reversion.

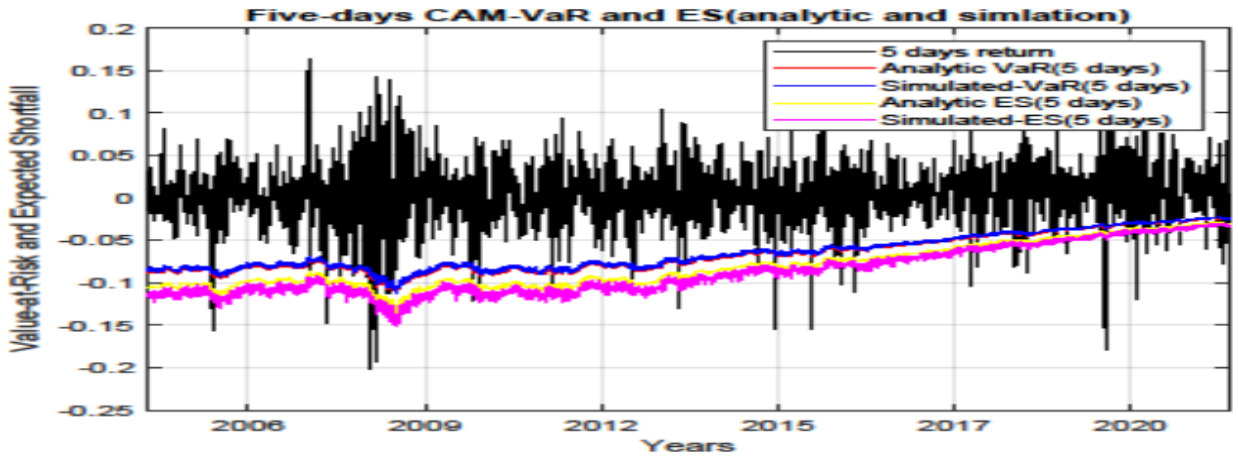


Figure-5.54: Analytic and Simulated VaR and ES forecasts for 5- days ahead under Conditional analytic Markovian (CAM) model apparently in the absence of mean reversion for Microsoft returns. Moreover, in figure-5.3 shows that Microsoft index is clearly increasing over 2006-2022 and lacks evidence of mean reversion.

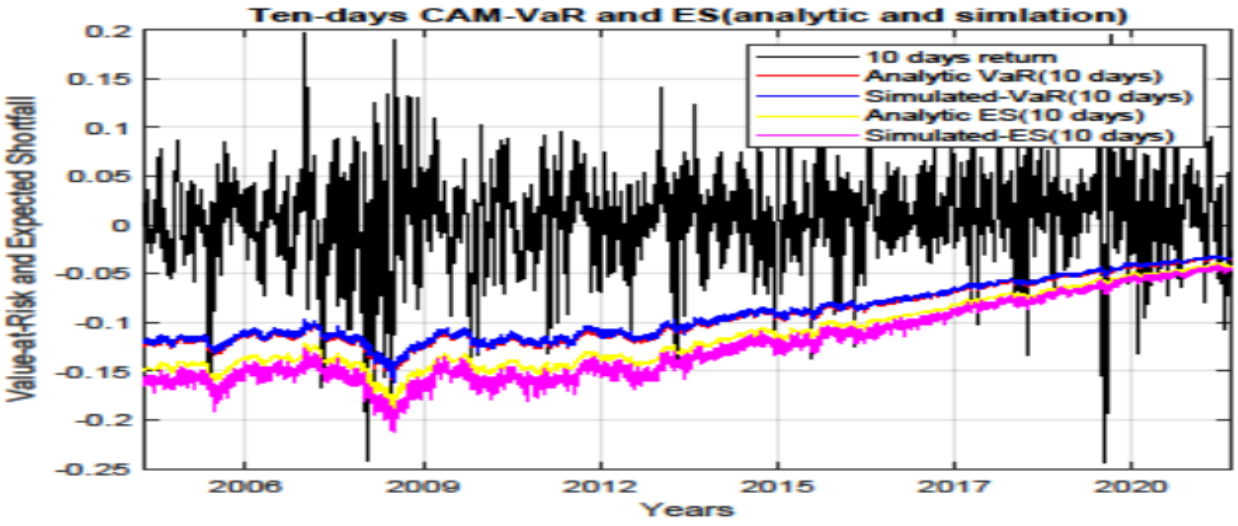


Figure-5.55: Analytic and Simulated VaR and ES forecasts for 10- days ahead under Conditional Analytic Markovian (CAM) model apparently in the absence of mean reversion for Microsoft returns. Moreover, in figure-5.3 shows that Microsoft index is clearly increasing over 2006-2022 and lacks evidence of mean reversion.

Figures 5.40–5.51 present the yearly back-testing performance that the Black–Scholes model exhibits the weakest risk forecasting performance. In contrast, the Conditional EVT model consistently provides the strongest back-testing performance throughout the sample period. For GE, IBM, Coca-Cola, and BMW, the yearly number of VaR violations remains closer to the theoretical expectation, while the yearly average ES severity ratios are generally the lowest among the three competing models. The Conditional Analytic Markov (CAM) model also performs consistently well across all four assets. Although it occasionally records slightly more VaR failures and marginally higher ES severity ratios than the Conditional EVT model, it substantially outperforms the Black–Scholes approach. The CAM model effectively captures structural shifts between low- and high-volatility market regimes, resulting in smoother and more stable risk forecasts over time. Its performance remains robust across different industries and varying market environments, indicating that regime-switching dynamics contribute significantly to improving risk estimation. These findings confirm that accounting for volatility clustering, heavy-tailed distributions, and changing market regimes is essential for reliable VaR and ES estimation over long evaluation periods and across diverse equity markets.

Figures 5.52–5.55 compare the analytic and Monte Carlo simulated forecasts, the figures demonstrate a high degree of arrangement between the analytic and simulated VaR and ES forecasts across all investment horizons. The close intersection between the two sets of forecasts indicates that the analytical approximation used in the CAM model accurately reproduces the results obtained from computationally intensive Monte Carlo simulations. These findings indicate that, for assets such as Microsoft, whose prices exhibit persistent growth rather than mean-reverting behaviour, the analytic CAM model offers an efficient and robust approach for both short- and long-horizon market risk forecasting, producing accurate VaR and ES estimates without sacrificing computational efficiency.

VaR violation	JPM (723)							BAC (694)						
	1-day	2-day		5-day		10-day		1-day	2-day		5-day		10-day	
		Param	Square	Param	Square	Param	Square		Param	Square	Param	Square	Param	Square
Expected	18	18	18	18	18	18	18	17	17	17	17	17	17	17
BS	10	8	8	6	6	2	2	8	5	5	3	3	3	3
GARCH-EVT	5	5	7	4	9	3	2	2	2	8	4	12	4	3
CAM	18	21	23	16	18	11	17	17	17	17	16	20	11	16

Table-5.9: Shows multi-period ahead VaR violations at the 97.5% level in different models during the period 2016-2018 for financial assets.

VaR violation	AIG (730)							PRU (731)						
	1-day	2-day		5-day		10-day		1-day	2-day		5-day		10-day	
		Param	Square	Param	Square	Param	Square		Param	Square	Param	Square	Param	Square
Expected	18	18	18	18	18	18	18	18	18	18	18	18	18	18
BS	7	9	9	9	9	15	15	15	12	12	12	12	11	11
GARCH-EVT	6	5	11	15	25	13	23	6	7	11	9	11	7	10
CAM	13	15	15	14	14	17	17	22	22	22	19	24	16	23

Table-5.10: Shows multi-period ahead VaR violations at the 97.5% level in different model during the period 2016-2018 for financial assets.

VaR violation	IBM (1254)							GE (1242)						
	1-day	2-day		5-day		10-day		1-day	2-day		5-day		10-day	
		Param	Square	Param	Square	Param	Square		Param	Square	Param	Square	Param	Square
Expected	105	105	105	105	105	105	105	102	102	102	102	102	102	102
BS	62	57	57	55	55	56	56	189	202	202	204	204	206	206
GARCH-EVT	34	28	60	43	80	23	80	28	29	47	73	60	47	59
CAM	127	123	125	132	139	133	141	99	102	106	106	110	108	117

Table-5.11: Shows multi-period ahead VaR violations at the 97.5% level in different model during the period 2016-2018 for non-financial assets.

VaR violations	Coca-Cola (1248)							BMW (1264)						
	1-day	2-day		5-day		10-day		1-day	2-day		5-day		10-day	
		Param	Square	Param	Square	Param	Square		Param	Square	Param	Square	Param	Square
Expected	103	103	103	103	103	103	103	106	106	106	106	106	106	106
BS	44	44	44	40	40	25	25	54	61	61	51	51	46	46
GARCH-EVT	32	27	40	38	38	22	48	14	14	37	38	47	27	47
CAM	81	80	82	67	72	61	67	85	92	104	60	89	58	86

Table-5.12: Shows multi-period ahead VaR violations at 97.5% level in different model during the period 2016-2018 for non-financial assets. Parenthesis shows the number of observations.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (234)	BS	G (0.16)	P (0.23)	P (0.18)	P (0.33)	P (0.40)	P (0.77)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.04)	F (0.01)	P (0.24)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.46)	P (0.72)	P (0.71)	P (0.13)	P (0.20)	P (0.08)	P (0.38)	P (0.37)
2017 (250)	BS	G (0.04)	P (0.08)	F (0.04)	P (0.27)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.18)	P (0.14)	P (0.27)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
2018 (239)	BS	G (0.44)	P (0.68)	P (0.67)	P (0.65)	P (0.82)	P (0.64)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.06)	P (0.09)	P (0.05)	P (0.65)	P (0.15)	P (0.85)	P (0.50)	P (0.50)
	CAM	Y (0.98)	F (0.03)	P (0.06)	P (0.63)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.13: At the 97.5% level, Value-at Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **JPM with a 1-day horizon** for the years 2016–2018 using the different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (233)	BS	G (0.16)	P (0.23)	P (0.19)	P (0.30)	F (0.03)	F (0.02)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.04)	F (0.01)	P (0.24)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.47)	P (0.72)	P (0.72)	P (0.10)	P (0.20)	P (0.08)	F (0.44)	F (0.43)
2017 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.18)	P (0.14)	P (0.73)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
2018 (240)	BS	G (0.44)	P (0.67)	P (0.67)	P (0.65)	P (0.82)	P (0.64)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.28)	P (0.40)	P (0.37)	P (0.65)	P (0.63)	P (0.71)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.01)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.14: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **JPM, 2-days** horizon for the year (2016-2018) **under parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (233)	BS	G (0.16)	P (0.23)	P (0.19)	P (0.30)	F (0.03)	F (0.02)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.06)	P (0.10)	P (0.06)	P (0.25)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.63)	P (0.94)	P (0.94)	P (0.10)	P (0.31)	P (0.12)	F (0.30)	F (0.30)
2017 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.30)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.24)	P (0.36)	P (0.32)	P (0.73)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
2018 (240)	BS	G (0.44)	P (0.67)	P (0.67)	P (0.65)	P (0.82)	P (0.64)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.44)	P (0.67)	P (0.67)	P (0.65)	P (0.82)	P (0.64)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.01)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.15: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **JPM, 2-days horizon** for the year (2016-2018) under **square-root-rule multi-period** characterization using three different Models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (230)	BS	G (0.07)	P (0.11)	P (0.06)	P (0.22)	P (0.18)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.07)	P (0.11)	P (0.06)	P (0.22)	P (0.18)	P (0.85)	P (0.50)	P (0.50)
2017 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (243)	BS	G (0.27)	P (0.39)	P (0.36)	P (0.73)	P (0.08)	F (0.04)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.27)	P (0.39)	P (0.36)	F (0.03)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.16: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **JP Morgan, 5-days** horizon for the year **(2016-2018)** under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (230)	BS	G (0.07)	P (0.11)	P (0.06)	P (0.22)	P (0.18)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.02)	F (0.04)	F (0.01)	P (0.26)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.07)	P (0.11)	P (0.06)	P (0.22)	P (0.18)	P (0.85)	P (0.50)	P (0.50)
2017 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (243)	BS	G (0.27)	P (0.39)	P (0.36)	P (0.73)	P (0.08)	F (0.04)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.84)	P (0.42)	P (0.45)	P (0.65)	F (0.00)	F (0.00)	P (0.33)	P (0.33)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table—5.17: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **JPM, 5-days** horizon for the **year (2006-2018**) under **square-root-rule** multi-period characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (225)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
2017 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (248)	BS	G (0.05)	P (0.08)	F (0.04)	F (0.01))	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.08)	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	Y (0.97)	P (0.05)	P (0.07)	P (0.68)	F (0.00)	F (0.00)	F (0.04)	P (0.06)

Table-5.18: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **JPM, 10-days** horizon for the **year (2006-2018**) under **parametric** multi-period characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (225)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.02)	F (0.04)	F (0.01)	P (0.29)	P (0.05)	P (0.92)	P (0.50)	P (0.50)
2017 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01))	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (248)	BS	G (0.05)	P (0.08	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	F (0.01)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.68)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.19: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of JP Morgan, 10-days** horizon for the year (2016-2018**) under square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (210)	BS	G (0.10)	P (0.15)	P (0.10)	P (0.40)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.03)	P (0.06)	F (0.02)	P (0.40)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.10)	P (0.15)	P (0.10)	P (0.40)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2017 (247)	BS	G (005)	P (0.08)	F (0.04)	P (0.76)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.08)	F (0.04)	P (0.76)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.25)	P (0.37)	P (0.34)	P (0.24)	P (0.59)	P (0.71)	P (0.50)	P (0.50)
2018 (237)	BS	G (0.29)	P (0.42)	P (0.39)	P (0.63)	P (0.65)	P (0.71)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	Y (0.98)	F (0.03)	P (0.05)	P (0.63)	P (0.13)	P (0.52)	F (0.00)	F (0.01)

Table-5.20: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Bank of America (BAC), 1-days** horizon for the year (2016-2018) using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (206)	BS	G (0.03)	P (0.06)	F (0.02)	P (0.40)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.03)	P (0.06)	F (0.02)	P (0.40)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.03)	P (0.06)	F (0.02)	P (0.40)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
2017 (247)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.76)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.25)	P (0.37)	P (0.34)	P (0.76)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2018 (241)	BS	G (0.15)	P (0.22)	P (0.17)	P (0.63)	P (0.38)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.01)	P (0.71)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.01)	F (0.02)	P (0.63)	F (0.00)	F (0.00)	F (0.00)	F (0.01)

Table-5.21: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Bank of America (BAC), 2-days** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (206)	BS	G (0.03)	P (0.06)	F (0.02)	P (0.40)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.03)	P (0.06)	F (0.02)	P (0.40)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.03)	P (0.06)	F (0.02)	P (0.40)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
2017 (247)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.76)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.08)	F (0.04)	P (0.76)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.25)	P (0.37)	P (0.34)	P (0.76)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2018 (241)	BS	G (0.15)	P (0.22)	P (0.17)	P (0.63)	P (0.38)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.45)	P (0.69)	P (0.68)	P (0.63)	P (0.19)	P (0.08)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.01)	F (0.02)	P (0.63)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.22: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Bank of America (BAC), 2-days** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (206)	BS	G (0.03)	P (0.06)	F (0.02)	P (0.42)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.02)	F (0.00)	F (0.02)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.03)	P (0.06)	F (0.02)	P (0.42)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
2017 (247)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.13)	P (0.19)	P (0.15)	P (0.76)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2018 (241)	BS	G (0.05)	P (0.09)	P (0.05)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.27)	P (0.40)	P (0.37)	F (0.02)	P (0.08)	F (0.04)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.01)	F (0.02)	P (0.70)	F (0.00)	F (0.00)	F (0.00)	F (0.01)

Table-5.23: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Bank of America (BAC), 5-days** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (206)	BS	G (0.03)	P (0.06)	F (0.02)	P (0.42)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.03)	P (0.06)	F (0.02)	P (0.42)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.03)	P (0.06)	F (0.02)	P (0.42)	P (0.07)	P (0.92)	P (0.50)	P (0.50)
2017 (247)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.13)	P (0.19)	P (0.15)	P (0.76)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.25)	P (0.37)	P (0.34)	P (0.76)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2018 (241)	BS	G (0.05)	P (0.09)	P (0.05)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.84)	P (0.41)	P (0.43)	P (0.63)	F (0.04)	F (0.01)	P (0.27)	P (0.28)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.63)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.24: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Bank of America (BAC), 5-days** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (201)	BS	G (0.00)	F (0.02)	F (0.00)	F (0.02)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.02)	F (0.00)	F (0.02)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.02)	F (0.00)	F (0.02)	F (0.00)	P (1)	P (0.50)	P (0.50)
2017 (247)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00))	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (246)	BS	G (0.13)	P (0.19)	P (0.15)	F (0.01)	F (0.02)	F (0.02)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.13)	P (0.19)	P (0.15)	F (0.01)	F (0.02)	F (0.02)	P (0.50)	P (0.50)
	CAM	Y (0.97)	F (0.04)	P (0.07)	P (0.68)	F (0.00)	F (0.00)	F (0.03)	P (0.50)

Table-5.25: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Bank of America (BAC), 10-day** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (201)	BS	G (0.00)	F (0.02)	F (0.00)	F (0.02)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.02)	F (0.00)	F (0.02)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.02)	F (0.00)	F (0.02)	F (0.00)	P (1)	P (0.50)	P (0.50)
2017 (247)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (246)	BS	G (0.13)	P (0.19)	P (0.15)	F (0.01)	F (0.02)	F (0.02)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.13)	P (0.19)	P (0.15)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.71)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.26: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Bank of America (BAC), 10-days** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (239)	BS	G (0.01)	F (0.03)	F (0.01)	P (0.21)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.06)	P (0.09)	P (0.05)	P (0.21)	P (0.15)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.28)	P (0.41)	P (0.38)	P (0.44)	P (0.09)	F (0.04)	P (0.50)	P (0.50)
2017 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.82)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.08)	F (0.04)	P (0.82)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.05)	P (0.08)	F (0.04)	P (0.82)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
2018 (242)	BS	G (0.27)	P (0.39)	P (0.36)	P (0.37)	P (0.62)	P (0.71)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.14)	P (0.20)	P (0.16)	P (0.37)	P (0.36)	P (0.78)	P (0.50)	P (0.50)
	CAM	G (0.73)	P (0.69)	P (0.70)	P (0.65)	P (0.75)	P (0.51)	P (0.29)	P (0.29)

Table-5.27: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **American Investment Group (AIG), 1-day** horizon for the year (2016-2018) using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (239)	BS	G (0.01)	F (0.03)	F (0.01)	P (0.21)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.01)	P (0.21)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.15)	P (0.22)	P (0.17)	P (0.22)	F (0.02)	F (0.02)	P (0.50)	P (0.50)
2017 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.82)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.05)	P (0.08)	F (0.04)	P (0.82)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2018 (242)	BS	G (0.59)	P (0.97)	P (0.97)	P (0.37)	F (0.01)	F (0.00)	P (0.46)	P (0.44)
	GARCH-EVT	G (0.27)	P (0.39)	P (0.36)	P (0.37)	P (0.61)	P (0.71)	P (0.50)	P (0.50)
	CAM	Y (0.95)	P (0.10)	P (0.13)	P (0.65)	F (0.00)	F (0.00)	F (0.04)	P (0.06)

Table-5.28: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **American Investment Group (AIG), 2-day** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (239)	BS	G (0.01)	F (0.03)	F (0.01)	P (0.21)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.15)	P (0.22)	P (0.17)	P (0.22)	F (0.02)	F (0.02)	P (0.50)	P (0.50)
	CAM	G (0.15)	P (0.22)	P (0.17)	P (0.22)	F (0.02)	F (0.02)	P (0.50)	P (0.50)
2017 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.82)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.08)	F (0.04)	P (0.82)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.05)	P (0.08)	F (0.04)	P (0.82)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2018 (242)	BS	G (0.59)	P (0.97)	P (0.97)	P (0.37)	F (0.01)	F (0.00)	P (0.46)	P (0.44)
	GARCH-EVT	G (0.73)	P (0.70)	P (0.71)	P (0.65)	F (0.03)	F (0.01)	P (0.37)	P (0.37)
	CAM	Y (0.95)	P (0.10)	P (0.13)	P (0.65)	F (0.00)	F (0.00)	F (0.03)	P (0.06)

Table-5.29: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of American Investment Group (AIG), 2-day** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (235)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.04)	F (0.01)	P (0.28)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.04)	F (0.01)	P (0.23)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
2017 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (246)	BS	G (0.90)	P (0.24)	P (0.27)	F (0.02)	F (0.00)	F (0.00)	P (0.50)	P (0.07)
	GARCH-EVT	Y (0.99)	F (0.00)	F (0.01)	P (0.73)	F (0.00)	F (0.00)	P (0.05)	F (0.04)
	CAM	Y (0.99)	F (0.00)	F (0.01)	P (0.73)	F (0.00)	F (0.00)	F (0.02)	F (0.00)

Table-5.30: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **American Investment Group (AIG), 5-day** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (235)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.46)	P (0.71))	P (0.70)	P (0.28)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.04)	F (0.01)	P (0.23)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
2017 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.08)	F (0.04)	P (0.82)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (246)	BS	G (0.90)	P (0.24)	P (0.27)	F (0.02)	F (0.00)	F (0.00)	P (0.05)	P (0.07)
	GARCH-EVT	R (0.99)	F (0.00)	F (0.00)	P (0.73)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	CAM	Y (0.99)	F (0.00)	F (0.01)	P (0.73)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.31: Backtesting results for Value-at-Risk (VaR) and Expected Shortfall (ES) of **American Investment Group (AIG), 5-day** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (230)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
2017 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	P (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (251)	BS	Y (0.99)	F (0.00)	F (0.00)	F (0.02)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	Y (0.98)	F (0.02)	F (0.03)	F (0.02)	F (0.00)	F (0.00)	F (0.04)	P (0.06)
	CAM	R (0.99)	F (0.00)	F (0.00)	F (0.02)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.32: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of American Investment Group (AIG), 10-day** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (230)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.02)	F (0.04)	F (0.01)	P (0.26)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
2017 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.07)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (251)	BS	Y (0.99)	F (0.00)	F (0.00)	F (0.02)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	R (1)	F (0.00)	F (0.00)	P (0.73)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	CAM	R (0.99)	F (0.00)	F (0.00)	F (0.02)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.33: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **American Investment Group (AIG), 10-days** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (238)	BS	G (0.28)	P (0.41)	P (0.38)	P (0.39)	P (0.09)	F (0.04)	P (0.41)	P (0.40)
	GARCH-EVT	G (0.06)	P (0.10)	P (0.05)	P (0.39)	P (0.16)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.45)	P (0.69)	P (0.68)	P (0.39)	P (0.19)	P (0.08)	P (0.23)	P (0.24)
2017 (251)	BS	G (0.04)	P (0.08)	F (0.04)	P (0.30)	P (0.12)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.24)	P (0.35)	P (0.32)	P (0.73)	P (0.57)	P (0.71)	P (0.50)	P (0.50)
2018 (242)	BS	G (0.91)	P (0.22)	P (0.25)	P (0.65)	P (0.37)	P (0.40)	P (0.05)	P (0.07)
	GARCH-EVT	G (0.59)	P (0.98)	P (0.98)	P (0.65)	P (0.85)	P (0.57)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.01)	P (0.55)	F (0.00)	F (0.02)	F (0.00)	F (0.00)

Table-5.34: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **Prudential Capital (PRU), 1-day** horizon for the year (2016-2018) using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (237)	BS	G (0.15)	P (0.22)	P (0.17)	P (0.27)	P (0.39)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.04)	F (0.01)	P (0.23)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.15)	P (0.22)	P (0.17)	P (0.27)	P (0.39)	P (0.78)	P (0.50)	P (0.50)
2017 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.08)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2018 (243)	BS	G (0.91)	P (0.22)	P (0.26)	P (0.58)	P (0.32)	P (0.32)	P (0.06)	P (0.08)
	GARCH-EVT	G (0.59)	P (0.97)	P (0.97)	P (0.58)	P (0.85)	P (0.58)	P (0.50)	P (0.50)
	CAM	R (0.99)	F (0.00)	F (0.00)	P (0.58)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.35: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **Prudential Capital (PRU), 2-days** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (237)	BS	G (0.15)	P (0.22)	P (0.17)	P (0.27)	P (0.39)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.06)	P (0.10)	P (0.05)	P (0.36)	P (0.16)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.15)	P (0.22)	P (0.17)	P (0.27)	P (0.39)	P (0.78)	P (0.50)	P (0.50)
2017 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.08)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2018 (243)	BS	G (0.91)	P (0.22)	P (0.26)	P (0.58)	P (0.32)	P (0.32)	P (0.06)	P (0.08)
	GARCH-EVT	G (0.91)	P (0.22)	P (0.26)	P (0.58)	P (0.32)	P (0.32)	P (0.18)	P (0.19)
	CAM	R (0.99)	F (0.00)	F (0.00)	P (0.58)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.36: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **Prudential Capital (PRU), 2-days** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (234)	BS	G (0.16)	P (0.23)	P (0.18)	P (0.19)	P (0.40)	P (0.77)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.16)	P (0.23)	P (0.18)	P (0.19)	P (0.40)	P (0.77)	P (0.50)	P (0.50)
2017 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (246)	BS	G (0.90)	P (0.24)	P (0.27)	P (0.65)	F (0.00)	F (0.00)	P (0.07)	P (0.09)
	GARCH-EVT	G (0.83)	P (0.44)	P (0.46)	P (0.65)	F (0.04)	F (0.01)	P (0.34)	P (0.33)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.63)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.37: Backtesting results for Value-at-Risk (VaR) and Expected Shortfall (ES) of **Prudential Capital (PRU), 5-day** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (234)	BS	G (0.16)	P (0.23)	P (0.18)	P (0.19)	P (0.40)	P (0.77)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.06)	F (0.10)	P (0.06)	P (0.27)	P (0.17)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.30)	P (0.43)	P (0.41)	P (0.19)	P (0.66)	P (0.70)	P (0.50)	P (0.50)
2017 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (246)	BS	G (0.90)	P (0.24)	P (0.27)	P (0.65)	F (0.00)	F (0.00)	P (0.07)	P (0.09)
	GARCH-EVT	G (0.90)	P (0.24)	P (0.27)	P (0.63)	F (0.00)	F (0.00)	P (0.15)	P (0.17)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.63)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.38: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) **of Prudential Capital (PRU), 5-day** horizon for the year (2016-2018) under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (229)	BS	G (0.07)	P (0.11)	P (0.06)	P (0.13)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	P (0.00)	F (0.01)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.07)	P (0.11)	P (0.06)	P (0.13)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2017 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2018 (251)	BS	G (0.89)	P (0.27)	P (0.30)	P (0.73)	F (0.00)	F (0.00)	P (0.13)	P (0.15)
	GARCH-EVT	G (0.56)	P (0.91)	P (0.91)	P (0.73)	F (0.01)	F (0.00)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.39: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **Prudential Capital (PRU), 10-day** horizon for the year (2016-2018) under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2016 (229)	BS	G (0.07)	P (0.11)	P (0.06)	P (0.13)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.02)	F (0.04)	P (0.01)	P (0.13)	F (0.04)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.07)	P (0.11)	P (0.06)	P (0.13)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2017 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.04)	P (0.08)	F (0.04)	P (0.06)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2018 (251)	BS	G (0.89)	P (0.27)	P (0.30)	P (0.73)	F (0.00)	F (0.00)	P (0.13)	P (0.15)
	GARCH-EVT	G (0.89)	P (0.27)	P (0.30)	P (0.65)	F (0.00)	F (0.00)	P (0.13)	P (0.15)
	CAM	R (0.99)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)

Table-5.40: Backtesting results at the 97.5% level for Value-at-Risk (VaR) and Expected Shortfall (ES) of **Prudential Capital (PRU), 10-day** horizon for the year (2016-2018) **under square-root-rule** multi-period characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.04)	P (0.08)	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.24)	P (0.35)	P (0.32)	P (0.33)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
	CAM	G (0.56)	P (0.91)	P (0.91)	P (0.33)	P (0.29)	P (0.11)	P (0.44)	P (0.43)
2008 (252)	BS	Y (0.99)	F (0.00)	F (0.00)	F (0.04)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	F (0.03)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (252)	BS	G (0.12)	P (0.18)	P (0.13)	P (0.91)	P (0.32)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	F (0.02)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.55)	P (0.90)	P (0.90)	P (0.33)	P (0.85)	P (0.58)	P (0.41)	P (0.40)
2010 (250)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.30)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.24)	P (0.36)	P (0.32)	P (0.33)	P (0.58)	P (0.71)	P (0.50)	P (0.50)

Table-5.41: At the 97.5% level, Value-at Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **IBM** with a **1-day** horizon for the years 2006–2010 using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.04)	P (0.08)	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.04)	P (0.08)	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.18)	P (0.14)	P (0.09)	F (0.02)	F (0.01)	P (0.50)	P (0.50)
2008 (252)	BS	Y (0.97)	P (0.05)	P (0.08)	F (0.03)	F (0.00)	F (0.00)	F (0.00)	F (0.01)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	F (0.03)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	CAM	R (0.99)	F (0.00)	F (0.00)	F (0.03)	F (0.00)	F (0.02)	F (0.00)	F (0.00)
2009 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.18)	P (0.13)	P (0.82)	P (0.32)	P (0.78)	P (0.50)	P (0.50)
2010 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.18)	P (0.14)	P (0.36)	P (0.33)	P (0.78)	P (0.50)	P (0.50)

Table-5.42: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **IBM** with a **2-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models**.**

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.04)	P (0.08)	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.12)	P (0.18)	P (0.14)	P (0.97)	F (0.02)	F (0.01)	P (0.50)	P (0.50)
	CAM	G (0.24)	P (0.35)	P (0.32)	P (0.09)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2008 (252)	BS	Y (0.97)	P (0.05)	P (0.08)	F (0.03)	F (0.00)	F (0.00)	F (0.00)	F (0.01)
	GARCH-EVT	G (0.12)	P (0.18)	P (0.13)	P (0.05)	P (0.32)	P (0.78)	P (0.50)	P (0.50)
	CAM	R (0.99)	F (0.00)	F (0.00)	F (0.03)	F (0.00)	F (0.02)	F (0.00)	F (0.00)
2009 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	F (0.02)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.24)	P (0.35)	P (0.32)	P (0.82)	P (0.57)	P (0.71)	P (0.50)	P (0.50)
2010 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.18)	P (0.14)	P (0.36)	P (0.33)	P (0.78)	P (0.50)	P (0.50)

Table-5.43: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **IBM** with a **2-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.04)	P (0.08)	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.40)	P (0.60)	P (0.59)	P (1)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.40)	P (0.60)	P (0.59)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2008 (252)	BS	Y (0.99)	F (0.00)	F (0.00)	P (0.10)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.81)	P (0.49)	P (0.51)	P (0.10)	F (0.00)	F (0.00)	P (0.38)	P (0.38)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.08)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (252)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.91)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.91)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
2010 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.44)	F (0.03)	P (0.92)	P (0.50)	P (0.50)

Table-5.44: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **IBM** with a **5-d**ay horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models**.**

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.04)	P (0.08)	F (0.04)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.40)	P (0.60)	P (0.59)	P (1)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.40)	P (0.60)	P (0.59)	F (0.01)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2008 (252)	BS	Y (0.99)	F (0.00)	F (0.00)	P (0.10)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	Y (0.97)	P (0.05)	P (0.08)	P (0.10)	F (0.00)	F (0.00)	P (0.16)	P (0.18)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.08)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (252)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.91)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.91)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
2010 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.44)	F (0.03)	P (0.92)	P (0.50)	P (0.50)

Table-5.45: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **IBM** with a **5-d**ay horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models**.**

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GAECH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.01)	F (0.03)	F (0.00)	F (0.01)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.24)	P (0.35)	P (0.32)	F (0.01)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
2008 (252)	BS	R (1)	F (0.00)	F (0.00)	P (0.13)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.55)	P (0.90)	P (0.90)	F (0.03)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.13)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (252)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.91)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.04)	P (0.08)	F (0.04)	P (0.91)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2010 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)

Table-5.46: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **IBM** with a **10-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models**.**

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.01)	F (0.03)	F (0.00)	F (0.01)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.24)	P (0.35)	P (0.32)	P (0.96)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
	CAM	G (0.24)	P (0.35)	P (0.32)	F (0.01)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
2008 (252)	BS	R (1)	F (0.00)	F (0.00)	P (0.13)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.55)	P (0.90)	P (0.90)	F (0.03)	F (0.00)	F (0.00)	P (0.49)	P (0.47)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.13)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (252)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.91)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.04)	P (0.08)	F (0.04)	P (0.91)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
2010 (250)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)

Table-5.47: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **IBM** with a **10-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (245)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.36)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.36)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.36)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2007 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.08)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	P (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.05)	P (0.08)	F (0.04)	P (0.08)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
2008 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	P (0.33)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.12)	P (0.18)	P (0.14)	P (0.52)	P (0.32)	P (0.78)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	P (0.70)	F (0.00)	F (0.00)
2009 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.10)	F (0.00)	P(0.13)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.10)	F (0.00)	F (0.01)	F (0.00)	F (0.00)
2010 (246)	BS	G (0.90)	P (0.24)	P (0.27)	P (0.63)	P (0.39)	P (0.40)	P (0.08)	P (0.09)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	F (0.03)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.41)	P (0.63)	P (0.62)	P (0.39)	P (0.80)	P (0.64)	P (0.50)	P(0.50)

Table-5.48: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **GE** with **1-day** horizon for the years 2006–2010 using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (245)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.09)	F(0.04)	P (0.36)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.08)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.19)	P (0.14)	P (0.10)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
2008 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.24)	P (0.35)	P (0.32)	P (0.52)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2010 (246)	BS	Y (0.95)	P (0.11)	P (0.14)	P (0.63)	F (0.00)	F (0.00)	P (0.05)	P (0.07)
	GARCH-EVT	G (0.05)	F (0.00)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.26)	P (0.37)	P (0.34)	P (0.40)	F (0.00)	F (0.00)	P (0.50)	P (0.50)

Table-5.49: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **GE** with **2-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (245)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.36)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.08)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.19)	P (0.14)	P (0.10)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
2008 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.24)	P (0.35)	P (0.32)	P (0.52)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.65)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.97)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2010 (246)	BS	Y (0.95)	P (0.11)	P (0.14)	P (0.63)	F (0.00)	F (0.00)	P (0.05)	P (0.07)
	GARCH-EVT	G (0.05)	P (0.09)	F (0.04)	P (0.39)	P (0.14)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.26)	P (0.37)	P (0.34)	P (0.40)	F (0.00)	F (0.00)	P (0.50)	P (0.50)

Table-5.50: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **GE** with **2-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (245)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.09)	F (0.04)	P (0.41)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (249)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.07)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.07)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.07)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2008 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.70)	P (0.76)	P (0.77)	P (0.33)	P (0.38)	P (0.17)	P (0.48)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.12)	P (0.18)	P (0.14)	P (0.91)	F (0.02)	F (0.01)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2010 (246)	BS	G (0.83)	P (0.44)	P (0.46)	P (0.39)	F (0.00)	F (0.00)	P (0.20)	P (0.19)
	GARCH-EVT	G (0.05)	P (0.09)	F (0.04)	P (0.39)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.13)	P (0.19)	P (0.15)	P (0.39)	F (0.02)	F (0.02)	P (0.50)	P (0.50)

Table-5.51: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **GE** with **5-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (245)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.26)	P (0.38)	P (0.35)	P (0.36)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (249)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.07)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.07)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2008 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.56)	P (0.91)	P (0.91)	P (0.52)	F (0.00)	F (0.00)	P (0.44)	P (0.43)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.04)	P (0.08)	F (0.04)	P (0.94)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2010 (246)	BS	G (0.83)	P (0.44)	P (0.46)	P (0.39)	F (0.00)	F (0.00)	P (0.19)	P (0.20)
	GARCH-EVT	G (0.05)	P (0.09)	F (0.04)	P (0.39)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.13)	P (0.19)	P (0.15)	P (0.39)	F (0.02)	F (0.02)	P (0.50)	P (0.50)

Table-5.52: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **GE** with **5-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models**.**

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (245)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.52)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.24)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.40)	P (0.60)	P (0.59)	P (0.97)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.27)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2010 (246)	BS	G (0.83)	P (0.44)	P (0.46)	P (0.38)	F (0.00)	F (0.00)	P (0.23)	P (0.22)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.15)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.28)	F (0.03)	P (0.92)	P (0.50)	P (0.50)

Table-5.53: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **GE** with **10-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models**.**

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (245)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.05)	P (0.09)	F (0.04)	P (0.41)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.52)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.40)	P (0.60)	P (0.59)	P (0.49)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.33)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (251)	BS	R (1)	F (0.00)	F (0.00)	P (0.24)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.04)	P (0.08)	F (0.04)	P (0.96)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.27)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2010 (246)	BS	G (0.83)	P (0.44)	P (0.46)	P (0.38)	F (0.00)	F (0.00)	P (0.22)	P (0.23)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.38)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	G (0.05)	P (0.09)	F (0.04)	P (0.28)	F (0.00)	F (0.00)	P (0.50)	P (0.50)

Table-5.54: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **GE** with **10-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (246)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.97)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (250)	BS	Y (0.99)	F (0.00)	F (0.01)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.12)	P (0.18)	P (0.14)	P (0.11)	F (0.00)	P (0.78)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.22)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (249)	BS	G (0.12)	P (0.19)	P (0.14)	P (0.24)	F (0.00)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.19)	P (0.14)	P (0.24)	F (0.00)	P (0.78)	P (0.50)	P (0.50)
2010 (252)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.95)	F (0.00)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.95)	F (0.00)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.18)	P (0.13)	P (0.95)	F (0.00)	P (0.78)	P (0.50)	P (0.50)

Table-5.55: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **Coca-Cola** with **1-day** horizon for the years 2006–2010 using three different models**.**

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (246)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (250)	BS	Y (0.99)	F (0.00)	F (0.00)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.24)	P (0.36)	P (0.32)	P (0.39)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (0.99)	F (0.00)	F (0.00)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (249)	BS	G (0.12)	P (0.19)	P (0.14)	P (0.73)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.19)	P (0.14)	P (0.73)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
2010 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00))	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.97)	F (0.02)	P (0.92)	P (0.50)	P (0.50)

Table-5.56: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **Coca-Cola** with **2-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (246)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (250)	BS	Y (0.99)	F (0.00)	F (0.00)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.40)	P (0.61)	P (0.60)	P (0.39)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (0.99)	F (0.00)	F (0.00)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (249)	BS	G (0.12)	P (0.19)	P (0.14)	P (0.73)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.12)	P (0.19)	P (0.14)	P (0.73)	P (0.33)	P (0.78)	P (0.50)	P (0.50)
2010 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00))	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.97)	F (0.02)	P (0.92)	P (0.50)	P (0.50)

Table-5.57: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **Coca-Cola** with **2-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (246)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (250)	BS	Y (0.97)	P (0.05)	P (0.08)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.71)	P (0.76)	P (076)	P (0.39)	F (0.00)	F (0.00)	P (0.32)	P (0.32)
	CAM	Y (0.99)	F (0.00)	F (0.01)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.96)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.05)	P (0.08)	F (0.04)	P (0.96)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
2010 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)

Table-5.58: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **Coca-Cola** with **5-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (246)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (250)	BS	Y (0.97)	P (0.05)	P (0.08)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.71)	P (0.76)	P (0.76)	P (0.39)	F (0.00)	F (0.00)	P (0.27)	P (0.27)
	CAM	Y (0.99)	F (0.00)	F (0.00)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (249)	BS	G (0.05)	P (0.08)	F (0.04)	P (0.96)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.05)	P (0.08)	F (0.04)	P (0.96)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
2010 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)

Table-5.59: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **Coca-Cola** with **5-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (246)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (250)	BS	G (0.82)	P (0.47)	P (0.49)	P (0.47)	F (0.00)	F (0.00)	F (0.02)	F (0.04)
	GARCH-EVT	G (0.04)	P (0.08)	F (0.04)	P (0.47)	P (0.13)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.94)	P (0.12)	P (0.16)	P (0.47)	F (0.00)	F (0.00)	F (0.00)	F (0.01)
2009 (249)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.92)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.92)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2010 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)

Table-5.60: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **Coca-Cola** with **10-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (246)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (251)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (250)	BS	G (0.82)	P (0.47)	P (0.49)	P (0.47)	F (0.00)	F (0.00)	F (0.04)	F (0.02)
	GARCH-EVT	G (0.71)	P (0.76)	P (0.76)	P (0.47)	F (0.00)	F (0.00)	P (0.35)	P (0.36)
	CAM	Y (0.97)	P (0.05)	P (0.08)	P (0.44)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (249)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.92)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.92)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2010 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)

Table-5.61: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **Coca-Cola** with **10-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.30)	F (0.03)	P (0.92)	P (0.50)	P (0.50)
2007 (252)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.09)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.09)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
2008 (253)	BS	Y (0.98)	F (0.02)	F (0.04)	P (0.68)	F (0.00)	F (0.01)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.94)	P (0.13)	P (0.17)	P (0.68)	P (0.26)	P (0.36)	F (0.01)	F (0.03)
2009 (254)	BS	G (0.81)	P (0.50)	P (0.52)	P (0.24)	P (0.62)	P (0.46)	P (0.17)	P (0.19)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.69)	P (0.79)	P (0.79)	P (0.24)	P (0.79)	P (0.52)	P (0.32)	P (0.32)
2010 (256)	BS	G (0.04)	P (0.07)	F (0.03)	P (0.29)	P (0.11)	P (0.85)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.11)	P (0.17)	P (0.12)	P (0.29)	P (0.30)	P (0.78)	P (0.50)	P (0.50)

Table-5.62: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **BMW** with **1-day** horizon for the years 2006–2010 using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (252)	BS	G (0.01)	F (0.03)	F (0.00)	F (0.01)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	F (0.01)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	F (0.01)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
2008 (253)	BS	R (0.99)	F (0.00)	F (0.00)	P (0.07)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	R (0.99)	F (0.00)	F (0.00)	P (0.07)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (254)	BS	G (0.89)	P (0.28)	P (0.31)	P (0.24)	F (0.00)	F (0.00)	P (0.11)	P (0.14)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.89)	P (0.28)	P (0.31)	P (0.24)	F (0.00)	F (0.00)	P (0.15)	P (0.17)
2010 (256)	BS	G (0.11)	P (0.17)	P (0.12)	P (0.28)	F (0.02)	F (0.01)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.23)	P (0.33)	P (0.30)	P (0.28)	F (0.07)	F (0.04)	P (0.50)	P (0.50)

Table-5.63: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **BMW** with **2-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

In summary, Tables 5.9–5.12 present empirical evidence that the Conditional Analytic Markov (CAM) model yields the most precise and resilient multi-period VaR forecasts for both financial and non-financial assets. The findings indicate that the CAM model reliably yields VaR violation counts that closely align with theoretical expectations for the majority of assets and forecasting periods. For financial institutions, CAM produces nearly the anticipated 17–18 exceedances for JP Morgan, Bank of America, AIG, and Prudential Capital, especially at the one-day horizon and inside the parametric multi-period framework. The violation frequencies consistently correspond with theoretical expectations, demonstrating better calibration and dependable coverage probability. The

GARCH–EVT model effectively captures heavy-tailed risk but demonstrates increased variability in forecasting performance across different assets and investment horizons. Thus, GARCH–EVT competently captures extreme market fluctuations, its calibration exhibits greater inconsistency across various assets and forecasting horizons compared to the CAM model. The superiority of the CAM model is particularly noticeable among non-financial assets. For IBM, CAM generates annual violation counts between around 123 and 141, which are significantly nearer to the theoretical benchmark than the somewhat underestimated numbers provided by the Black–Scholes model.

		VaR Back-testing						ES Back-testing	
Year (Observations)	**Model**	**TL**	**BIN**	**POF**	**TUFF**	**CC**	**CCI**	**UCN**	**UNT**
2006 (249)	**BS**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
	GARCH-EVT	**G (0.01)**	**F (0.03)**	**F (0.00)**	**F (0.01)**	**F (0.03)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
2007 (252)	**BS**	**G (0.01)**	**F (0.03)**	**F (0.00)**	**F (0.01)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	GARCH-EVT	**G (0.01)**	**F (0.03)**	**F (0.00)**	**F (0.01)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.01)**	**F (0.03)**	**F (0.00)**	**F (0.01)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
2008 (253)	**BS**	**R (0.99)**	**F (0.00)**	**F (0.00)**	**P (0.70)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**F (0.00)**
	GARCH-EVT	**G (0.01)**	**F (0.03)**	**F (0.00)**	**P (0.28)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	CAM	**R (0.99)**	**F (0.00)**	**F (0.00)**	**P (0.70)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**F (0.00)**
2009 (254)	**BS**	**G (0.89)**	**P (0.28)**	**P (0.31)**	**P (0.24)**	**F (0.00)**	**F (0.00)**	**P (0.14)**	**P (0.11)**
	GARCH-EVT	**G (0.04)**	**P (0.08)**	**F (0.04)**	**P (0.24)**	**P (0.12)**	**P (0.85)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.89)**	**P (0.28)**	**P (0.31)**	**P (0.24)**	**F (0.00)**	**F (0.00)**	**P (0.18)**	**P (0.16)**
2010 (256)	**BS**	**G (0.11)**	**P (0.17)**	**P (0.12)**	**P (0.28)**	**F (0.02)**	**F (0.01)**	**P (0.50)**	**P (0.50)**
	GARCH-EVT	**G (0.01)**	**F (0.03)**	**F (0.00)**	**F (0.00)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.38)**	**P (0.57)**	**P (0.56)**	**P (0.28)**	**P (0.17)**	**P (0.07)**	**P (0.50)**	**P (0.50)**

Table-5.64: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **BMW** with **2-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.04)	P (0.08)	F (0.04)	P (0.88)	P (0.12)	P (0.85)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (253)	BS	Y (0.99)	F (0.00)	F (0.00)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.24)	P (0.34)	P (0.31)	P (0.36)	P (0.07)	F (0.04)	P (0.50)	P (0.50)
	CAM	R (0.99)	F (0.00)	F (0.00)	P (0.39)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (254)	BS	G (0.69)	P (0.79)	P (0.79)	P (0.30)	F (0.00)	F (0.00)	P (0.34)	P (0.34)
	GARCH-EVT	G (0.11)	P (0.17)	P (0.13)	P (0.33)	F (0.02)	F (0.01)	P (0.50)	P (0.50)
	CAM	G (0.89)	P (0.28)	P (0.31)	P (0.27)	F (0.00)	F (0.00)	P (0.15)	P (0.17)
2010 (256)	BS	G (0.01)	F (0.03)	F (0.00)	P (0.26)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.01)	F (0.03)	F (0.00)	P (0.26)	F (0.02)	P (0.92)	P (0.50)	P (0.50)

Table-5.65: At the 97.5% level, VaR and ES back-testing outcomes of **BMW** with **5-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

Likewise, for General Electric, the CAM model documents approximately 99–117 violations, closely aligning with the anticipated 102 exceedances, while the Black–Scholes model significantly overestimates violations at times of heightened volatility. Tables 5.13–5.40 present the out-of-sample back-testing results of the 97.5% VaR and ES for four major financial institutions—JP Morgan (JPM), Bank of America (BAC), American International Group (AIG), and Prudential Capital (PRU)—over the period 2016–2018. The tables compare the performance of the Black–Scholes (BS), GARCH–EVT, and CAM models across 1-, 2-, 5-, and 10-day forecasting horizons, considering both parametric and square-root-rules multi-period approaches. The comparison between parametric multi-period

forecasting and the square-root-rules that the parametric approach generally provides more reliable VaR and ES forecasts. Across the 2-, 5-, and 10-day horizons, the parametric specification more frequently satisfies the VaR back-testing criteria and produces more consistent ES validation results than the square-root-rules approximation. Among the competing approaches, the CAM model consistently delivers the most robust and statistically reliable performance, passing the largest number of regulatory back-testing procedures across assets, years, and investment horizons. The GARCH–EVT model offers competitive performance, particularly in capturing heavy-tailed market behaviour, but exhibits greater variability in calibration.

		VaR Back-testing						ES Back-testing	
Year (Observations)	**Model**	**TL**	**BIN**	**POF**	**TUFF**	**CC**	**CCI**	**UCN**	**UNT**
2006 (249)	**BS**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
	GARCH-EVT	**G (0.01)**	**F (0.03)**	**F (0.00)**	**P (0.30)**	**F (0.03)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.01)**	**F (0.03)**	**F (0.00)**	**P (0.30)**	**F (0.03)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
2007 (252)	**BS**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
	GARCH-EVT	**G (0.01)**	**F (0.03)**	**F (0.00)**	**F (0.01)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.01)**	**F (0.03)**	**F (0.00)**	**F (0.01)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
2008 (253)	**BS**	**Y (0.99)**	**F (0.00)**	**F (0.00)**	**P (0.39)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**F (0.00)**
	GARCH-EVT	**G (0.24)**	**P (0.34)**	**P (0.31)**	**P (0.36)**	**P (0.07)**	**F (0.04)**	**P (0.50)**	**P (0.50)**
	CAM	**Y (0.99)**	**F (0.00)**	**F (0.00)**	**P (0.36)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**F (0.00)**
2009 (254)	**BS**	**G (0.69)**	**P (0.79)**	**P (0.79)**	**P (0.30)**	**F (0.00)**	**F (0.00)**	**P (0.34)**	**P (0.34)**
	GARCH-EVT	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.54)**	**P (0.88)**	**P (0.88)**	**P (0.30)**	**F (0.00)**	**F (0.00)**	**P (0.50)**	**P (0.50)**
2010 (256)	**BS**	**G (0.01)**	**F (0.03)**	**F (0.00)**	**P (0.26)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	GARCH-EVT	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.04)**	**P (0.07)**	**F (0.03)**	**P (0.26)**	**P (0.12)**	**P (0.90)**	**P (0.50)**	**P (0.50)**

Table-5.66: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **BMW** with **5-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Year (Observations)	Model	VaR Back-testing						ES Back-testing	
		TL	BIN	POF	TUFF	CC	CCI	UCN	UNT
2006 (249)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2007 (252)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
2008 (253)	BS	R (0.99)	F (0.00)	F (0.00)	P (0.44)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
	GARCH-EVT	G (0.39)	P (0.59)	P (0.57)	P (0.41)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	CAM	R (1)	F (0.00)	F (0.00)	P (0.44)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2009 (254)	BS	G (0.38)	P (0.58)	P (0.57)	P (0.41)	F (0.00)	F (0.00)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.01)	F (0.03)	F (0.00)	P (0.44)	F (0.02)	P (0.92)	P (0.50)	P (0.50)
	CAM	Y (0.99)	F (0.00)	F (0.01)	P (0.30)	F (0.00)	F (0.00)	F (0.00)	F (0.00)
2010 (256)	BS	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	GARCH-EVT	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)
	CAM	G (0.00)	F (0.01)	F (0.00)	F (0.00)	F (0.00)	P (1)	P (0.50)	P (0.50)

Table-5.67: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **BMW** with **10-day** horizon for the years 2006–2010 under **parametric multi-period** characterization using three different models.

The findings therefore support the adoption of CAM as the preferred model for multi-period market risk measurement, with GARCH–EVT serving as an effective alternative, while confirming the limitations of the Black–Scholes model for modern financial risk management. Tables 5.41–5.68 present the out-of-sample back-testing results of 97.5% VaR and ES for the non-financial assets GE, IBM, Coca-Cola, and BMW over the period 2006–2010. The analysis compares the performance of the BS, GARCH–EVT, and CAM models across 1-, 2-, 5-, and 10-day forecasting horizons, considering both parametric and square-root-rules multi-period characterizations. Overall, the result demonstrate that

dynamic volatility models consistently outperform the traditional Black–Scholes model in forecasting market risk for non-financial assets. Across all four companies and multiple investment horizons, CAM and GARCH–EVT models satisfy a greater proportion of the regulatory back-testing criteria, reflecting their superior ability to capture time-varying volatility and changing market conditions. The results also highlight the impact of the 2008 Global Financial Crisis on model performance. During 2008, all models experience greater difficulty satisfying the VaR back-testing criteria because of the exceptionally high market volatility.

		VaR Back-testing						ES Back-testing	
Year (Observations)	**Model**	**TL**	**BIN**	**POF**	**TUFF**	**CC**	**CCI**	**UCN**	**UNT**
2006	**BS**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
(249)	**GARCH-EVT**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
2007	**BS**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
(252)	**GARCH-EVT**	**G (0.04)**	**P (0.08)**	**F (0.04)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
2008	**BS**	**R (0.99)**	**F (0.00)**	**F (0.00)**	**P (0.44)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**F (0.00)**
(253)	**GARCH-EVT**	**G (0.39)**	**P (0.59)**	**P (0.57)**	**P (0.41)**	**F (0.00)**	**F (0.00)**	**P (0.50)**	**P (0.50)**
	CAM	**R (1)**	**F (0.00)**	**F (0.00)**	**P (0.39)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**F (0.00)**
2009	**BS**	**G (0.38)**	**P (0.58)**	**P (0.57)**	**P (0.41)**	**F (0.00)**	**F (0.00)**	**P (0.50)**	**P (0.50)**
(254)	**GARCH-EVT**	**G (0.01)**	**F (0.03)**	**F (0.00)**	**P (0.44)**	**F (0.02)**	**P (0.92)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.38)**	**P (0.58)**	**P (0.57)**	**P (0.41)**	**F (0.00)**	**F (0.00)**	**P (0.50)**	**P (0.50)**
2010	**BS**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
(256)	**GARCH-EVT**	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**
	CAM	**G (0.00)**	**F (0.01)**	**F (0.00)**	**F (0.00)**	**F (0.00)**	**P (1)**	**P (0.50)**	**P (0.50)**

Table-5.68: At the 97.5% level, Value-at-Risk (VaR) and Expected Shortfall (ES) back-testing outcomes of **BMW** with **10-day** horizon for the years 2006–2010 under **square-root-rule multi-period** characterization using three different models.

Nevertheless, the GARCH–EVT model generally records better Binomial and POF test performance than the Black–Scholes model, reflecting its ability to incorporate heavy-tailed return behaviour through Extreme Value Theory. The CAM model similarly adapts to the heightened volatility by capturing regime shifts, although both dynamic models continue to face challenges under the most severe market conditions. These findings indicate that while no model is immune to unprecedented market stress, incorporating conditional volatility substantially improves forecasting performance.

5.6 Conclusion

A method of risk forecasts conditioned on current level of the asset price alone (rather than depending on a good amount of historical asset prices required for the risk forecasts to be conditioned on current level of volatility) has been proposed. Markovian in nature it has the potential to accommodate mean reversion and leverage effects of assets evolution into the risk forecasts; while keeping the simplicity of applying it with analytic Black and Scholes formulas for estimating both VaR and ES and for making forecasts at any period ahead. In terms of back-testing evidence of VaR and ES (for both single and multi-period forecasts) it systematically out-performs traditional GARCH-EVT model of conditional volatility which is non-Markovian in nature. Moreover, since GARCH-EVT have the evidence of out-performing other conditional risk forecasts with other GARCH dynamics (those not applied with extreme value innovations), we guess our proposed method has the potential to become a very popular model in real applications among all models. As CAM-VaR and CAM-ES do not require any optimization or numerical search and come with analytic formulas for both single and multi-period forecasts, we claim our method to be the most parsimonious model in the risk management literature (at least among those methods that reflect upon mean-reversion and leverage effects in making the forecasts of VaR and ES). So, it should be under policy maker's attention in order to popularize effective risk management to a wider level at banks as well as at non-financial sectors.

Chapter-6

Hypothetical Hedging performance of an emerging option market.

6.1 Introduction

Option trading commenced in the United States in 1973 at the Chicago Board Options Exchange. Few could have anticipated that in the succeeding decades, it would significantly influence the practice and philosophy of finance. Currently, the options market has emerged as a significant element of the financial markets. The option exemplifies effective innovation in finance, stimulating the sector. Other prevalent forms of derivatives include futures and forward contracts.

Hedging is essential in the financial sector. It is utilized to mitigate prospective investment losses. The preeminent hedging strategy for financial derivatives is delta hedging, derived from the modeling framework established by (Black & Scholes, 1973). The paper demonstrates that, given specific assumptions, investment risk can be eliminated through continuous trading of the underlying asset and the risk-free asset. The Black and Scholes model's unreasonable assumptions regarding the underlying price process and the impracticality of continuous trading result in market incompleteness, preventing this strategy from fully mitigating risk in practice. Consequently, scholars have suggested hedging frameworks that are relevant in scenarios of market incompleteness and incorporate more realistic characteristics of the underlying asset model. Delta is the absolutely essential hedge characteristic and, wonderfully, it is the most simply adjustable, necessitating merely a transaction in the underlying asset. Since the inception of exchange-traded options markets in 1973, delta hedging has been essential to the management of options portfolios. Option traders commonly alter delta to approximate zero by trading the underlying asset.

(DUAN, 1995) traded options are easily accessible in market prices. The GARCH option pricing model allows for the inference of implied GARCH parameters from market data. The estimated parameter values can be employed in a manner akin to the implied volatility of the Black-Scholes model. Determining the implicit GARCH parameters by Monte Carlo

simulation is, nonetheless, computationally demanding. To make it simpler the work, it is necessary to establish a numerical approximation method for calculating the GARCH option price. Analytically approximating the distribution function of the terminal asset price under the locally risk-neutral pricing measure appears to be a promising method.

Ultimately, employing the GARCH option pricing model in delta hedging introduces an additional layer of flexibility to the dynamic hedging process. In a low-variance environment, GARCH delta hedging necessitates a reduced option position compared to the Black-Scholes model, whereas it requires an expanded position in a high-variance environment. This supplementary dynamic attribute may significantly enhance risk management.

(Hsieh & Ritchken, 2005) examined the efficiency of two GARCH models: Duan's NGARCH model and the HN model. The NGARCH model is significant in its own context and also functions as an approximation for specific stochastic volatility models produced by two orthogonal Wiener processes. The HN model is noteworthy because its volatility updating framework allows for the derivation of analytical solutions for European options. This article has juxtaposed these two models against one another and in relation to the Black-Scholes model. The findings demonstrate that both GARCH models effectively account for a substantial portion of the maturity and strike price bias linked to the Black-Scholes model. The NGARCH model seems to exceed the HN model, particularly in pricing deep out-of-the-money contracts.

The out-of-sample efficacy of the GARCH models, particularly the NGARCH model, seems promising. The ability of models derived from historical option data to elucidate option prices over extended durations suggests that these models effectively summarize critical aspects of the option pricing mechanism. In extreme instances, where the NGARCH parameters are not re-estimated for quarters or even years, the model consistently performs at levels typically no inferior to in-sample Black-Scholes for in-the-money and at-the-money options, while for out-of-the-money options, the NGARCH model demonstrates evidently superior performance. The expense of re-evaluating the

parameters of a GARCH process is relatively low. Consequently, we do not advise employing the model to price options utilizing parameter estimations derived from a remote time frame. Our assertion is that, in the absence of frequent model updates, the performance of GARCH models remains satisfactory, particularly when compared to that of a Black-Scholes model. The sustained efficacy of an NGARCH model suggests it effectively captures critical factors influencing option prices.

The primary goal of this project is to compare the performance of four GARCH models such as NGARCH, GARCH, GARCH-Jump and GARCH-Jump constant as a proxy of the Black-Scholes model in terms of delta hedging and delta-gamma hedging effectiveness. We evaluated delta and delta gamma with 1-day, 3.5-days and 7-days hedging performance of the four GARCH models by comparing their hypothetical option prices and hedging errors with those of the Black-Scholes model. In section 6.2, we present family of Generalized Auto Regressive Conditional Heteroskedastic (GARCH) models and Black-Sholes (BS) model. Section 6.3 shows parameter estimation of family of GARCH models. Section 6.4 presents data. Section 6.5 illustrates empirical result with discussion and finally section 6.6 contains conclusion.

6.2 Research Methodology

This section emphasizes the pertinent theoretical framework employed in addressing our research concerns. The theoretical framework provides a comprehensive theoretical description, preparing the reader with essential information for understanding the succeeding method part. This section highlights and examines the Black-Scholes Option Pricing Model, GARCH model, NGARCH model, GARCH Jump model, and GARCH Jump Constant model.

6.2.1 The Black-Scholes Option Pricing Models Model

We can now derive the Black-Scholes partial differential equation for a call option on a non-dividend paying stock with a strike price K and maturity T. We assume that the stock price adheres to a geometric Brownian motion, such that

$$dS_t = \mu S_t\, dt + \sigma S_t dW_t \tag{6.1}$$

Here, W_t represents a normal Brownian motion. We additionally assume that interest rates remain unchanged, such that 1 unit of currency deposited in the account at time 0 will be valued $B_t := exp(rt)$ at time t. We will denote the value of the call option at time t. Itô's lemma indicates that

$$dC(S,t) = \left(\mu S_t \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}\right) dt + \sigma S_t \frac{\partial C}{\partial S} dW_t \tag{6.2}$$

We will now analyze a self-financing trading strategy in which, at each time t, we hold x_t units of the cash account and y_t units of the stock. Therefore, the time variable t in this approach is represented by P_t.

$$P_t = x_t B_t + y_t S_t. \tag{6.3}$$

We will select x_t and y_t to ensure the strategy replicates the option's value. The self-financing hypothesis indicates that

$$\begin{aligned} dP_t &= x_t dB_t + y_t dS_t \\ &= r x_t B_t dt + y_t(\mu S_t\, dt + \sigma S_t dW_t) \\ &= (r x_t B_t + y_t \mu S_t)\, dt + y_t \sigma S_t dW_t) \end{aligned} \tag{6.5}$$

Note that (6.4) corresponds with our earlier concept of self-financing. Any variations in the portfolio's value are exclusively due to the performance of the underlying securities, specifically the cash-account and stock, rather than changes in the holdings x_t and y_t.

By re-examining our reasoning, we can equate the terms in (6.2) with their equivalent counterparts in (6.5) to derive

$$y_t = \frac{\partial C}{\partial S} \tag{6.6}$$

$$rx_t B_t = \frac{\partial C}{\partial t} + \frac{1}{2}\,\sigma^2 S^2\,\frac{\partial^2 C}{\partial S^2} \tag{6.7}$$

If we establish $C_0 = P_0$ as the beginning value of our self-financing method, then it follows that $C_t = P_t$ for all t, given that C and P exhibit identical dynamics. This is valid by design after we equated the terms in (6.2) with the comparable terms in (6.5). By substituting (6.6) and (6.7) into (6.3), we derive

$$rS_t\frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2}\,\sigma^2 S^2\,\frac{\partial^2 C}{\partial S^2} - rC = 0 \tag{6.8}$$

the Black-Scholes partial differential equation. To resolve (6.8), boundary conditions must also be provided. The conditions for our call option are as follows:

$$C(S,T) = max(S - K, 0),\ C(0,t) = 0 \text{ for all } t$$
$$\text{and } C(S,t) \to S \text{ as } S \to \infty.$$

The resolution to (6.8) about a call option is

$$C(S,T) = S_t N(d_1) - e^{-r(T-t)} K N(d_2) \tag{6.9}$$

$$\text{Where } d_1 = \frac{log(\frac{S_t}{K}) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}} \tag{6.10}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)}$$

$N(.)$ denotes the cumulative distribution function of the standard normal distribution. A method to verify (6.9) is to calculate the different partial derivatives utilizing (6.9), subsequently substituting them into (6.8) to ascertain if (6.8) is satisfied. The valuation of a European put option can now be readily determined using put-call parity (6.9).

6.2.2 GARCH Model

The most basic generalized autoregressive conditional heteroskedasticity (GARCH) model has been widely employed in the analysis of return time series. Recently, the GARCH option pricing model has been increasingly utilized to empirically assess its pricing efficacy. GARCH is a prevalent time series model utilized for modelling volatility clustering, extensively applied in financial market forecasting. The GARCH framework, derived from the ARCH model, incorporates the variance of the error component from

prior periods, hence improving the model's ability to reflect volatility persistence over time. The foundational equations of the simple GARCH model are delineated as follows:

$$r_t = \mu + \epsilon_t \tag{6.11}$$

$$\epsilon_t = \sigma_t z_t \quad , z_t \sim N(0,1)$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{6.12}$$

with $\omega > 0, \alpha > 0, \beta > 0$.

where r_t denotes the daily returns, and σ_t^2 denotes the conditional variance of ϵ_t, for $t = 1, \ldots, T$. (Bollerslev, 1986) showed that the necessary and sufficient condition for weak stationarity of the GARCH model ((6.11) and (6.12)) is with $\alpha + \beta < 1$.

6.2.3 NGARCH Model

The NGARCH model, also known as the asymmetric GARCH or A-GARCH adds a leverage parameter to the original GARCH (1,1) to capture the leverage effect, thus it is known as the simplest GARCH-model that contains volatility clustering and a leverage effect. The model was initially proposed by Engle & Ng (1993) and extended through LRNVR by Duan (1995) in order to price options. These models are more flexible than the basic GARCH model and allows for asymmetric behaviours in the volatility. The models are

$$r_t = \mu + \epsilon_t \tag{6.13}$$

$$\epsilon_t = \sigma_t z_t \quad , z_t \sim N(0,1)$$

$$\sigma_t^2 = \omega + \alpha\sigma_{t-1}^2 + \beta(\epsilon_{t-1} - \gamma\sqrt{\sigma_{t-1}^2}\ ) \tag{6.14}$$

Here γ is a nonnegative parameter that captures the negative correlation between return and volatility innovations. To ensure covariance stationarity of the NGARCH process, $\alpha + \beta(1 + \gamma^2)$ is assumed to be less than one. Notice that the negative value of ϵ_{t-1} will generate a larger value of conditional variance in the next period.

6.2.4 GARCH-Jump model

We propose a combined GARCH-Jump model to examine the structure of dynamic leaps and to encapsulate the dynamic volatility, including the time-varying volatility effects inside the jump dynamics component. The findings indicate that the mixed GARCH-Jump model surpasses the conventional GARCH model and demonstrates the presence of leaps in the empirical data across all three financial markets. To accommodate various volatility events, we establish distinct assumptions for jump intensity, mean value, and variance of jump amplitude for different GARCH-Jump models.

The mixed GARCH-Jump model is as follows

$$r_t = \mu + \emptyset_1 r_{t-1} + \epsilon_t \tag{6.15}$$

where r_t represents the stock return at time t, μ is a constant, $\emptyset_1$ represents the autoregressive coefficient of the stock return rate and ϵ_t is a random error term. The components of returns in which two stochastic innovations, $\epsilon_{t,1}$ and $\epsilon_{t,2}$, drive returns

$$\epsilon_t = \epsilon_{t,1} + \epsilon_{t,2} \tag{6.16}$$

where $\epsilon_{t,1}$ is a mean-zero innovation with a normal stochastic process, $\epsilon_{t,2}$ is a jump innovation specified so that it is also conditionally mean-zero, and $\epsilon_{t,1}$is contemporaneously independent of $\epsilon_{t,2}$. The component $\epsilon_{t,1}$ is:

$$\epsilon_{t,1} = \sigma_t z_t \tag{6.17}$$

And employ the simplest GARCH model of dynamic variance:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{6.18}$$

where σ_t^2 is the conditional variance; $\omega, \alpha, and\ \beta$ are unknown parameters.

The definition of ϵ_t include the anticipated jump component, so enabling it to influence and convey future volatility via the GARCH variance factor. To assurance of positive variance and stationarity, we impose the following usual constraints on the variance parameters: $\omega, \alpha, \beta > 0\ and\ \alpha + \beta < 1$.

Let n_t represent the count of jumps occurring between $t\ -1\ and\ t$, where the conditional jump size of the $k-th$ jump is denoted as $Y_{t,k}$, contingent upon the historical returns. The jump component affecting returns period t is $\sum_k^{n_t} Y_{t,k}$.

For specification of $\epsilon_{t,2}$, we introduce the random jump size variable $\sum_k^{n_t} Y_{t,k}$ which follows a normal distribution with a mean of θ and a variance of δ^2. Hence, the jump innovation associated with period t is expressed as associated with period *t* is expressed as

$$\epsilon_{t,2} = \sum_k^{n_t} Y_{t,k} - \theta\lambda_t \tag{6.19}$$

where $Y_{t,k} \sim N(\theta, \delta^2)$, the jump number n_t follows a Poisson distribution with jump intensity λ_t ($\lambda_t > 0$). That is, $n_t \sim Pois(\lambda_t)$ and the probability function is:

$$P(n_t = j) = \frac{\exp(-\lambda_t)\lambda_t^j}{j!} \tag{6.20}$$

The conditional jump intensity λ_t, which controls the jump dynamics, is assumed to follow the autoregressive process:

$$\lambda_t = a + b\lambda_{t-1} + c\epsilon_{t-1}^2 \tag{6.21}$$

where a controls the expected number of jumps per day, b controls the persistence of jump-clustering effect, and c controls the time-varying effects in jump dynamics.

6.2.5 GARCH-Jump Constant model

To accommodate various volatility events, we use distinct assumptions for jump intensity, mean value, and variance of jump amplitude for different GARCH-Jump models. In the GARCH-Jump Constant model, we speculate that the mean of jump occurrences and the mean and variance of jump magnitude are constants, specifically, $\lambda_t = \lambda > 0$, $\theta_t = \theta$, and $\sigma_t^2 = \delta^2 > 0$.

6.3 Parameter Estimation of GARCH Model with Jump

To find the parameters of the GARCH model with jump, we used maximum likelihood estimation method. As we assume that the likelihood function is Gaussian, therefore the

likelihood function and logarithmic likelihood function of the GARCH model with jump are expressed as follows:

$$f(\epsilon_t|n_t = j, \Theta) = \sum_{j=0}^{\infty} \frac{exp(-\lambda_t)\lambda_t^j}{j!} f\left(\epsilon_t|n_t = j, I_{t-1,}\, \Theta\right) \tag{6.22}$$

Where, $f\left(\epsilon_t|n_t = j, I_{t-1,}\, \Theta\right) = \frac{1}{\sqrt{2\pi\left(\sigma_t^2 + j\delta^2\right)}} exp\left(-\frac{(\epsilon_t + \theta\lambda_t - \theta j)^2}{2\left(\sigma_t^2 + j\delta^2\right)}\right)$

$$L(\,\Theta) = \sum_{t=1}^{n} \log f(\epsilon_t|n_t = j, \Theta) \tag{6.23}$$

Here Θ is a parameter set of the model. i.e. $\Theta = (\omega, \alpha, \beta, \delta, \theta, a, b, c)$ and $I_{t-1,}$ is the information set.

6.4 Data

The dataset comprises the daily open, high, low, and close prices of the Dow Jones Industrial Average (DJIA) for all trading days from January 2, 2000, to December 31, 2014, as well as S&P 500 index options data for call options for the ten-year period from January 2008 to December 2014. Unlike traditional option markets, such as the S & P 500 index option data, this project focuses on emerging countries where option trading is not yet available. Using the Dow Jones data, we derived hypothetical strike prices for each day by maintaining the same moneyness (the ratio of stock price to strike price) as observed in the options market on the same date. This approach allowed us to simulate strike prices typically found in option market data, creating a framework for analyzing option pricing and hedging in markets where such data is not readily available. Finally, Data selected for trading on Wednesdays, and contracts with maturities of fewer than eight (08) days were eliminated. Consequently, all price effects associated with expiration are removed from the dataset. Furthermore, liquidity issues are mitigated, as they frequently arise with long-term options. Moneyness is defined as the ratio of the underlying asset's price to the option's strike price. Consequently, the final sample has 466,522 observations on options.

Statistics	Dow Jones returns
Mean	0.0103
Maximum	52.1689
Minimum	-38.5413
Std. Dev	5.2230
Skewness	0.8589
Kurtosis	22.9648
ADF test	1
Observations	3772

Table-6.1: Summary statistics of Dow Jones stocks data from January 2, 2000 to December 31, 2014. Alternative hypothesis: the returns series are stationary.

Table 6.1 presented summary statistics of daily returns of Dow Jones stocks data to provide a general understanding of the nature of market return. The mean return is 0.0103 while the Std. Dev is more than 5. The return distribution is positively skewness, which indicates that the data values are right skewed distribution and high kurtosis value, shows that the data contains extreme values and have heavy tails than normal distribution. Finally, Augmented Dicky Fuller test show that null hypothesis is rejected and the returns series are stationary which shows ARCH effect, implying the presence of conditional heteroscedasticity in returns. To illustrate our methods, we use daily stock market indices of Dow Jones obtained from Google finance for the period from January 2, 2000, to December 31, 2014. Daily log returns are calculated as $r_t = (ln(S_t) - log(S_{t-1})) \times 100$, where S_t is the price index at time t.

6.5 Empirical Results with Discussion

We calibrated the volatility parameter (sigma) from the Black-Scholes model, taking the closing prices of the Dow Jones stock and the hypothetical strike prices. Additionally, we calibrated volatility using four different GARCH models: GARCH, NGARCH, GARCH-Jump, and GARCH-Jump Constant. These models were applied to a rolling window of historical Dow Jones stock prices to capture time-varying volatility dynamics.

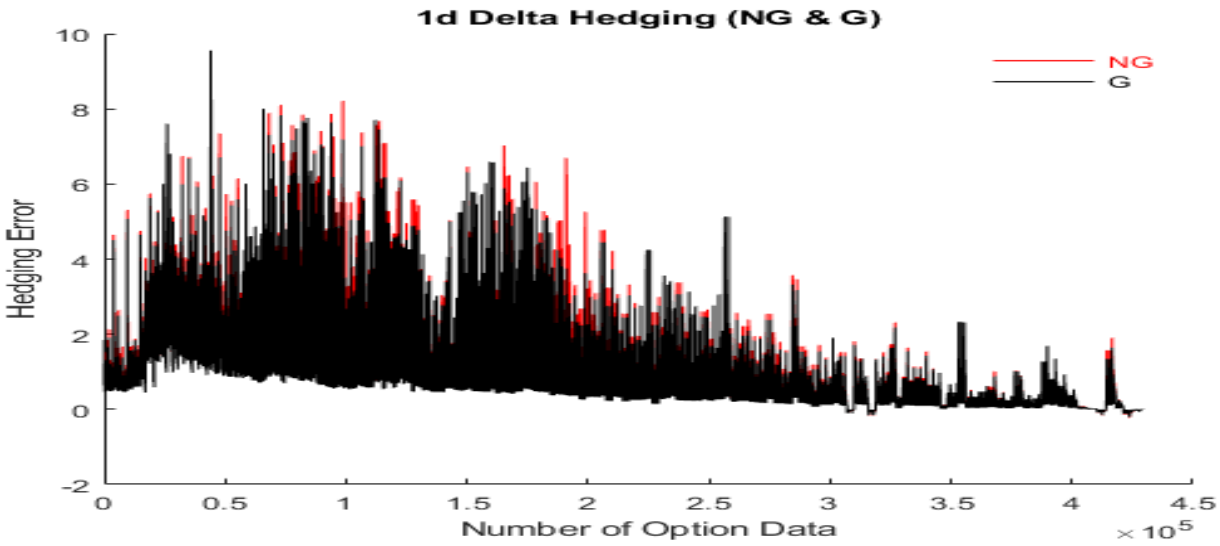


Figure-6.1: One trading day as risk management horizon for delta hedging in Wednesday's call option data using NGARCH and GARCH models during the period January 2, 2008 to December 31, 2014.

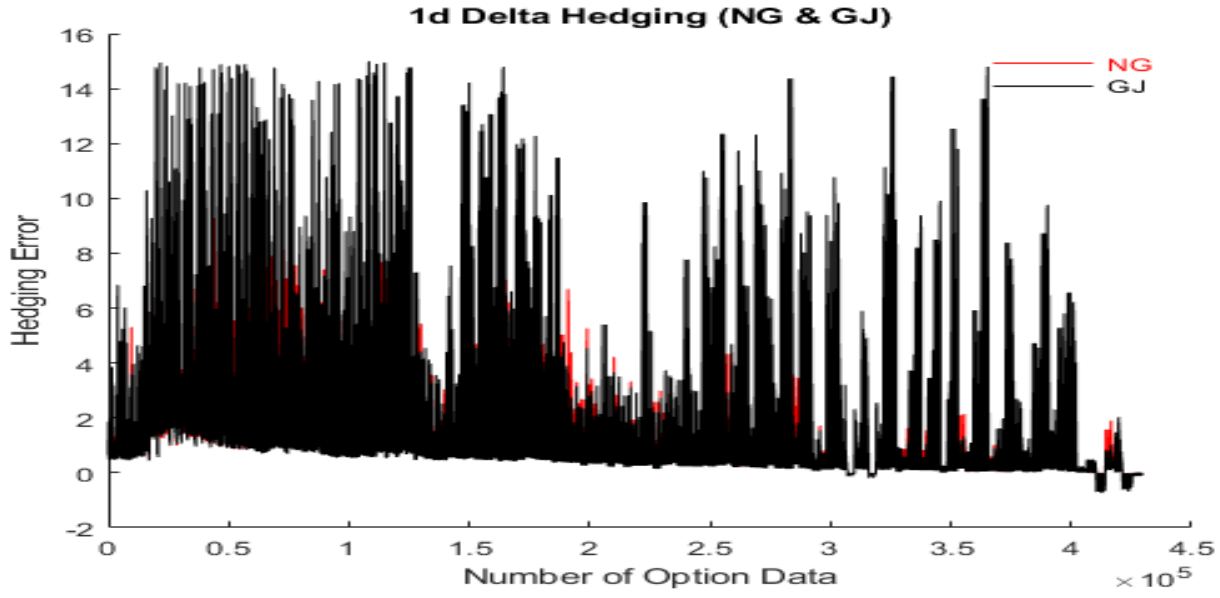


Figure-6.2: One day Delta hedging considering NGARCH and GARCH-Jump models for call option data during the period January 2008 to December 2014.

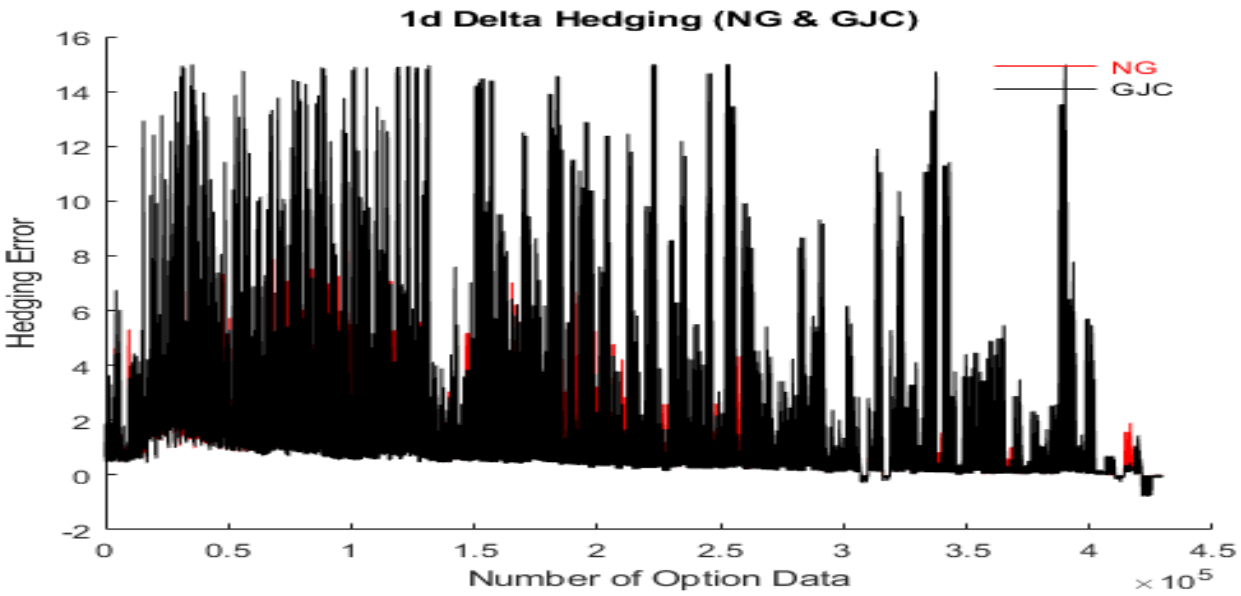


Figure-6.3: One day Delta hedging considering NGARCH and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

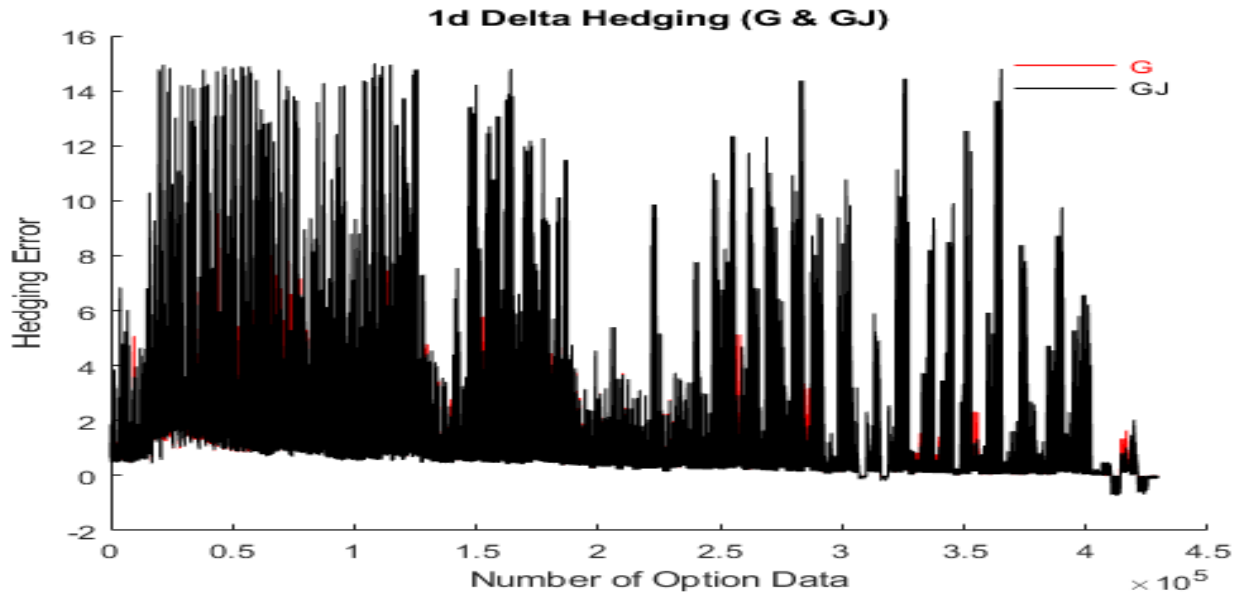


Figure-6.4: One day Delta hedging considering GARCH and GARCH-Jump models for call option data during the period January 2008 to December 2014.

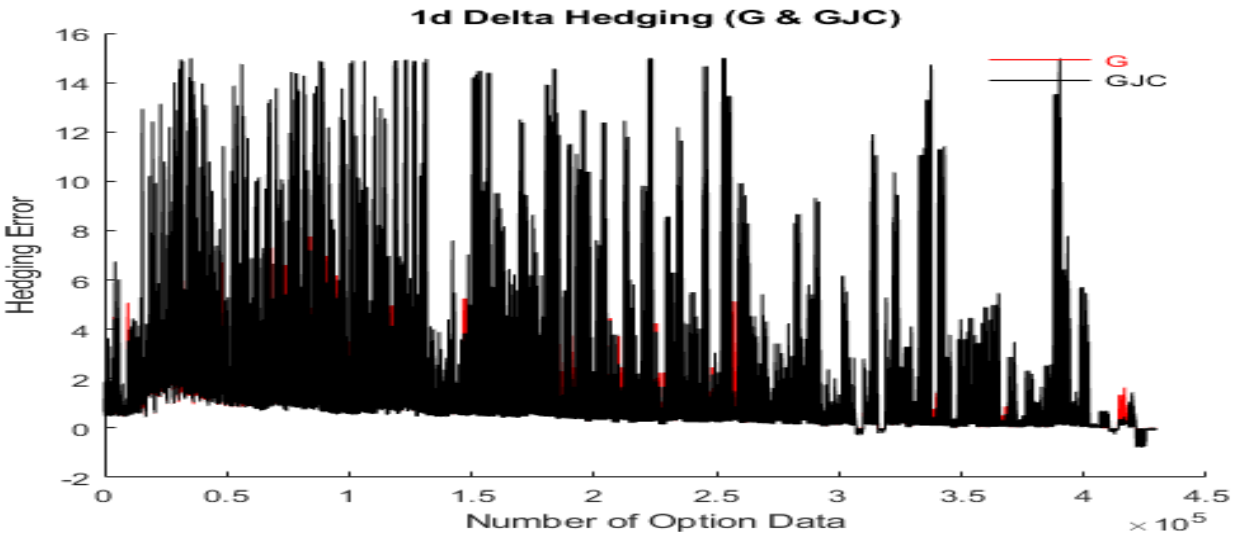


Figure-6.5: One day Delta hedging considering GARCH and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

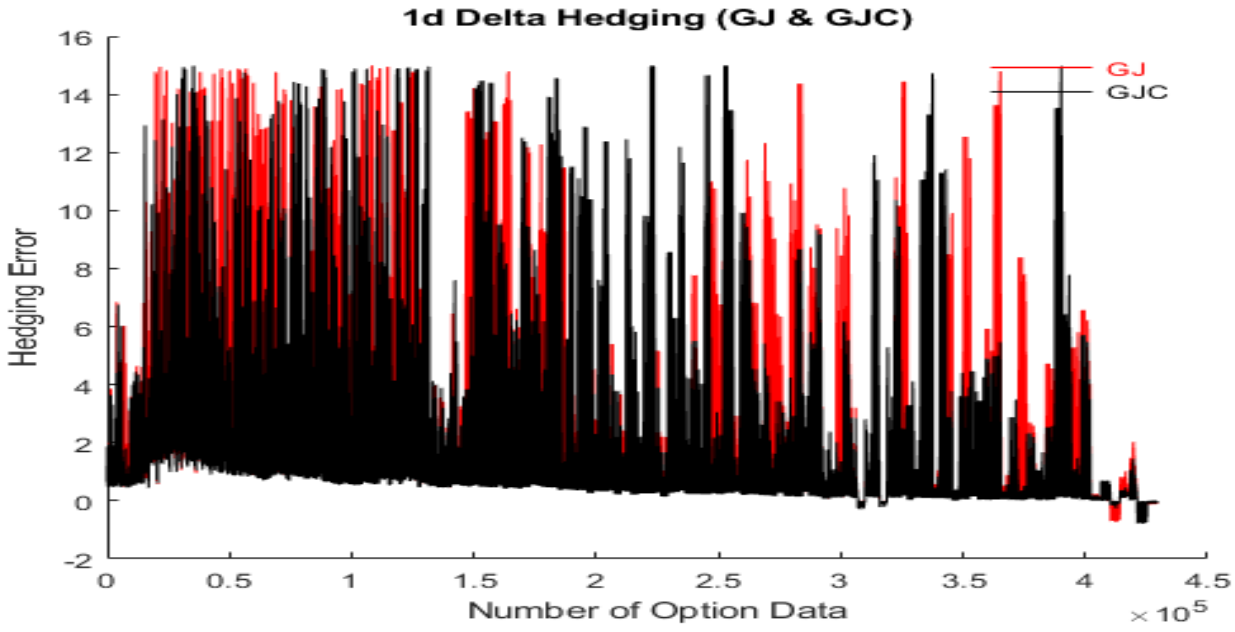


Figure-6.6: One day Delta hedging considering GARCH-Jump and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

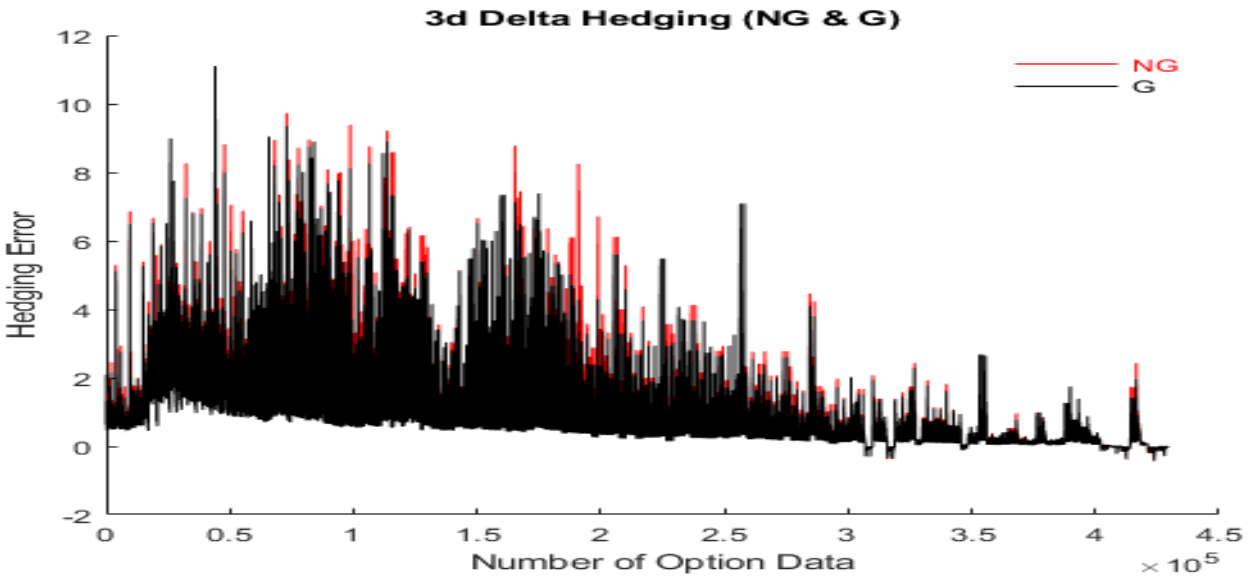


Figure-6.7: Three and half days Delta hedging considering NGARCH and GARCH models for call option data during the period January 2008 to December 2014.

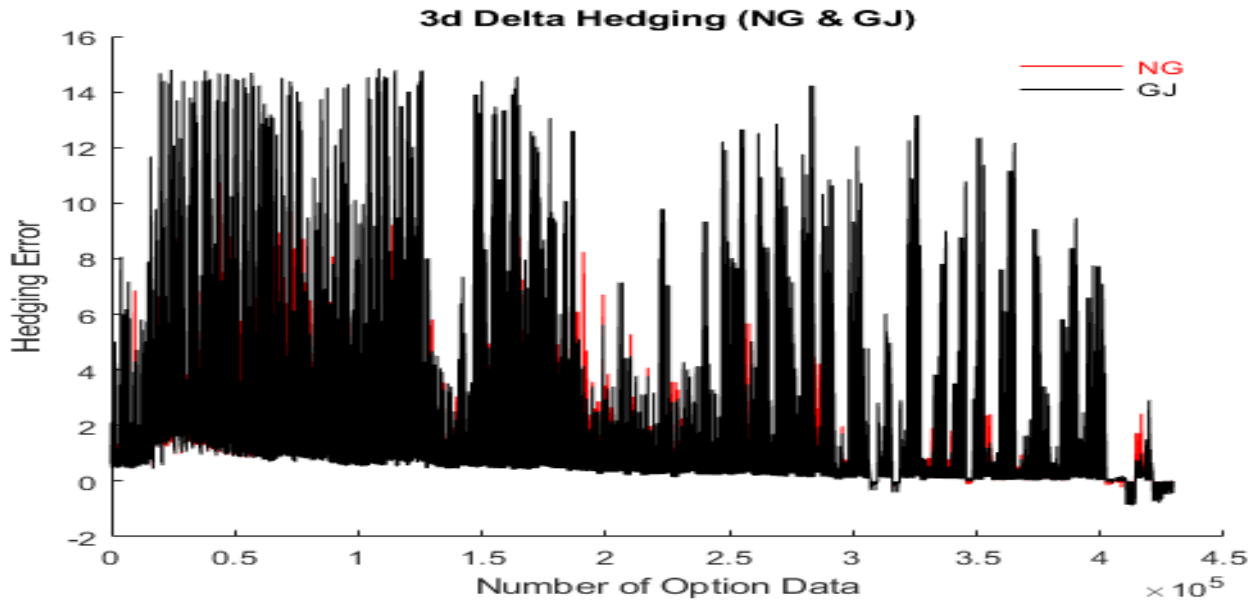


Figure-6.8: Three and half days Delta hedging considering NGARCH and GARCH-Jump models for call option data during the period January 2008 to December 2014.

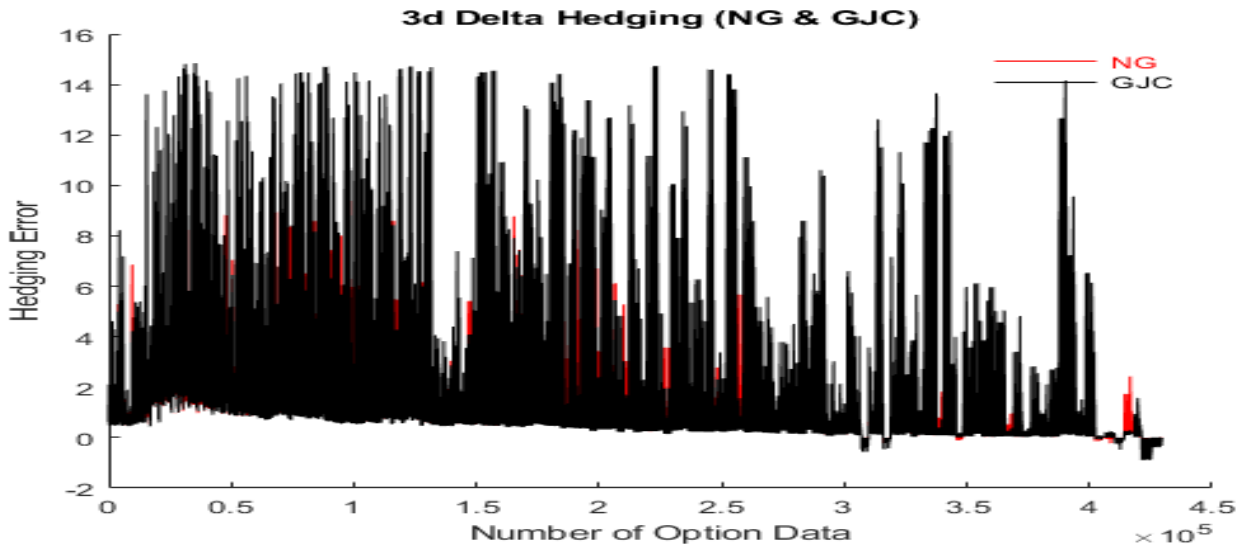


Figure-6.9: Three and half days Delta hedging considering NGARCH and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

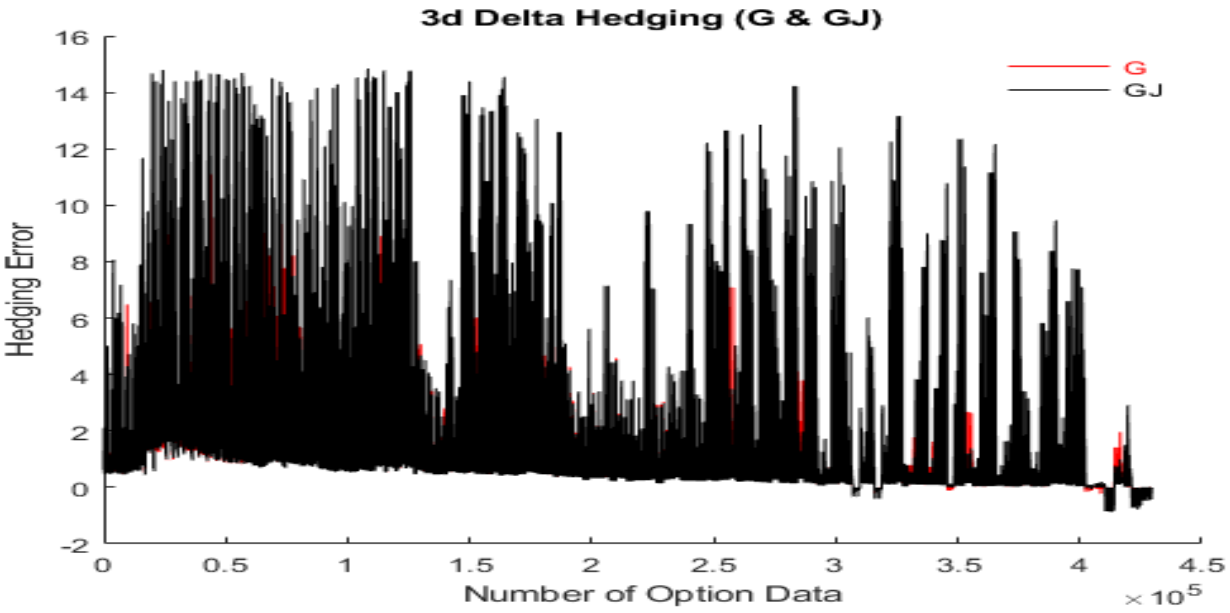


Figure-6.10: Three and half days Delta hedging considering GARCH and GARCH-Jump models for call option data during the period January 2008 to December 2014.

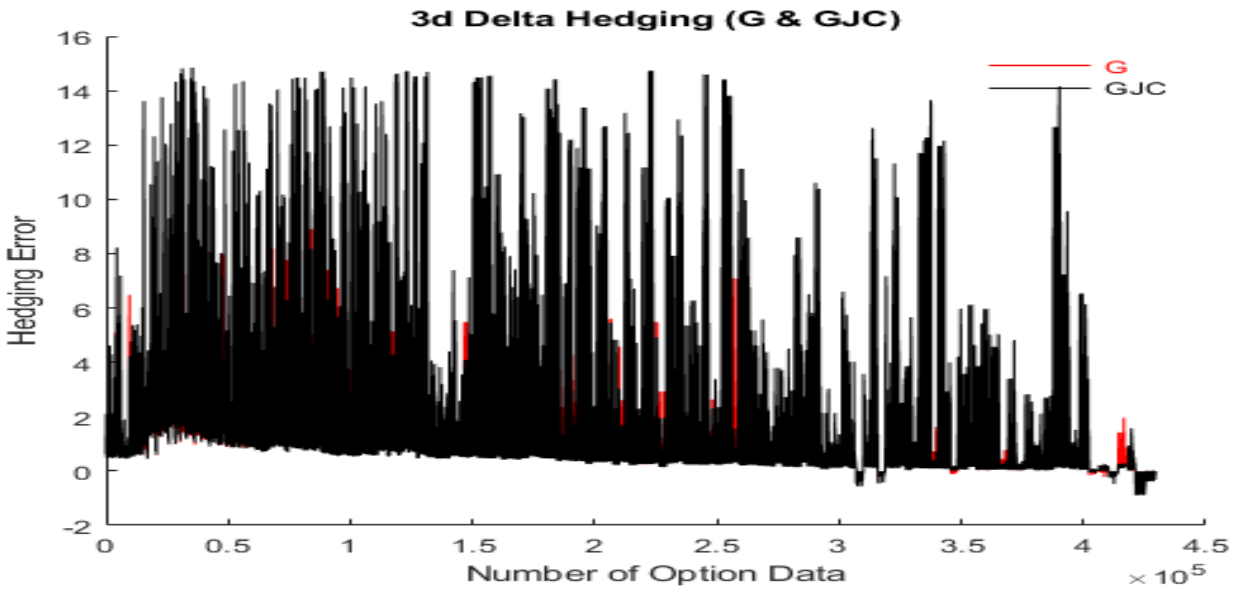


Figure-6.11: Three and half days Delta hedging considering GARCH and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

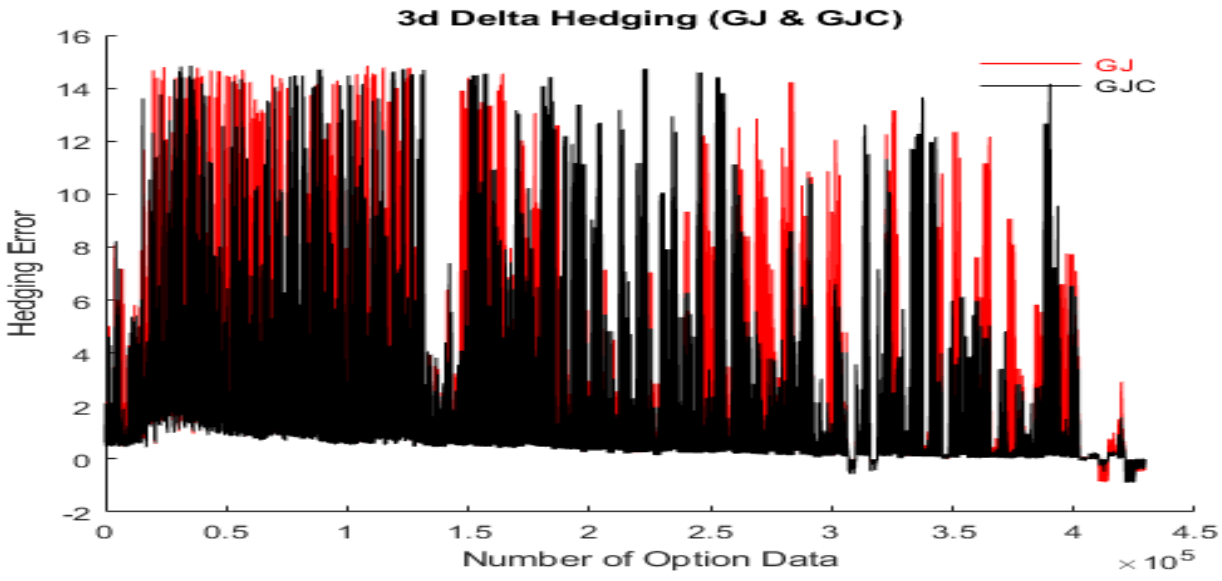


Figure-6.12: Three and half days Delta hedging considering GARCH-Jump and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

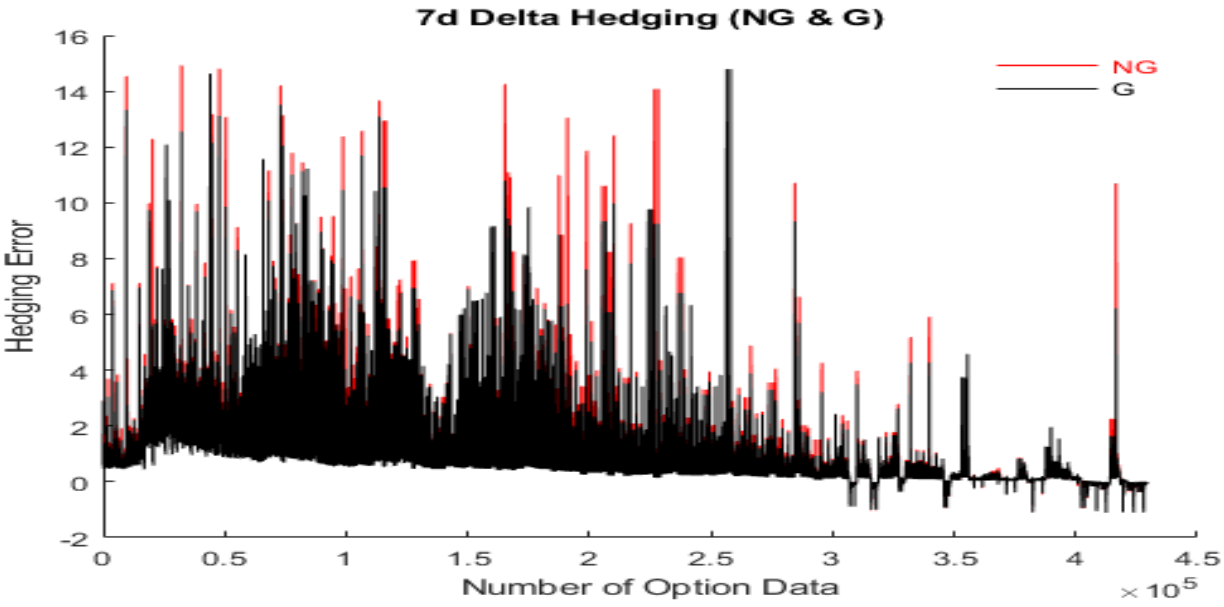


Figure-6.13: Seven days Delta hedging considering NGARCH and GARCH models for call option data during the period January 2008 to December 2014.

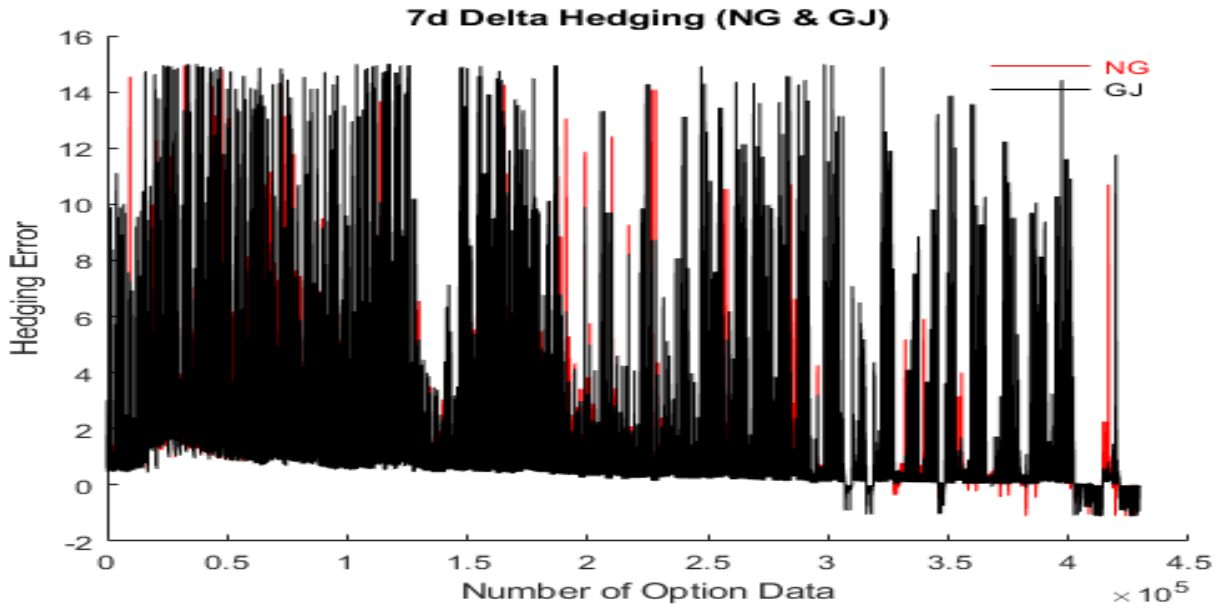


Figure-6.14: Seven days Delta hedging considering NGARCH and GARCH-Jump models for call option data during the period January 2008 to December 2014.

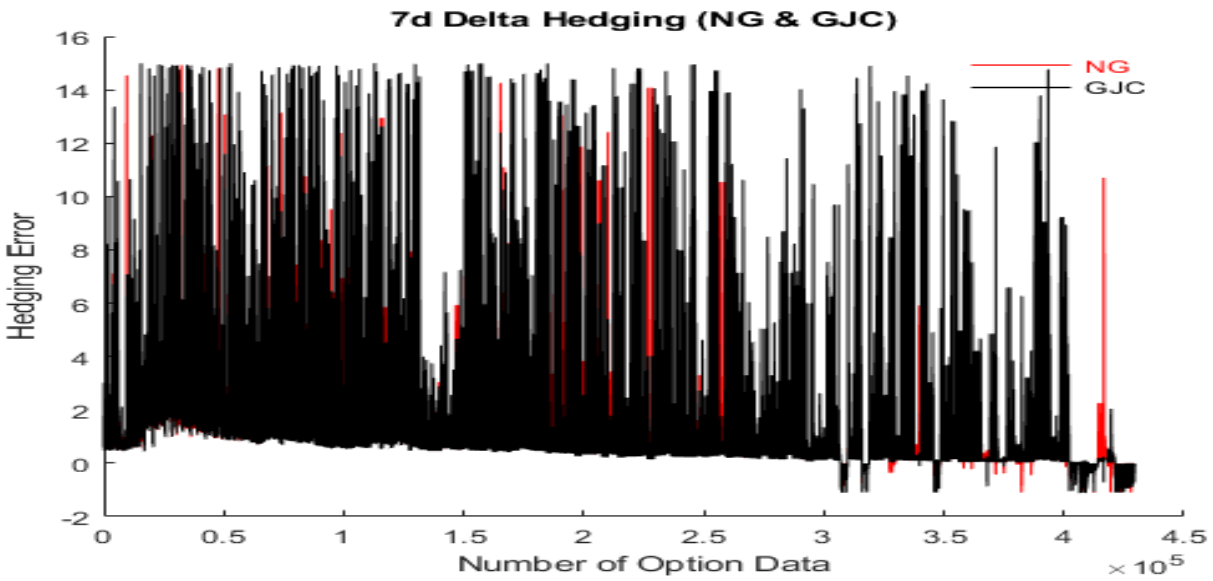


Figure-6.15: Seven days Delta hedging considering NGARCH and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

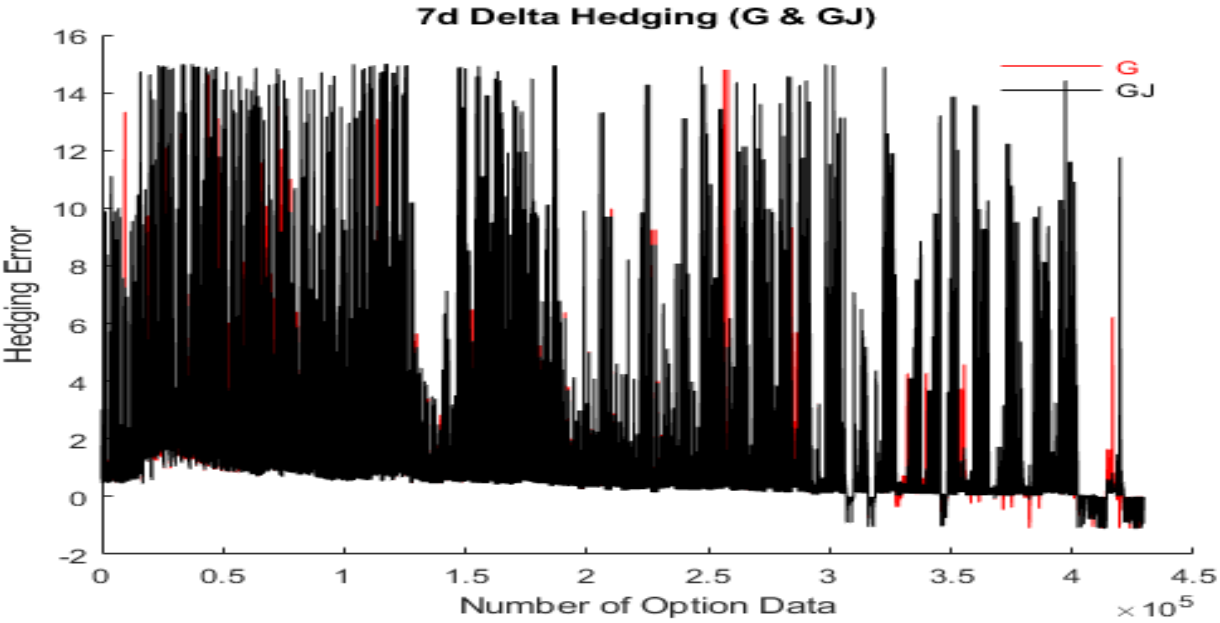


Figure-6.16: Seven days Delta hedging considering GARCH and GARCH-Jump models for call option data during the period January 2008 to December 2014.

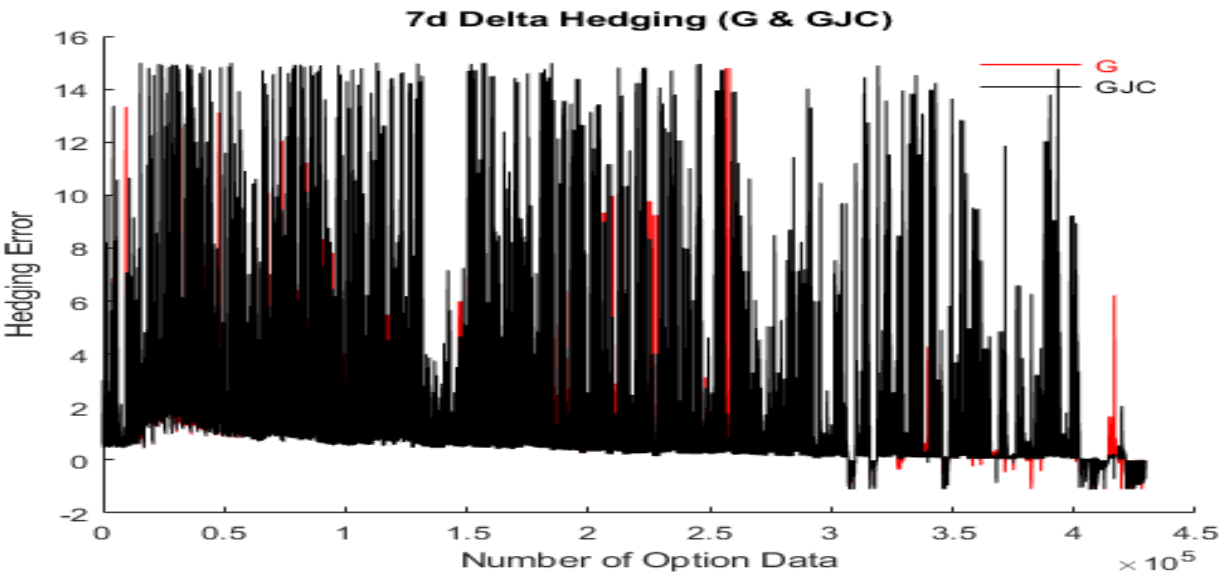


Figure-6.17: Seven days Delta hedging considering GARCH and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

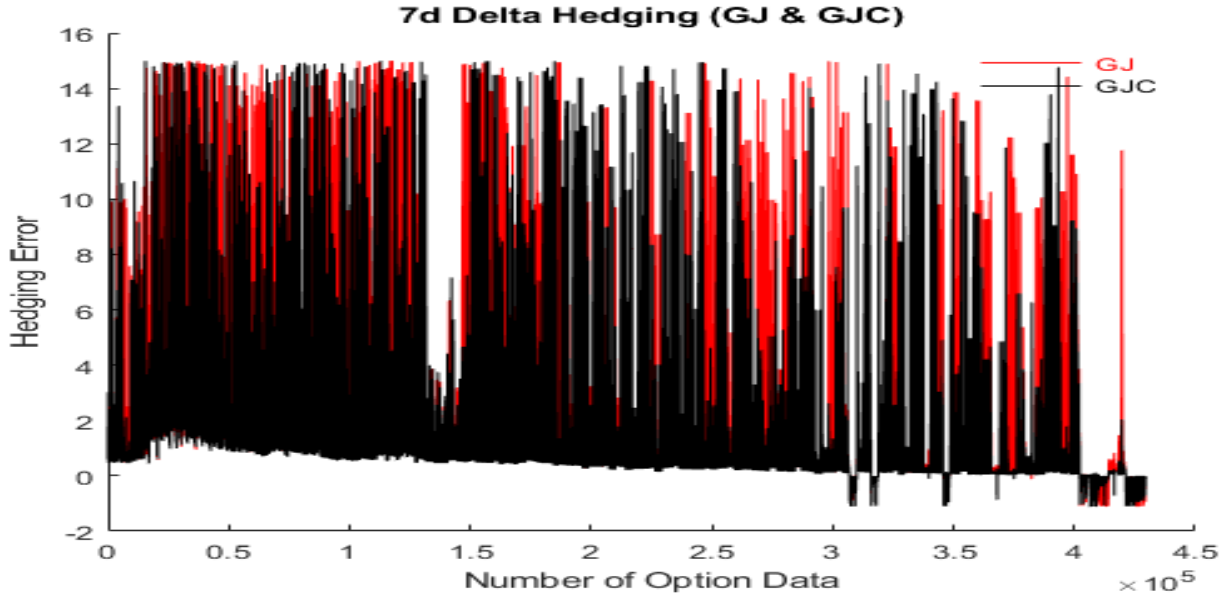


Figure-6.18: Seven days Delta hedging considering GARCH-Jump and GARCH-Jump Constant models for call option data during the period January 2008 to December 2014.

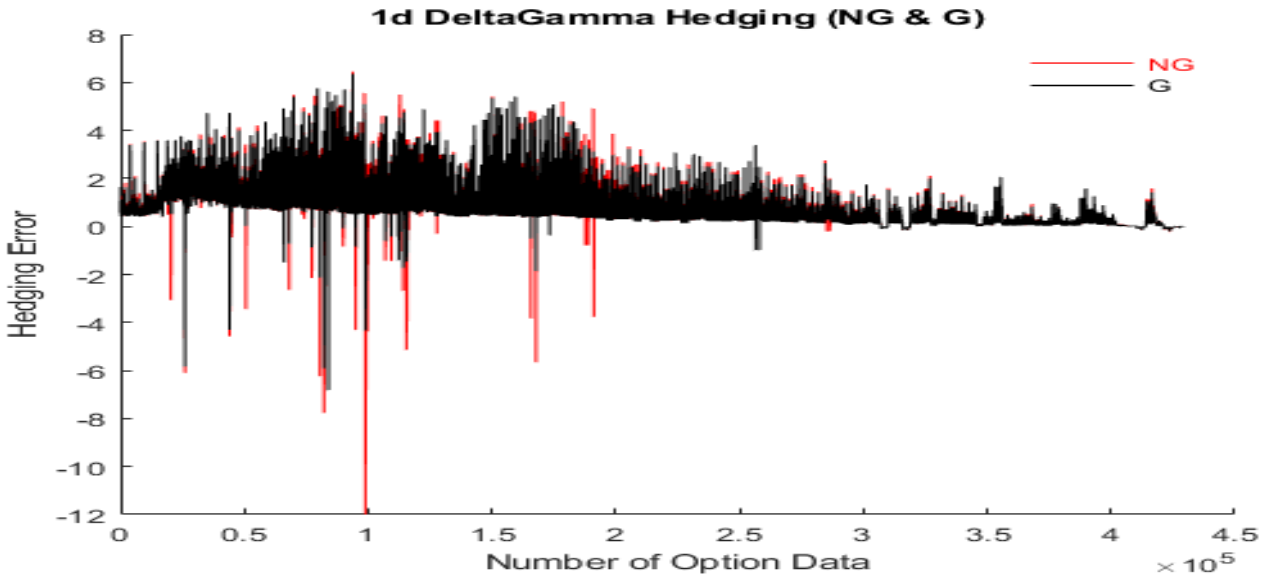


Figure-6.19: One day Delta-Gamma hedging considering NGARCH and GARCH models for call option data during the period January 2, 2008 to December 31, 2014.

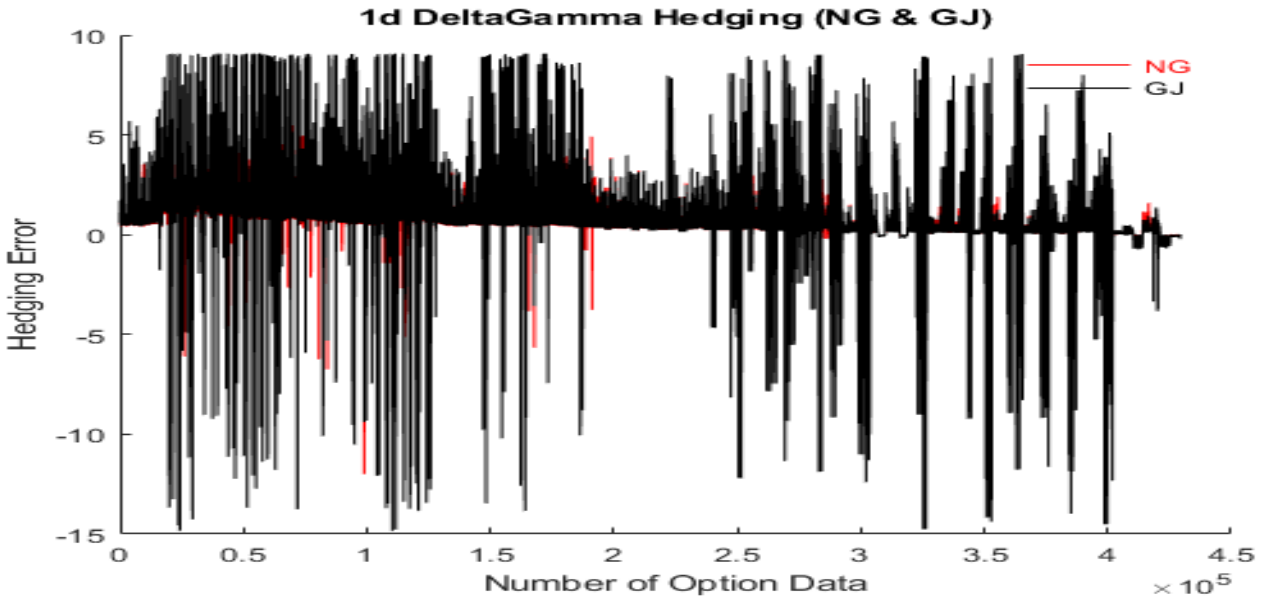


Figure-6.20: One day Delta-Gamma hedging considering NGARCH and GARCH Jump models for call option data during the period January 2, 2008 to December 31, 2014.

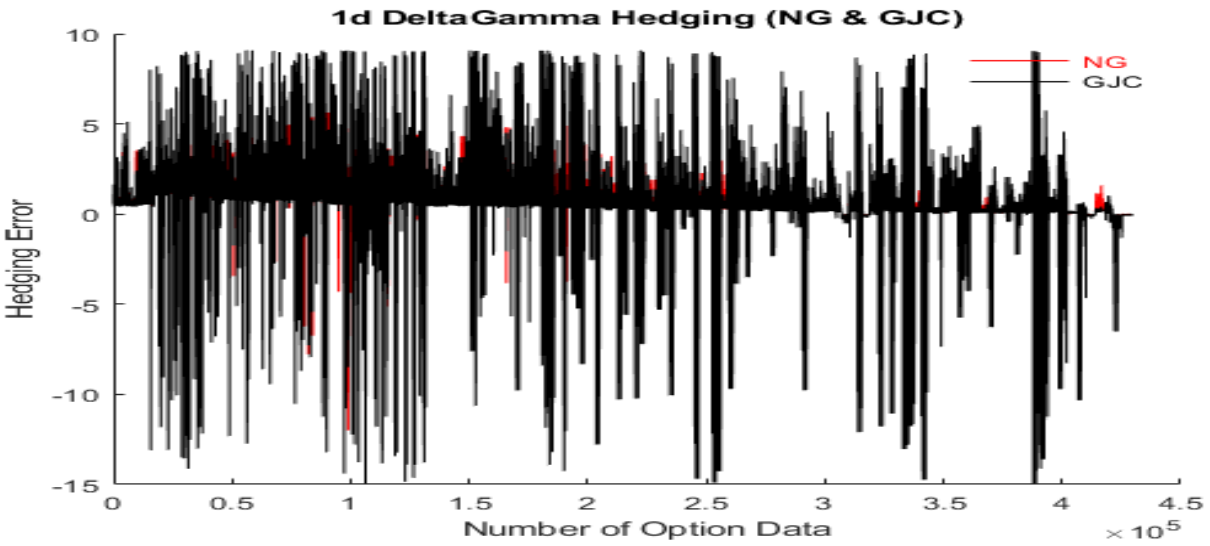


Figure-6.21: One day Delta-Gamma hedging considering NGARCH and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

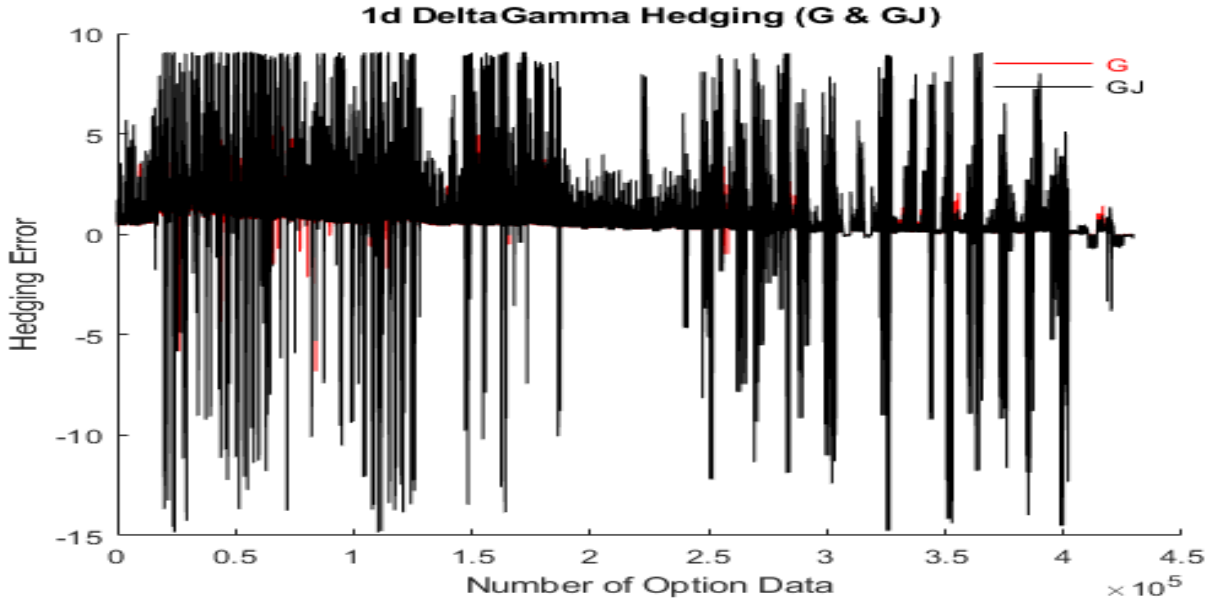


Figure-6.22: One day Delta-Gamma hedging considering GARCH and GARCH Jump models for call option data during the period January 2, 2008 to December 31, 2014.

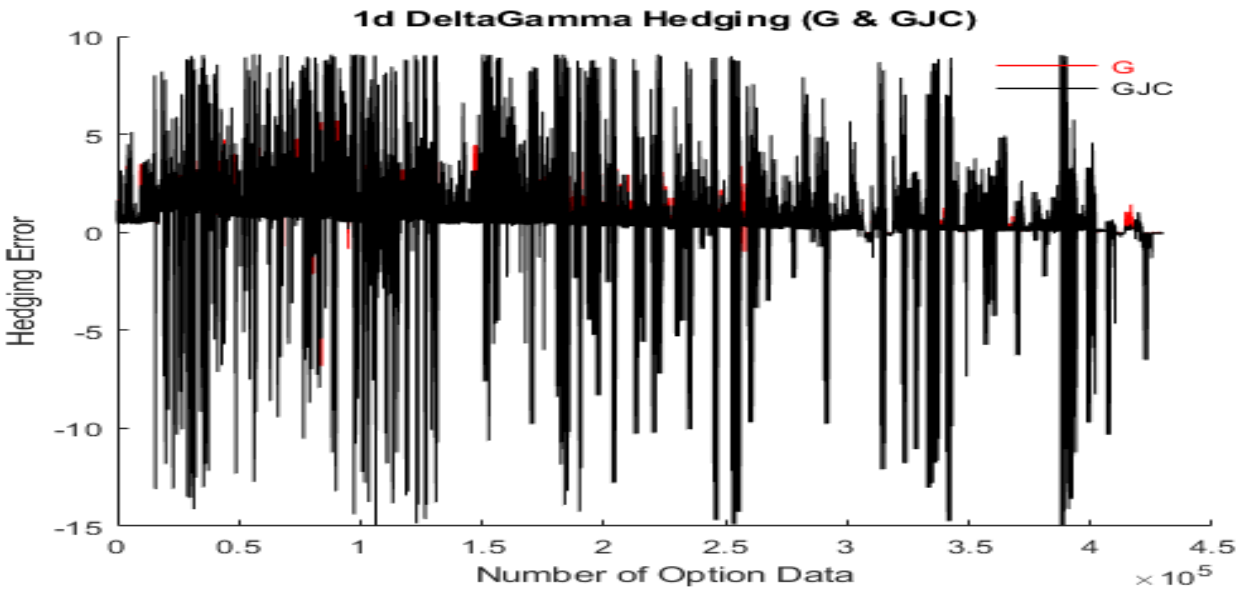


Figure-6.23: One day Delta-Gamma hedging considering GARCH and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

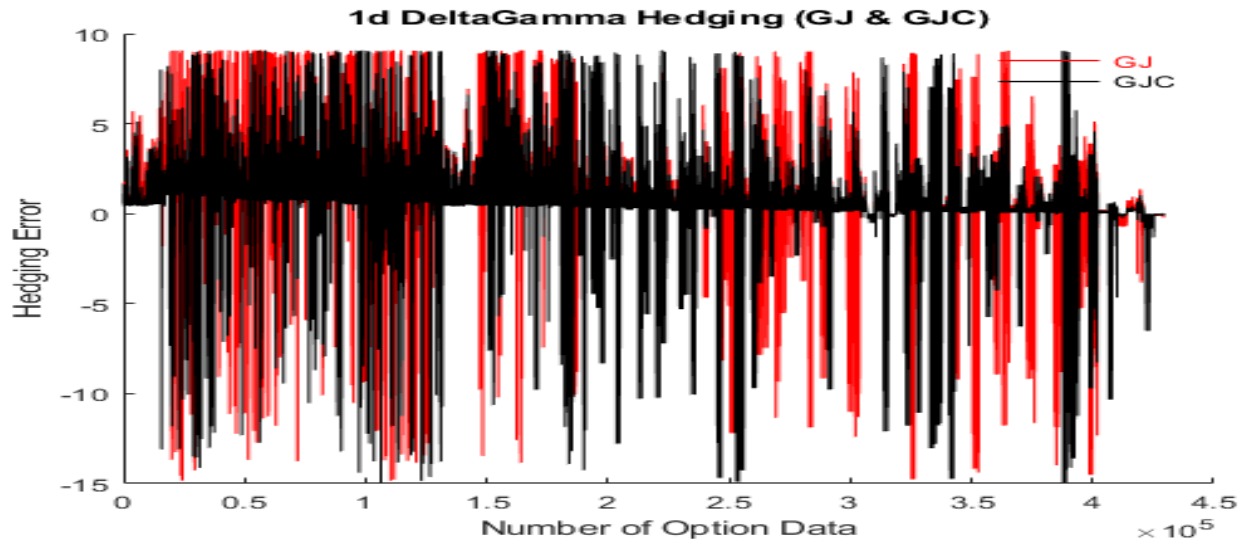


Figure-6.24: One day Delta-Gamma hedging considering GARCH Jump and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

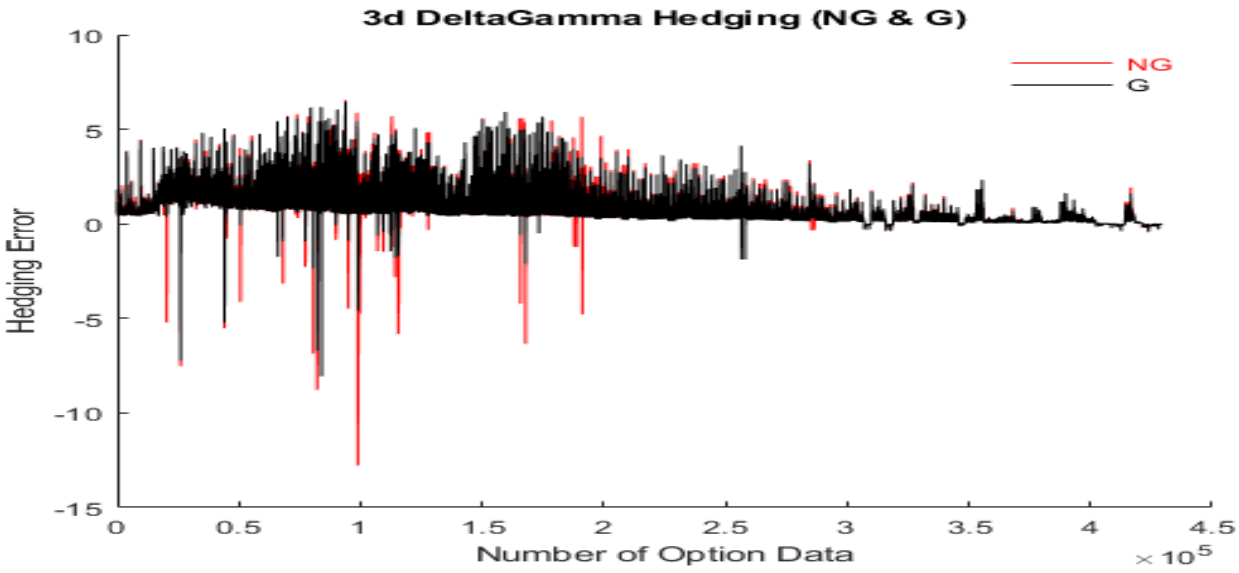


Figure-6.25: Three and half days Delta-Gamma hedging considering NGARCH and GARCH models for call option data during the period January 2, 2008 to December 31, 2014.

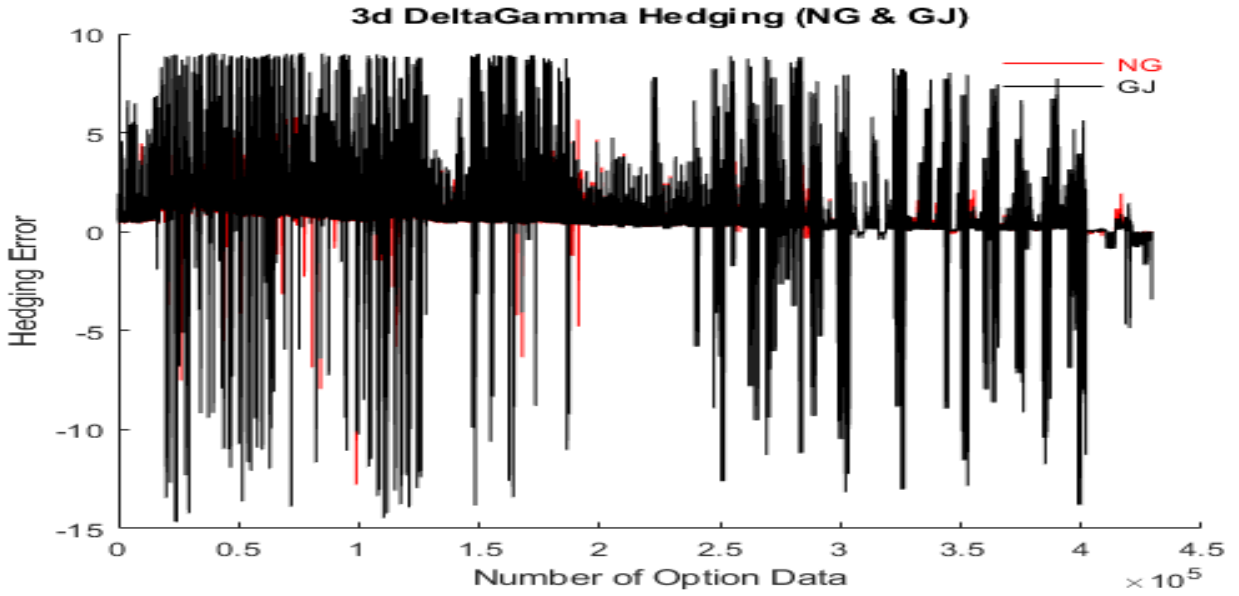


Figure-6.26: Three and half days Delta-Gamma hedging considering NGARCH and GARCH Jump models for call option data during the period January 2, 2008 to December 31, 2014.

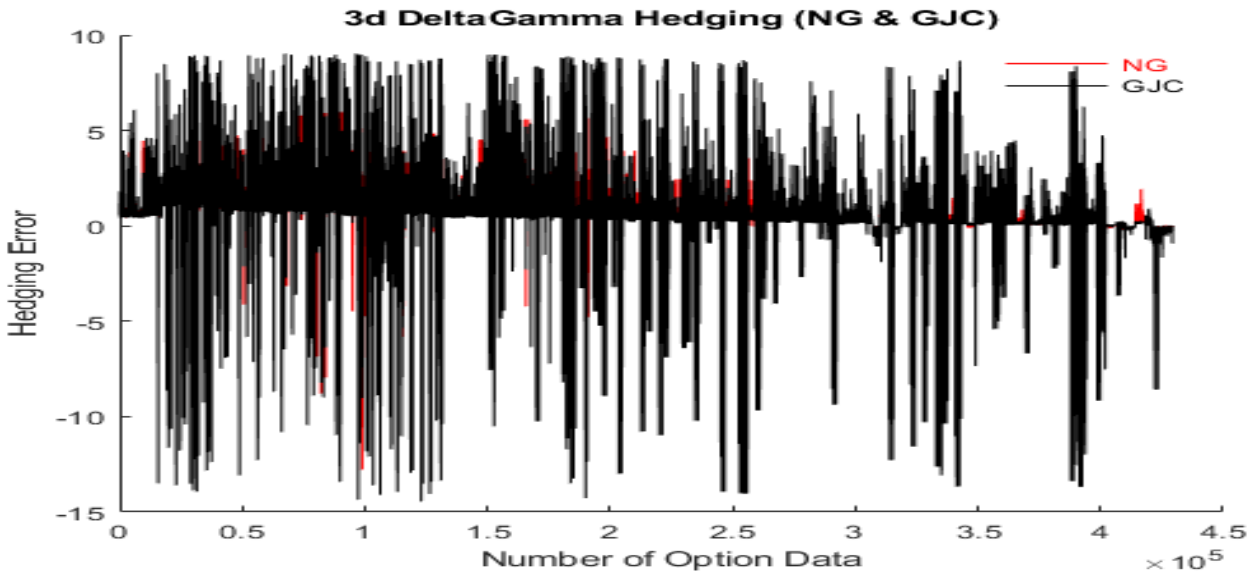


Figure-6.27: Three and half days Delta-Gamma hedging considering NGARCH and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

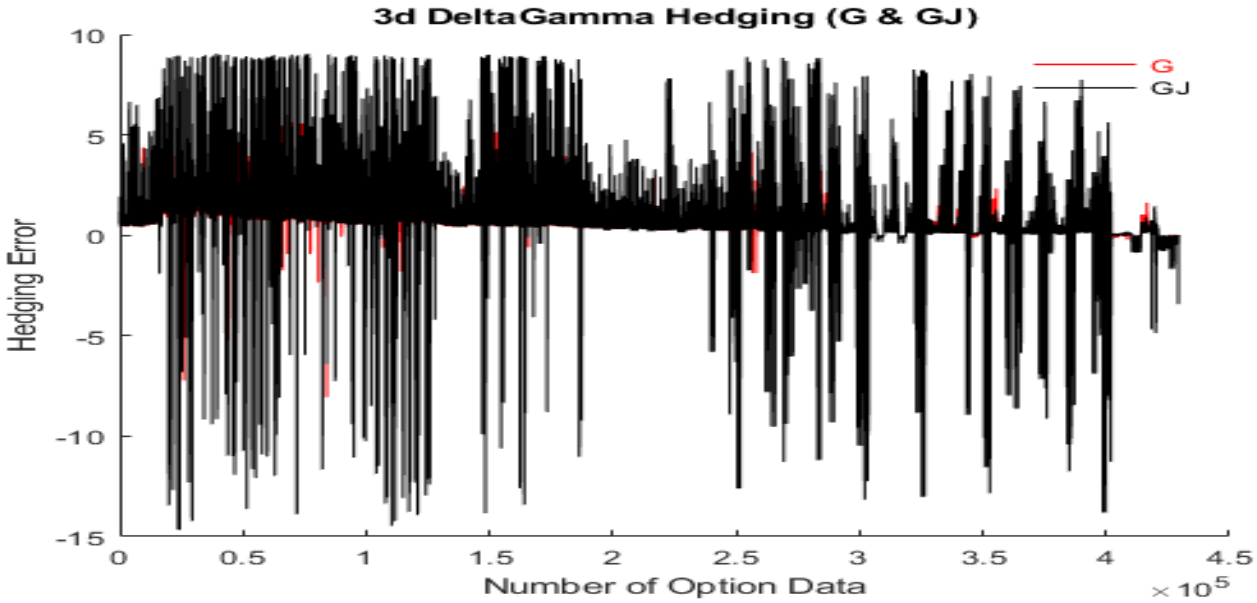


Figure-6.28: Three and half days Delta-Gamma hedging considering GARCH and GARCH Jump models for call option data during the period January 2, 2008 to December 31, 2014.

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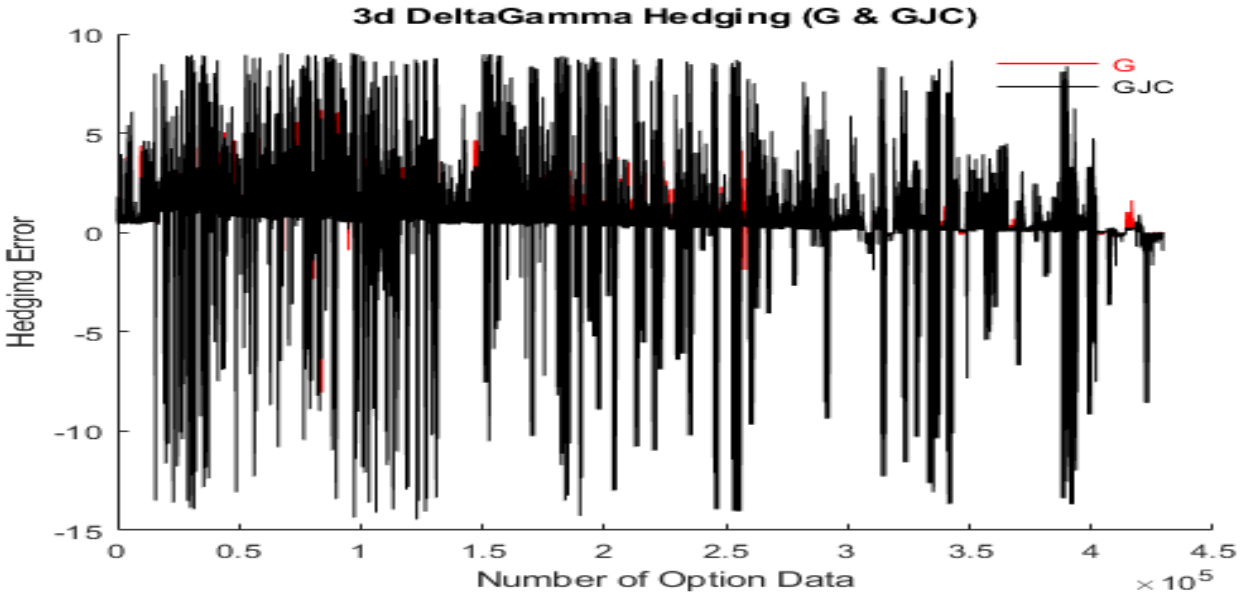


Figure-6.29: Three and half days Delta-Gamma hedging considering GARCH and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

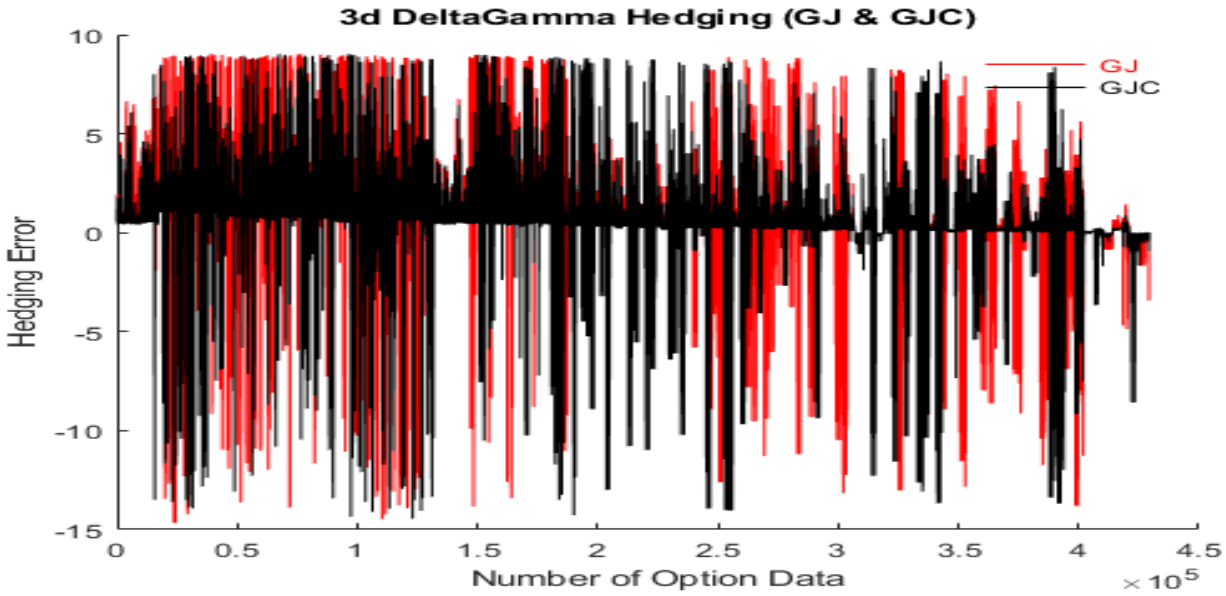


Figure-6.30: Three and half days Delta-Gamma hedging considering GARCH Jump and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

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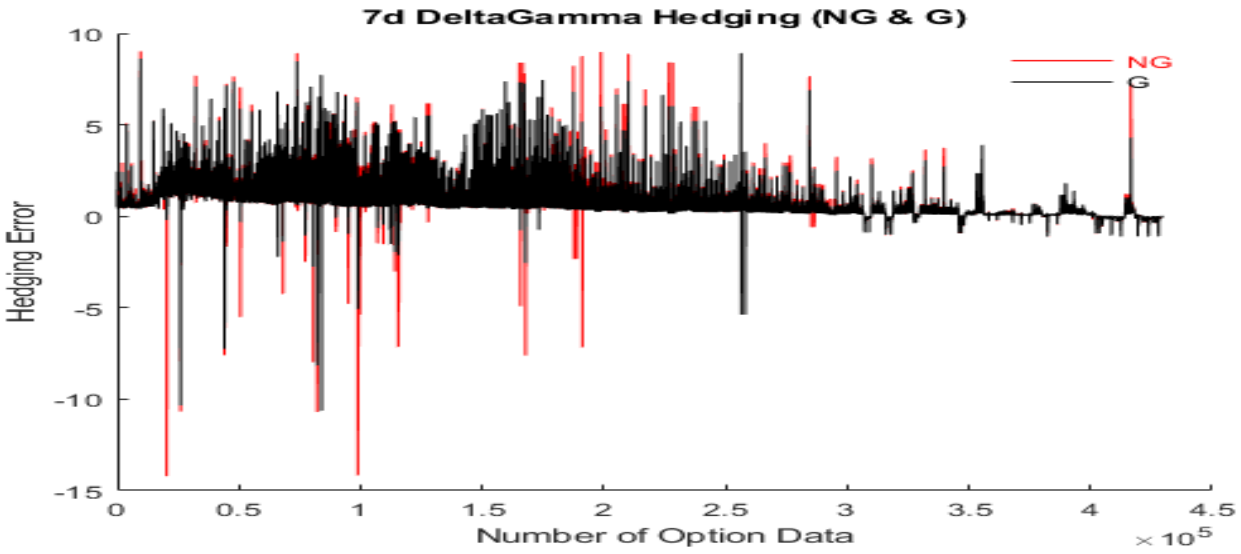


Figure-6.31: Seven days Delta-Gamma hedging considering NGARCH and GARCH models for call option data during the period January 2, 2008 to December 31, 2014.

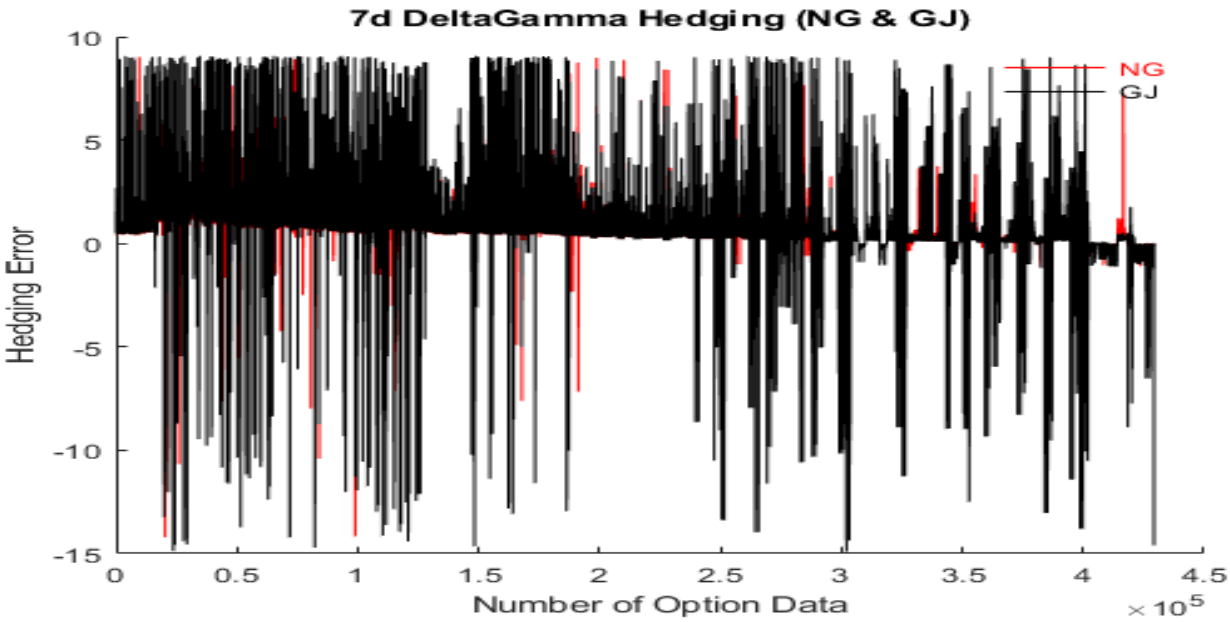


Figure-6.32: Seven days Delta-Gamma hedging considering NGARCH and GARCH Jump models for call option data during the period January 2, 2008 to December 31, 2014.

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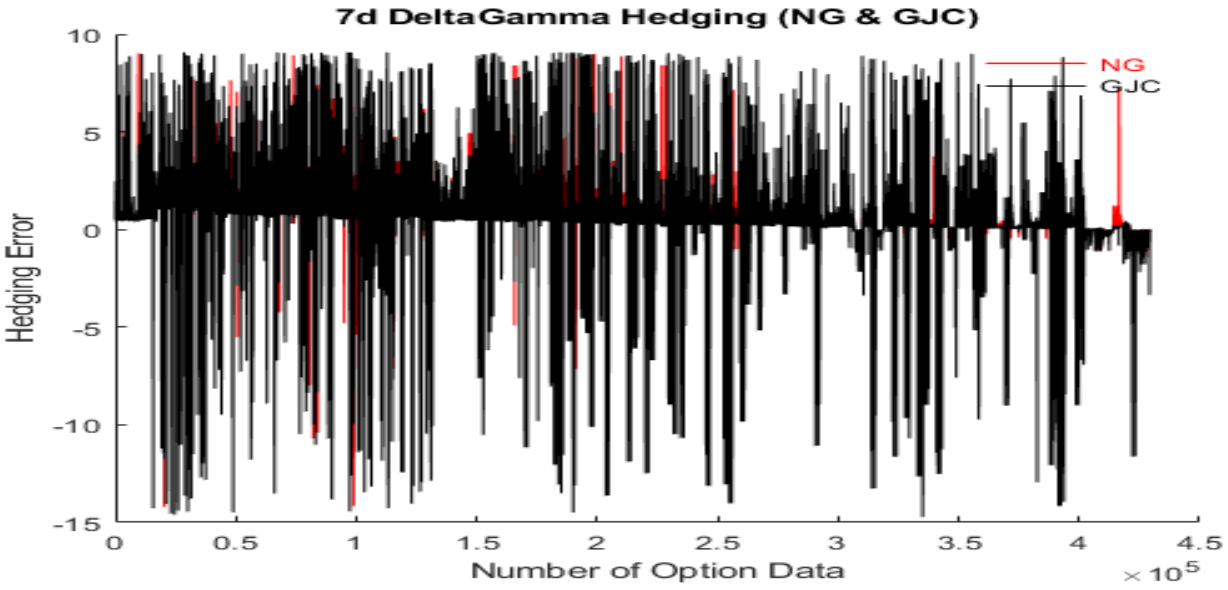


Figure-6.33: Seven days Delta-Gamma hedging considering NGARCH and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

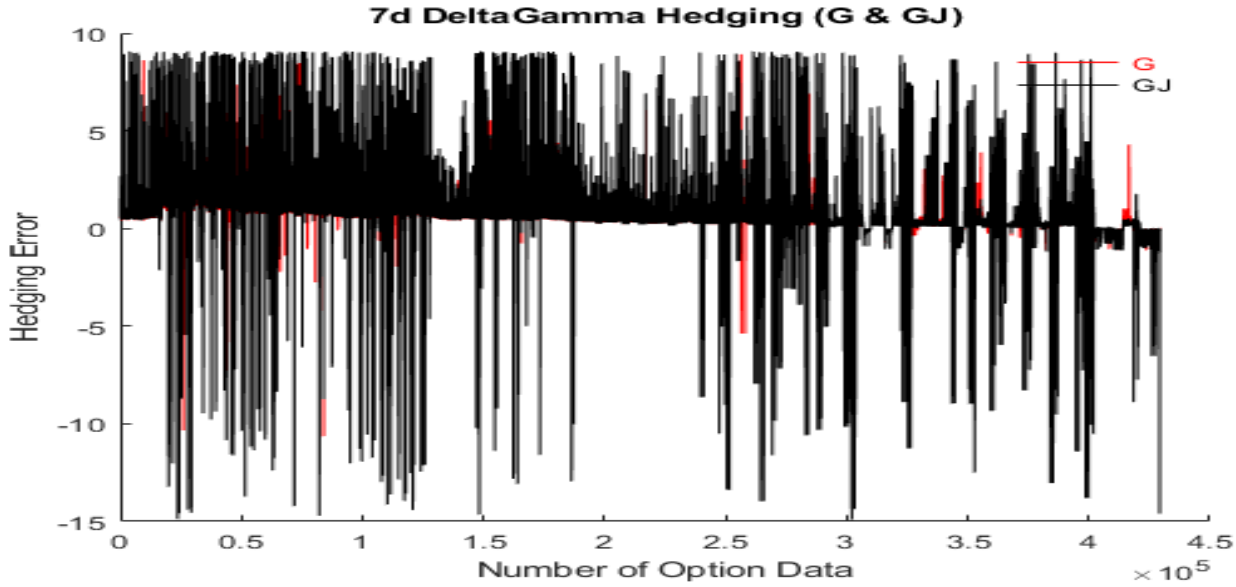


Figure-6.34: Seven days Delta-Gamma hedging considering GARCH and GARCH Jump models for call option data during the period January 2, 2008 to December 31, 2014.

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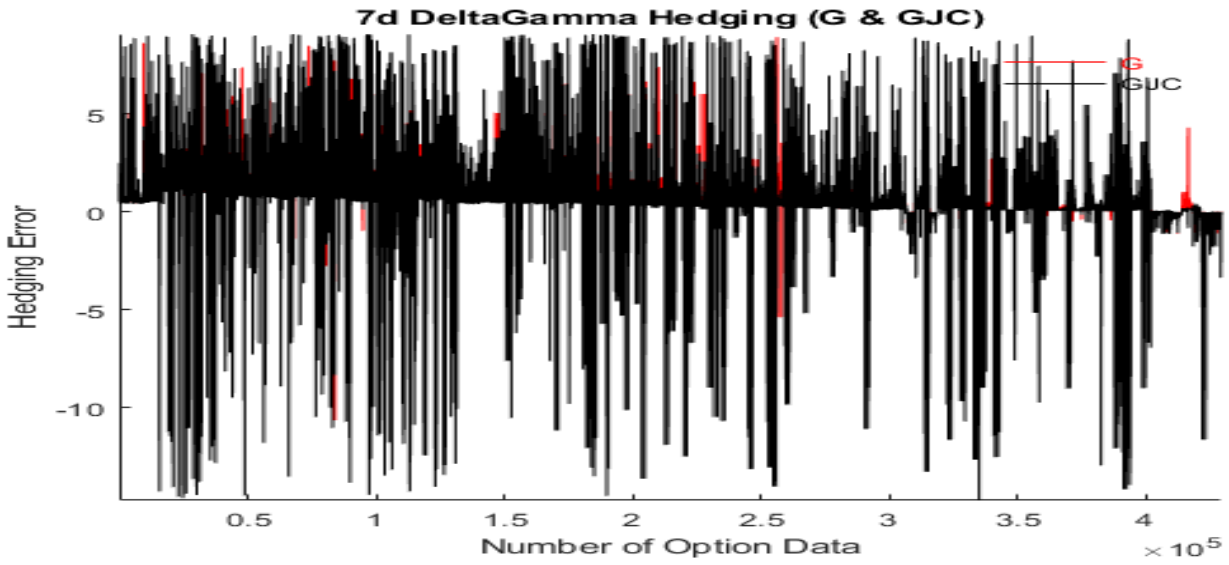


Figure-6.35: Seven days Delta-Gamma hedging considering GARCH and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

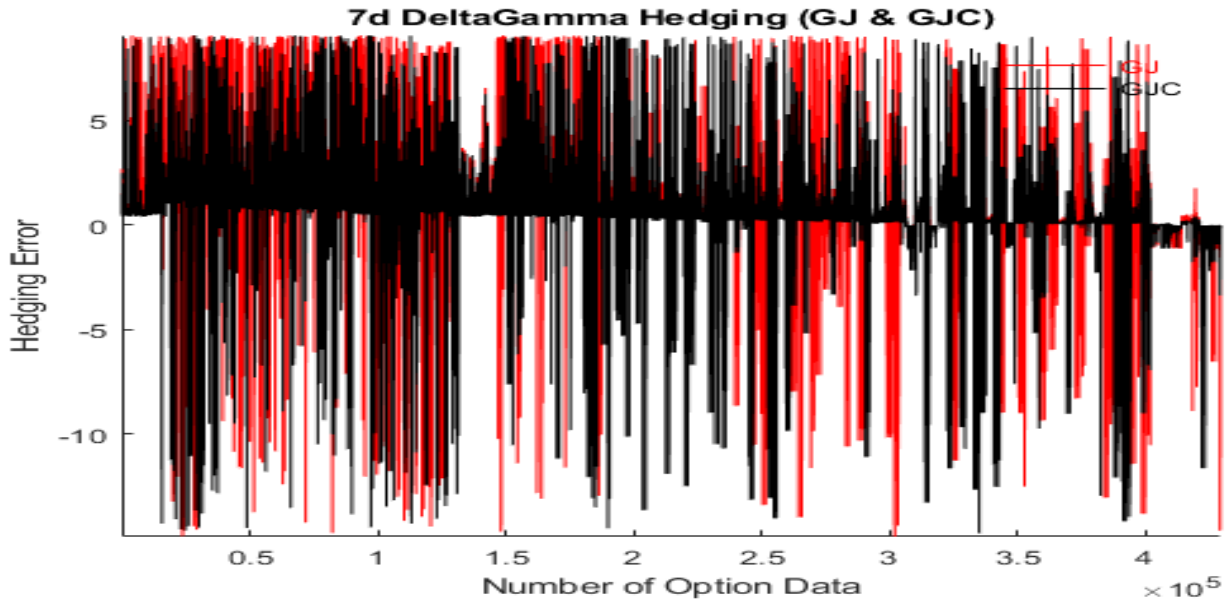


Figure-6.36: Seven days Delta-Gamma hedging considering GARCH Jump and GARCH Jump Constant models for call option data during the period January 2, 2008 to December 31, 2014.

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Comparing Models	1-Day	3.5-Days	7-Days
GARCH > NGARCH	20.627	19.668	18.643
NGARCH > GARCH Jump	26.633	25.869	25.039
NGARCH > GARCH Jump Constant	32.861	32.097	31.278
GARCH > GARCH Jump	32.465	31.699	30.864
GARCH > GARCH Jump Constant	36.453	35.686	34.863
GARCH Jump Constant > GARCH Jump	5.881	6.040	6.174

Table-6.2 shows success rate in percentage for 1-day, 3.5 -days and 7-days **delta-hedging** using NGARCH, GARCH, GARCH Jump, and GARCH Jump Constant. Here ">" indicates is better than.

Comparing Models	1-Day	3.5-Days	7-Days
GARCH > NGARCH	2.865	2.110	1.321
NGARCH > GARCH Jump	36.428	35.694	34.834
NGARCH > GARCH Jump Constant	27.270	26.594	25.859
GARCH > GARCH Jump	41.250	40.528	39.702
GARCH > GARCH Jump Constant	25.093	24.450	23.732
GARCH Jump Constant > GARCH Jump	18.925	19.045	18.911

Table-6.3 shows success rate in percentage for 1-day, 3.5 -days and 7-days **delta Gamma-hedging** using NGARCH, GARCH, GARCH Jump, and GARCH Jump Constant. Here ">" indicates is better than.

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In empirical analysis, we compare the hedging performance of NGARCH, GARCH, GARCH-Jump, and GARCH-Jump Constant models for Dow Jones call options over 1-day, 3.5-day, and 7-day risk management horizons. Figure 6.1 to 6.18 show the delta hedging performance in Wednesday's call option data using GARCH, NGARCH, GARCH-Jump, GARCH-Jump Constant models over 1-day, 3.5-days, and 7-days risk management horizons during the period January 2. 2008 to December 31, 20014. For delta hedging, Table 6.2 data confirm that GARCH exceeds an NGARCH in roughly 19–21% of instances and has significantly superior performance compared to both jump-based models. The GARCH-Jump Constant model exhibits marginally superior performance compared to the GARCH-Jump model, indicating that a constant jump specification yields more stable volatility predictions. The disparities in performance diminish as the hedging horizon extends, suggesting that the selection of the model becomes less significant over prolonged holding durations.

Figure 6.19 to 6.36 represents the delta-Gamma hedging performance in Wednesday's call option data considering several pair of models from GARCH, NGARCH, GARCH-Jump, GARCH-Jump Constant models over 1-day, 3.5-days, and 7-days risk management horizons during the period January 2. 2008 to December 31,20014. Table 6.3 describes the success rates associated with delta-gamma hedging. In contrast to pure delta hedging, delta-gamma hedging manages both first-order and second-order price sensitivity, offering protection against significant fluctuations in the underlying asset. The findings indicate that GARCH consistently surpasses NGARCH, albeit by a modest margin. GARCH has superior hedge efficiency compared to NGARCH in 2.87% of one-day observations, 2.11% of 3.5-day observations, and 1.32% of seven-day observations. This signifies that after hedging gamma risk, the performance disparity between the two volatility models diminishes significantly.

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Parameters	Mean	Standard Deviation	Minimum	Maximum
μ	0.0012	0.0013	-0.0045	0.0073
ω	1.9814e-06	1.631e-06	1.8139e-14	1.5453e-05
α	0.8868	0.0767	0.5038	0.9822
β	0.0234	0.0096	0.0084	0.1215
γ	1.8879	1.1457	-0.2751	6.4866

Table 6.4 shows the summary statistics of the parameters of NGARCH model that we calculated from the estimated NGARCH parameters for each Wednesday option contract during the period January, 2008 to December, 2014.Here number of option contract 1763.

Parameters	Mean	Standard Deviation	Minimum	Maximum
ω	5.0681e-06	3.2324e-06	1.3410e-06	1.2341e-05
α	0.0405	0.0055	0.0198	0.0521
β	0.9608	0.0055	0.9488	0.9794

Table-6.5: Summary statistics of the parameter estimation for GARCH option pricing model.

Parameters	Mean	Standard Deviation	Minimum	Maximum
ω	7.0413e-06	1.3497e-05	2.1086e-15	1.0562e-04
α	0.3323	0.3072	4.1106e-08	1.8880
β	0.5776	0.3000	1.2028e-04	0.9946
δ	0.0850	0.0415	0.0310	0.5081
θ	-0.0052	0.1222	-0.1019	0.0803
a	0.0379	0.0809	-0.0653	0.6714
b	0.4095	0.8034	-1.6020	1.0878
c	-0.0219	0.0626	-0.7178	0.2840

Table-6.6: Summary statistics of the parameter estimation for GARCH-Jump option pricing model.

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Table 6.4 presents the estimated parameters of the NGARCH model derived from 1,763 Wednesday option contracts. The mean value of α(0.8868) much exceeds that of β (0.0234), suggesting that current shocks exert a considerably greater influence on conditional volatility compared to the persistence of prior volatility. The positive mean value of γ (1.8879) corroborates the existence of substantial asymmetry or leverage effects in the volatility process. Negative market shocks elicit greater volatility responses than positive shocks of equivalent magnitude. The minimal value of ω indicates that the unconditional variance remains consistent across the sample period. Overall, the parameter estimations validate the presence of time-varying and asymmetric volatility in Dow Jones returns.The GARCH parameter estimates presented in Table 6.5 indicate significant volatility persistence. The average values of $\alpha = 0.0405$ and $\beta = 0.9608$ suggest that $\alpha + \beta \approx 1.0013$, signifying a markedly persistent volatility process. This outcome is characteristic of financial return series and indicates that volatility shocks diminish gradually over time. The superiority of β over α signifies that historical volatility has a greater role in predicting future volatility than recent developments.

Table 6.6 summarizes the parameter estimations for the model in daily market. The three parameters a, b, and c are crucial for delineating the distinct patterns of jump dynamics across diverse financial assets, while the parameters α and v are governed by equations (6.18) and (6.19), respectively, with the condition $\delta^2 > 0$. The typical jumps size θ is negative for the markets. The influence of jumps on the conditional mean of returns typically averages around zero, although this does not indicate that jumps do not alter the distribution of returns. Table 6.7 show a negative average jump-size parameter (θ = -0.0213) in the GARCH-Jump Constant model, signifying the occurrence of negative leaps in the market. Nonetheless, some parameters have exceedingly large standard deviations, especially λ and, α indicating significant estimation uncertainty. Nonetheless, this volatility, the GARCH-Jump Constant model routinely surpasses the conventional GARCH-Jump model. This indicates that implementing a consistent jump structure

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diminishes estimation noise and enhances the stability of volatility predictions. However, the model is still inferior to both GARCH and NGARCH, indicating that the advantages of simulating leaps do not compensate for the heightened parameter uncertainty costs.

Parameters	Mean	Standard Deviation	Minimum	Maximum
ω	1.5680e-05	1.3387e-04	5.7934e-17	0.0041
α	0.2903	2.2666	4.0294e-08	92.7199
β	0.7306	0.3041	5.7745e-08	1.4192
δ	0.1022	0.8394	-0.4095	27.6402
θ	-0.0213	0.0898	-2.1307	1.3883
λ	-2.3897	36.2399	-752.9070	3.8664

Table-6.7: Summary statistics of the parameter estimation for GARCH with Jump Constant option pricing model.

Over all, the findings indicate that the traditional GARCH model is the most efficient volatility model for both delta and delta-gamma hedging, however the incorporation of jump components does not enhance hedging performance over the sample period. Empirical evidence indicates that the traditional GARCH model yields the most accurate volatility estimates for both delta and delta-gamma hedging of Dow Jones call options. While NGARCH effectively captures leverage effects, its hedging performance is marginally inferior than that of GARCH. Both jump-based models exhibit significantly inferior performance, suggesting that the inclusion of jump dynamics does not improve actual hedging efficacy over the study period. The findings indicate that consistent and enduring volatility forecasts are more beneficial for risk management than intricate models aimed at capturing infrequent jump events. As a result, practitioners pursuing effective option hedging methods may favour the traditional GARCH framework over more complex jump-diffusion alternatives.

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6.6 Conclusion

We implement hypothetical strike price considering close price of DJIA as spot price in option data to compare among several complicated option pricing models GARCH, NGARCH, GARCH-Jump, GARCH-Jump Constant for pricing and hedging. For hypothetical delta hedging performance GARCH model shows better performances than other considering models, yet the differences among the models are not significant. Also, we investigate delta-gamma hedging performance of NGARCH, GARCH, GARCH-jump and GARCH-Jump constant models relative to the Black-Sholes (BS) option pricing model. Our result show that the GARCH model perform better than all other model in 1-day, 3.5-days and 7-days hedging horizon, for delta and delta gamma hedging irrespective of moneyness category. Although the current analysis is based on Dow Jones data, the ultimate goal of this project is to extend the framework to emerging markets, such as the Dhaka Stock Exchange (DSE), where option trading is not yet available. By applying this methodology to DSE data, we aim to evaluate the applicability of the framework in less developed financial markets and provide insight into the potential for introducing option trading in such markets. This extension will help bridge the gap between theoretical models and practical applications in emerging economies.

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Chapter 7

Concluding Remarks

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7 Concluding Remarks

This extensive research has formulated proposals, supported by empirical evidence from the derivative markets of developed countries, concerning which emerging nation should adopt what dynamic pricing models of their derivative contracts, in the absence of actual market derivative contract data pertaining to their own indices. The three key dimensions each contribute significantly to the advancement of stock market prediction and the potential for enhanced financial modelling. These dimensions have encompassed the utilization of risk measures value-at-risk, expected shortfall and backtesting of risk measures VaR and ES; analytic risk forecasts conditional on assets' current level, and the concept of hypothetical option models for emerging countries like Bangladesh. Within this many-sided research, several noteworthy innovations have taken center stage as follows:

(i) The utilization of risk measures value-at-risk, expected shortfall and backtesting of risk measure VaR and ES: We advise Bangladesh Bank and the top management of the financial institutions that successfully passed the expected shortfall backtests in our research to collaborate in formulating a policy that informs investors about the implications of the models employed in our study. We contend that implementing comparative efficacy assessments through readily available methods could aid generic traders in accurately forecasting the maximum potential loss associated with holding assets of registered commercial banks under adverse market conditions. This is achievable if investors employ these models alongside conventional backtesting protocols for projected shortfall, which functions as a mechanism for measuring their market risk exposure. Furthermore, it is imperative for the central bank of Bangladesh to enact laws that encourage retail investors to participate in concise risk-management training, in accordance with our proposed strategy. These procedures may also diminish the likelihood of a capital market bubble resulting in a stock market disaster owing to unforeseen fraud in the future.

(ii) Analytic risk forecasts conditional on assets' current level: An approach for risk forecasting based solely on the current asset price, rather than relying on a substantial

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historical dataset of asset prices necessary for forecasts conditioned on the present volatility level, has been proposed. Being Markovian, it may incorporate mean reversion and leverage effects of asset evolution into risk projections while maintaining the ease of application with analytical Black-Scholes formulas for estimating both VaR and ES, as well as for creating forecasts for any future period. The back-testing evidence indicates that VaR and ES (for both single- and multi-period forecasts) consistently surpass the performance of the classic GARCH-EVT model of conditional volatility, which is inherently non-Markovian. Furthermore, given that GARCH-EVT demonstrates superior performance compared to other conditional risk forecasts utilizing other GARCH dynamics (excluding those incorporating extreme value innovations), we posit that our suggested method possesses the potential to gain widespread adoption in practical applications among all models.

(iii) The concept of a hypothetical option model for an emerging country like Bangladesh: We implement a hypothetical strike price considering the close price of the DJIA as the spot price in option data to compare several complicated option pricing models: GARCH, NGARCH, GARCH-Jump, and GARCH-Jump Constant for pricing and hedging. For hypothetical delta hedging performance GARCH model shows better performance than other considered models, yet the differences among the models are not significant. Also, we investigate delta-gamma hedging performance of NGARCH, GARCH, GARCH-jump and GARCH-jump constant models relative to the Black-Scholes (BS) option pricing model. Our results show that the GARCH model performs better than all other models in 1-day, 3.5-day, and 7-day hedging horizons for delta and delta-gamma hedging, irrespective of moneyness category. Although the current analysis is based on Dow Jones data, the ultimate goal of this project is to extend the framework to emerging markets, such as the Dhaka Stock Exchange (DSE), where option trading is not yet available. By applying this methodology to DSE data, we aim to evaluate the applicability of the framework in less developed financial markets and provide insight into the potential for introducing

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option trading in such markets. This extension will help bridge the gap between theoretical models and practical applications in emerging economies.

7.1 Future Research Recommendations

This research establishes a foundation for improving derivative pricing and risk management in emerging markets lacking real-time option data. Future work should extend the Conditional Analytic Markov (CAM) model to other asset classes, support regulatory integration of VaR and ES disclosures, and assess the impact of investor risk instruction. Expanding the hypothetical option framework to markets like the Dhaka Stock Exchange can guide policymakers on readiness for derivatives trading. Additionally, evaluating GARCH-related models across various market conditions will inform model selection during volatility. Collectively, these directions aim to bridge theory and practice, enhancing financial resilience in developing economies. As in this study, we focus on both shorter and longer volatility forecasting horizons with multi-period parametric and square-root-rule characterization. For example, the Basel Committee recommends a 10-day horizon for calculating market risk capital requirements (Banks for International Settlements. Minimum capital requirements for market risk., 2016). Future research could extend this aanalysis by exploring the effects of intra-day trading in recent years which may influence volatility dynamics, suggesting that identifying the most suitable VaR and ES models should be seen as an ongoing process. While this study concentrates on stock volatility, future work could examine how GARCH models perform across other asset classes, such as interest rates or foreign exchange rates. Furthermore, evaluating the performance of multivariate GARCH models on diversified portfolios could provide deeper insights into risk management strategies across asset types. Another important aspect of this study is to investigate the contributions to risk measurement and hypothetical derivative modelling for emerging markets, future research should explore the application of non-Gaussian models, particularly the Normal Inverse Gaussian (NIG) and Variance Gamma (VG) distributions. These models can capture skewness and heavy tails more effectively than Gaussian-based approaches and are well-suited for modelling real-world asset returns.

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