# Optimal Production Planning Models for Garment Industries in Bangladesh under Stochastic Atmosphere

This thesis submitted to the University of Dhaka in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy in Mathematics


**Submitted by**

Sayma Suraiya
Registration No. 125
Session: 2020-2021
Department of Mathematics
University of Dhaka

**Under the Supervision of**
Prof. Dr. Mohammad Babul Hasan
Department of Mathematics
Faculty of Science
University of Dhaka

Department of Mathematics
Faculty of Science
University of Dhaka

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# Letter of Transmittal

July 25, 2025

**Professor Dr. Mohammad Babul Hasan**
Department of Mathematics
Faculty of Sciences
University of Dhaka

**Subject: Submission of PhD Thesis.**

Dear Sir,

With due respect and honor, I intend to let you know that I am privileged and delighted to submit my PhD thesis on 'Optimal Production Planning Models for Garment Industries in Bangladesh under Stochastic Atmosphere". It was quite challenging and academically exciting to work towards the completion of this PhD thesis. I have been able to apply some of the major theories and concepts to justify the models to get the outcomes of this thesis. I believe that the findings of this thesis are relevant to the extent of its subject matter. I also believe that the experience and skills that I have gathered in working on this thesis in a decision making perspective where I will be benefited throughout my profession and further education.

This PhD thesis has not been partially or completely written or addressed anywhere and is not being considered for another publication. I am thankful to you for your constant supervision and for granting me authorization to work on this thesis.

Sincerely,

**Sayma Suraiya**

PhD Candidate
Department of Mathematics
Faculty of Science
University of Dhaka

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# DECLARATION

I hereby proclaim that the research work titled with ‘Optimal Production Planning Models for Garment Industries in Bangladesh under Stochastic Atmosphere’ has been approved under Faculty of Science, Department of Mathematics, University of Dhaka in fulfillment of the Degree of Doctor of Philosophy. This thesis has been composed based on original research findings from field study developed by me together with references from published literature. This research has not been submitted in part or full to any other institute or for any other degree.

_____

**Sayma Suraiya**
Registration No. 125
Session: 2020-2021
Department of Mathematics
University of Dhaka

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# Certificate of the Supervisor

This is to certify that Sayma Suraiya has undertaken her PhD study under my supervision and guidelines, and therefore prepared the thesis 'Optimal Production Planning Models for Garment Industries in Bangladesh under Stochastic Atmosphere'. To the best of my knowledge, the researcher duly mentioned the other researchers' materials and sources used in her study. Further the thesis was not submitted to any other universities or institutions for any other degree or diplomas.

Thus, I am recommending that this thesis to be submitted to the Department of Mathematics, Faculty of Science of University of Dhaka in fulfillment of the requirements for the award of the degree of Doctor of Philosophy.

I also certify that there is no plagiarized content in this thesis and did not exceed the international standard.

.

**Professor Dr. Mohammad Babul Hasan**

Department of Mathematics

Faculty of Science

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# Dedication

This PhD dissertation is devotedly dedicated to my parents and my lovely family. Their constant support, inspiration, wisdom and confidence on me have been affected in every possible way throughout my life.

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# Optimal Production Planning Models for Garment Industries in Bangladesh under Stochastic Atmosphere

University of Dhaka

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# Acknowledgement

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Finally, I would like to end with a thanks note dedicated to all my colleagues and friends for their constant moral support throughout this research journey.

My wholehearted thanks to all of you.

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# Abstract

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# Abstract

A key pillar of Bangladesh's economy, the ready-made garment (RMG) industry makes a substantial contribution to the country's GDP, foreign exchange earnings, and job creation. This study presents a thorough modeling framework for production planning optimization in Bangladesh's RMG sector under both deterministic and stochastic circumstances. In order to maximize profit and optimize production planning of the RMG industry, this study develops first a deterministic linear programming (LP) model. The model finds the optimal product mix and output levels to effectively meet demand while lowering production costs and increasing profitability when it is applied to a real-world factory setting in Gazipur, Dhaka.

Uncertainties in global demand trends, variable manufacturing costs, volatile raw material prices, and dynamic international trade rules etc. are the most vital reasons of the continuously increased challenges to the industry. This research seeks to develop strong decision-making frameworks to support strategic and tactical decision-making under such uncertainty. Taking into consideration economic fluctuations, the deterministic LP is expanded in this stage into stochastic programming models (SLP) in which all significant factors are represented as random variables, including cost coefficients, demand levels, labor availability, and processing durations. Another stochastic model also formed in this research by combining all the scenarios, as it is not actually predictable which situation would be come. This approach provides a strong planning for RMG under a variety of circumstances by capturing holistic uncertainty.

A two-stage stochastic linear programming model (TSLP), in which only a chosen subset of parameters remains stochastic, is developed next in this study to reduce uncertainty. Following the revelation of parameter realizations, decisions taken in stage one are modified in stage two. This model improves tractability and increases adaptability. Expected Value of Perfect Information (EVPI) and Value of the Stochastic Solution (VSS), two important parameters in stochastic programming that measure the advantages of uncertainty modeling and perfect foresight, are used to assess the performance of deterministic, general stochastic, and two-stage stochastic models.

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In next, two separate two-stage stochastic (TSLP) models are formulated in this study, where the demand uncertainty represents various scenarios, including seasonal variations. In this case, the fluctuation of one product's demand is presented. Key uncertainties in demand, export prices, labor costs, raw material costs, and operational costs related to that specific product are included in scenarios that are created using historical data and probabilistic distributions. These models are then expanded into a multistage stochastic programming (MSLP) model, which simulates a decision process by allowing decision variables to change over several stages as demand unfolds and gradually adapting decisions at each stage to capture dynamic changes and integrating learning over time. By applying these models, a RMG factory can be able to decide the best production planning along with the profit and cost optimization. A comparison between the two-stage and multistage is presented next.

LINDO and AMPL (A Mathematical Programming Language) are used to solve all the deterministic and stochastic models, while Excel Solver is applied for preparing all the graphical presentations of this study. In terms of predicted profit, robustness to uncertainty, and overalsl operational efficiency, the stochastic models perform noticeably better than the deterministic model, according to computational results validated using real-world data gathered from RMG companies in Bangladesh.

By offering an integrated optimization framework that enables RMG stakeholders to use scenario-based planning, efficient, uncertainty-aware design tools from deterministic LP to comprehensive multistage stochastic programming. Optimization tools to increase profitability, the study adds both theoretically and practically significant, as global markets continue to vary. By presenting a thorough modeling framework designed to address the unique difficulties faced by Bangladesh's RMG industry in order to facilitate adaptable and proactive decision-making. In the geopolitically unstable environment, where flexibility and risk-aware planning are critical also in the post-COVID atmosphere, these findings are especially pertinent. And provide resiliency in a world economy that is becoming more and more uncertain.

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# Abbreviations and Acronyms

| | |
|---|---|
| RMG | Ready-Made Garments |
| LP | Linear Programming |
| DLP | Deterministic Linear Programming |
| SP | Stochastic Programming |
| SLP | Stochastic Linear Programming |
| SIP | Stochastic Integer Programming |
| TSLP | Two-Stage Stochastic Linear Programming |
| MSLP | Multi-Stage Stochastic Linear Programming |
| EVPI | Expected Value of Perfect Information |
| VSS | Value of Stochastic Solution |

# Chapter 1

# Introduction

# Chapter 1
# Introduction

## 1.1 Introduction

Bangladesh, a tiny nation in Southeast Asia with a dense population, is one of the world's growing economies. Bangladesh has faced significant swings in its economy, particularly in trade, industrial, fiscal, agricultural, and financial policies, over the last two and a half decades. Many sectors contribute to the growth of our economy during challenging periods. The ready-made garment (RMG) sector is one and the most important of these sectors. The leading representative of Bangladesh in the international market is the country's RMG industry. Bangladesh is ranked second only to China in terms of the overall manufacturing and export of clothing. Unquestionably, RMG export earnings have occupied a prominent position in the nation's overall export earnings since 1990s (Hassan, 2014). Of all the sectors, this industry has the most promise. According to secondary statistics, the major export sector of Bangladesh has grown significantly over the past 25 years, with RMG being the main product. Day by day, RMG's share in overall exports is growing. It now exports 84.58% of Bangladesh's total exports, up from 3.89% in 1983–1984 (Mia and Akter 2019). The nation's ready-made garment products exports were valued at $34.13 billion in 2018–2019 fiscal year, but it fell to US$27.95 billion in the FY 2019–2020 by 18.84 percent (The Financial Express, 2020). Now it comes into its form again. The total earnings are more than $47 billion in FY 2023-2024, which provided a noteworthy contribution of 10.35 percent to Bangladesh's GDP, signifying an increase of 10.27 percent compared to the preceding fiscal year. This sector is now playing a vital role in the nation’s economy. So, proper care should be taken of this industry.

In this chapter, the introduction of RMG of Bangladesh with history, background of this study, definitions of some relevant terms, basics of operations research, linear programming, stochastic programming, uncertainties, and optimizations are presented. The outline of the thesis is also be discussed at the end of this chapter.

## 1.2 History of the Garment Industry of Bangladesh

The Ready-Made Garment (RMG) industry in Bangladesh boasts a significant and transformative history. It has evolved into one of the primary pillars of the nation's economy, offering employment to millions and serving a crucial function in the export sector. Below is a summary of the historical development of the RMG industry in Bangladesh:

### 1.2.1 Initial Development (1950s–1970s)

While the textile industry in Bangladesh has deep roots, the ready-made garment sector began to emerge in the 1970s following the country's independence in 1971. At first, the industry was modest in scale, primarily consisting of local tailoring establishments. In the early years after gaining independence, this sector did not significantly impact the economy, as the nation faced challenges such as limited infrastructure and a nascent industrial framework (Bhattacharya & Rahman, 2000). During the 1970s, local entrepreneurs focused mainly on producing garments for the domestic market, establishing the first garment factories.

### 1.2.2 Development and Growth (1980s–1990s)

The 1980s marked a pivotal moment for the RMG sector, fueled by supportive government policies and the opening of international markets. Bangladesh's garment industry began exporting to global markets, especially to Western nations. In the 1980s, the government implemented export-oriented policies, including tax exemptions and incentives for industries focused on exports (Quddus & Rashid, 2000). These measures attracted foreign investment, leading to a rapid increase in garment production for export. Additionally, the establishment of export processing zones (EPZs) during this decade provided various advantages to garment manufacturers. The 1990s saw further expansion of the RMG sector, benefiting from the quota system established under the Multi-Fiber Arrangement (MFA), which limited textile imports from developing countries into western markets (Ahmed, 2004). Bangladesh capitalized on its ability to produce garments at lower costs due to inexpensive labor and minimal overhead expenses which resulting a significant increase in export orders from countries such as the United States, Germany, and the United Kingdom.

### 1.2.3 Global Competitiveness and Challenges (2000s)

As the ready-made garment (RMG) sector in Bangladesh expanded, it encountered both opportunities and obstacles. The early 2000s represented a crucial time when the industry achieved notable global prominence, yet it also faced heightened competition and scrutiny. In 2004, the gradual elimination of the quota system under the Multifiber Arrangement (MFA) intensified competition. Nevertheless, Bangladesh maintained its competitive edge due to its low labor costs and increasing production capabilities (Khosla, 2013). From 2005 to 2010, the industry continued its upward trajectory, positioning Bangladesh among the leading garment exporters globally, alongside China and India. This sector also began to diversify, with a growing number of factories engaged in the production of knitwear, sweaters, woven wear, and denim. However, this growth was accompanied by significant challenges, including issues related to labor rights, inadequate working conditions, and insufficient safety standards in numerous factories. The tragic Rana Plaza disaster in 2013, which resulted in the deaths of over 1,100 workers due to the collapse of a garment factory, drew international attention to these pressing concerns.

### 1.2.4 Contemporary RMG Sector (2010s–Present)

In spite of ongoing challenges, the RMG industry in Bangladesh has continued to adapt and modernize. Incidents such as the Rana Plaza collapse, sustainability, and compliance have been mounting pressure on the industry to enhance working conditions, safety measures, and environmental sustainability. The garment sector in Bangladesh has made significant progress in this era in implementing improved safety protocols, and international buyers now expect adherence to environmental and labor standards (Lopez-Acevedo & Robertson, 2016). The country is now manufacturing more sophisticated apparel, including knitwear, sportswear, woven wear, and denim. Diversification and Innovation of Bangladesh are also witnessing an increase in the production of eco-friendly clothing and the adoption of sustainable manufacturing practices.

Table 1.1 shows the statistics of RMG products' export of different periods compare to the total export of Bangladesh as well as the contribution to GDP. The graphical representation is also displayed in Figure 1.1.

| Period | Export of RMG (USD billion) | Total Export of Bangladesh (USD billion) | % of RMG's to Total Export | Estimated contribution of RMG to GDP |
|---|---|---|---|---|
| 1983–84 | 0.03157 | 0.811 | 3.89% | 0.16% |
| 1990–91 | 0.86682 | 1.71755 | 50.47% | 4.60% |
| 1999–2000 | 4.34941 | 5.7522 | 75.61% | 10.05% |
| 2009–10 | 12.49672 | 16.2046 | 77.12% | 10.71% |
| 2019–20 | 27.94919 | 33.6741 | 83.00% | 10.71% |
| 2022–23 | 46.99161 | 55.5587 | 84.58% | 9.3% |

Table 1.1: RMG export and total export of Bangladesh with the contribution of RMG to GDP. Source: (BGMEA, 2024)

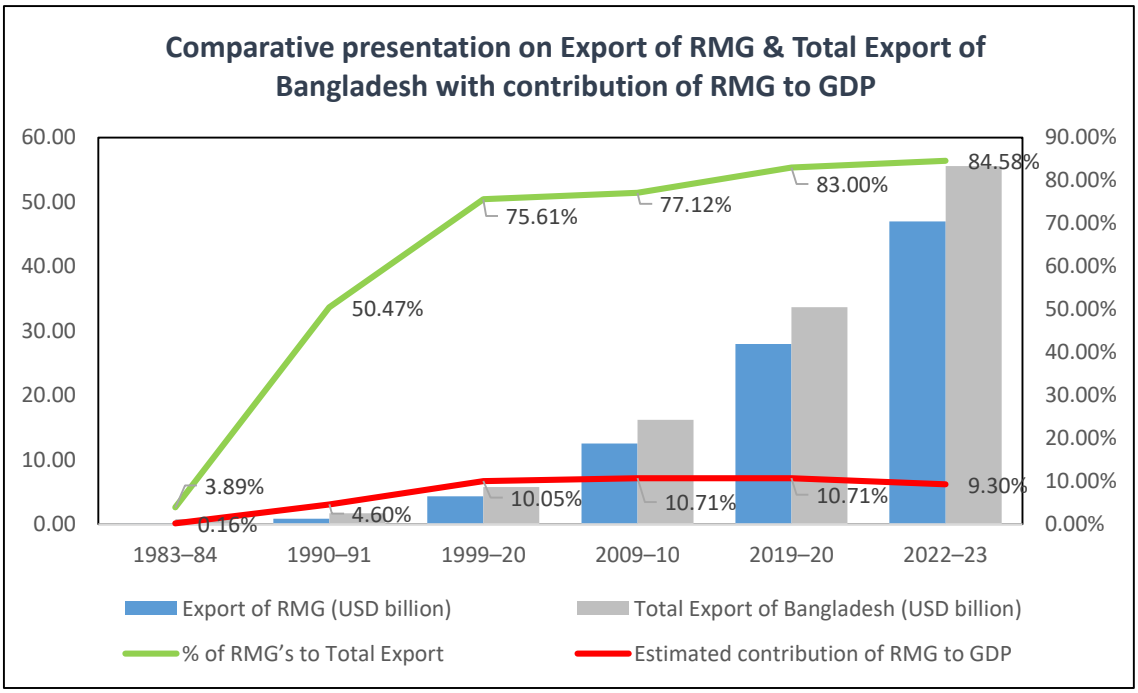


Figure 1.1: Graphical Presentation of RMG Export and Total Export of Bangladesh with the contribution of RMG to GDP (Bangladesh Bank, 2024)

## 1.3 Background of the study

In this section, the background and the motivation of this research will be presented.

It is recognized that Bangladesh's industrial development has been guided by a number of industrial policies after becoming independent, such as the industrial policy of 1973, the new industrial policy of 1999, and several more policies—the most recent being the industrial policy of 2010 (Yunus & Yamagata, 2012). Since the nation's independence, the number of clothing industries in the nation has grown significantly. The economy still depends a lot on the garment sector for employment in manufacturing, foreign exchange reserves, and women's empowerment (Jia et al., 2020). According to data from the 2018–19 fiscal year, there were 4621 garment factories and 4280000 people were employed, up from 384 in 1984–85 (BGMEA, 2023). Bangladesh's unemployment rate has decreased because of this industry, which offers the most employment opportunities. Approximately 10 million people's lives depend on this business either directly or indirectly. This sector empowered women in addition to creating jobs (Arnob et al., 2020). Bangladeshi women were able to forge a new identity as industrial workers because of this sector. Their status in their own families, in society, and ultimately in terms of societal sustainability has all changed as a result of this. There are cascading consequences of the RMG sector's expansion on other industries in Bangladesh. The growth of the RMG industry's allied sectors also produced a great deal of work opportunities for both skilled and unskilled workers. As a result, this industry indirectly contributes to job growth in other sectors by serving as suppliers of produced or traded goods to the RMG sector. It is important to take good care of this industry to maximize export earnings and, consequently, ensure maximum production since it contributes significantly to the nation’s economy as well as the creation of jobs both directly and indirectly

Bangladesh faces fierce competition in the global textile sector from other nations that produce cheap clothing. China, Bangladesh, Vietnam, India, Hong Kong, Turkey, Germany, Mexico, Italy, and Indonesia are the top 10 exporters of textiles. In the global RMG market, Bangladesh's main rivals are China, Vietnam, and India (Islam, 2023).

Table 1.2 and Figure 1.2 below show the contribution of Bangladesh compare to rival countries for the year 2023.

| Country | Export earnings in $billion (2023) | Global market share (%) |
|---|---|---|
| China | US$159 | 31.6% |
| Bangladesh | US$47 | 7% |
| Vietnam | US$35 | 6% |
| Turkey | US$20 | 3.9% |
| India | US$18 | 3.6% |
| Hongkong | US$15 | 3% |

Table 1.2: The global contribution in apparel export, 2023. Source: (Islam, 2023)

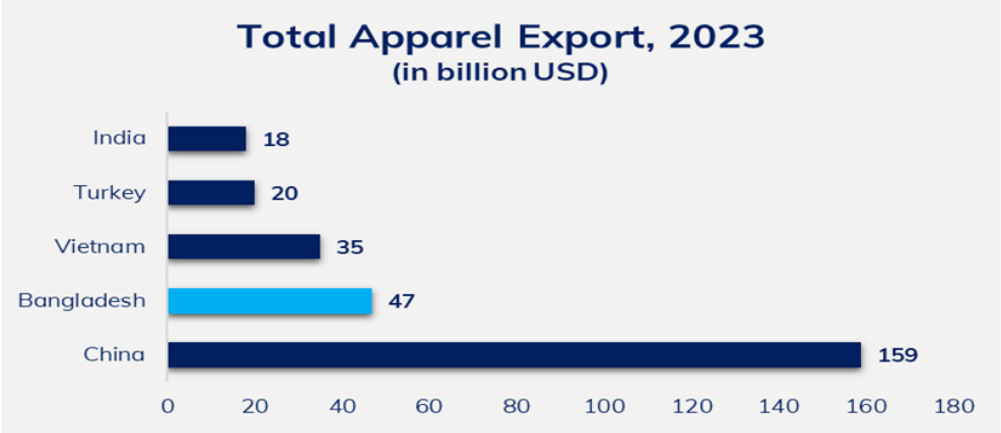


Figure 1.2: The global apparel export, 2023 (WTO's World Statistical Review, 2023)

Most of the RMG goods of Bangladesh exports to the United States of America and other members of the European Union (EU), despite several government and non-government initiatives to find and tap into other sectors. Approximately 64% of Bangladesh's total readymade product exports went to the nations in the European Union in 2017–18, while 17% went to the US (Hossain et al., 2019). In recent years, there has been a downward trend in the U.S. market also our exports to major destinations, including the European Union, Canada, and some other obsolete markets, have risen negatively (Shakirullah et al., 2020). That's why the necessary steps should be taken to upgrade this negative trend without any further delay. Since the demand for apparel, textiles, wool,

and fibers is rising and this is driving rapid growth in the garment manufacturing sector worldwide, nations, particularly emerging nations, are making great efforts to reach a high level of growth in the ready-made garment business in order to expedite their economic development. However, environmental sustainability, which is today one of the most important challenges facing the globe and a significant challenge for businesses, is meaningfully impacted by the rapid development of industrial sectors. (Smith, 2016).

Resources are more in demand in the contemporary era due to growing global economic and demographic trends. By 2021, the BGMEA declared its export objective. Even if the aim was attainable at the time, it was not realized as a result of several events, including the 2008 financial crisis, the Tazreen fire catastrophe, the collapse of Rana Plaza, the TransPacific Partnership Agreement (TPP), and America's withdrawal from the generalized system of preference (GSP) (Arnob et al., 2020). Therefore, especially in light of the current global crisis, a competent supply chain is essential to addressing compliance concerns. Furthermore, the worldwide mismatch between supply and demand is widened by the COVID-19 pandemic and the Russia-Ukraine conflict (Zakeri et al., 2022). Rising prices for energy, raw materials, and logistics have presented significant challenges for the manufacturing industry. As a result, sales of the products have decreased and customer satisfaction has increased. The economy of Bangladesh has been primarily driven by the ready-made garments (RMG) sector. According to Mridha et al. (2022), buyer-oriented businesses dominate our RMG sector. Customer satisfaction with the product's low cost is therefore the main priority. Since the gap between supply and demand is widening daily, optimizing resources has become a top priority to lower manufacturing costs. And that’s why the concern ground of this research is to offer an optimal production planning by minimizing the gap while ensuring the best use of available resources.

The lack of manufacturing inputs in the garment industry is a persistent problem that lowers capacity utilization and ultimately production. However, enterprise-level management decisions that boost output—either by cutting costs or by optimizing output—are what drive economic development and in turn, raise productivity in the real sector. Consequently, managers of businesses are always searching for the best course of action that matches their objectives, which are mostly profit maximization. The growth of the industry puts pressure on management to figure

out how best to organize, direct, and regulate output at the various levels of the economy. Due to this pressure, management theories were developed to analyze the business environment, tackling the practical issues resulting from industry operations, and tackling the external difficulties posed by the industry's operating environment (Ghuge et al., 2025). Efficient supply chain management with proper utilization of scarce resources in the apparel industry of Bangladesh is necessary to face its future challenges in formidable global competition. Currently, operations research is a science that is frequently utilized to assist in solving or solving issues connected to a business's operational issues (Nurmayanti & Sudrajat, 2021). Nowadays, a lot of business actors or companies use operations research science to manage their operations, handle operational issues, and identify the best course of action when choosing a business. The use of scientific tools, procedures, and methods to address issues with corporate operations and identify the best answers is known as operations research (Filders et al., 2008). In the realm of business, an individual dealing with optimum problems—such as minimizing expenses or optimizing profit margins—will consistently encounter limits. A challenge that a businessman encounters is the insufficiency of resources to generate ideal solutions and accomplish a predetermined objective within the company. A quantitative method of modeling and evaluating decision-making difficulties has been created in response to these obstacles. This operations research can use linear programming to address application challenges. A company will eventually run into issues figuring out how much has to be produced in order to maintain optimal earnings. The work of Ekwonwune and Edebatu (2019) shows the employment of linear programs. The application of LPP models is such a quantitative strategy, which determines the best course of action by mathematical methods. The Soviet mathematician A.N. Kolmogorov created it before World War II (Woubante, 2017). A linear program is one of the many scientific branches that mathematics today unquestionably stimulates and is extremely helpful for society's survival. It is a widely used technique in many domains, such as operations research, economics, engineering, and finance etc. It can be applied to a variety of problems, such as resource allocation, production planning, transportation, and network optimization (Aliu et al. 2023; Syifa et al. 2023; Shamsuzzaman et al. 2023). There are two approaches for finishing linear programs: the graphical technique and the simplex method. As stated by Asmara et al. (2018), a linear program is a general planning model used to determine the best course of action for allocating limited resources in an allocation dilemma. In this study, a

linear programming model is established to determine the best production planning by using the available resources. One mathematical approach for tackling or optimizing anything that is constrained by particular limits is linear programming.

The case industry of this study is a renowned garments factoty of Gazipur, Dhaka produces different types of woven and knit products to meet the global demand. To fulfill the worldwide requisition, the rapid growth of this industry is a must for ours. But in real-life situations, uncertainty is a common aspect. Deterministic optimization models, according to Nagar and Jain (2008), do not accurately represent the dynamic behavior of the majority of real-world applications. Due to uncertainty in demand, supply, costing, pricing etc. proper maximum production or optimization in profit or costing is not possible. Ignoring the consequences of uncertainty, modeling can lead to poor conclusions. There is a large collection of decision problems where uncertainty is an intrinsic property that is crucial to consider (Narum, 2020). To overcome this, some general stochastic models on different uncertain conditions with the addition of a deterministic one a two-stage and multi-stage stochastic programming models considering different scenarios are developed in this study to offer optimal production planning for optimizing profit with the best utilization of available resources. By Senna et al. (2016), Stochastic LP takes into consideration the fluctuation of characteristics like costs, demands, and resource availability, making it possible to construct more realistic scenarios. The purpose of stochastic LP is to handle scenarios in which model parameters are unpredictable. It offers solutions that work effectively in a variety of potential settings and are not just ideal in one. When making decisions and preparing strategically in the face of unpredictable future situations, this resilience is extremely helpful. By analyzing scenarios, decision makers can make better judgments by understanding the influence of uncertainty on the solution and by thoroughly analyzing many situations.

Two different two-stage stochastic models (TSLP) are also formulated in this study, considering two distinct cases where the actual sold amount of RMG industry is considered with demand fluctuation. Often, the most prevalent in the literature, a model might be rendered more realistic by including another decision stage. Thus, these TSLP models are extended to MSLP models in Chapter 6 where the same two cases are measured. The outcomes of the two-stage model and the three-stage model are compared later. MSLP models allow for new judgments at each time step depending on the uncertainty recognized thus far, extending the two-stage stochastic programming

models (Ahmed et al., 2003). In production planning, it's essential to make decisions about production quantities, scheduling, and resource allocation while considering uncertain factors.

## 1.4 Some Important Terms

This section is covered by some relevant definitions and concepts for this study.

### 1.4.1 Uncertainty

"Uncertainty" is the absence of total certainty or a situation in which the facts, results, or specifics are not entirely known or foreseeable. Making decisions is a constant throughout life, as people attempt to allocate their limited resources to the greatest possible course of action to achieve the best possible results. It frequently results in an inability to make accurate predictions or judgments when knowledge is few or future events are uncertain (Ben-Tal et al., 2009).
Uncertainty can manifest in a variety of ways, depending on the situation below:

**Mathematical and Scientific Uncertainty:** This term in statistics refers to the level of assurance in measurements or forecasts is known as mathematical uncertainty.

- **Economic Uncertainty**: Unpredictable financial circumstances that frequently affect markets and investment are referred to as economic uncertainty.
- **Psychological or Personal Uncertainty**: A type of uncertainty or indecision over future events or life choices is referred to as psychological or personal uncertainty.
- **Philosophical Uncertainty**: Questions about knowledge, truth, and what can be known in the end are frequently known as philosophical uncertainty.

There are also various types of uncertainty in the real-life world. Like weather-related, demand in supply chain, financial uncertainty - return in financial market, market-related uncertainty, noisy, biased, incomplete data in machine learning, model uncertainty, state uncertainty in sequential decision making, competition, technology-related, acts of God, etc. Numerous academic fields and aspects of life are fundamentally impacted by uncertainty, which affects everything from risk

assessment and planning to policy and decision-making. In fact, uncertainty appears everywhere, which implies that uncertainty = reality.

### 1.4.2 Optimization

Optimization refers to the process of selecting the greatest option from a range of accessible choices or coming up with the best result. Optimization includes a variety of disciplines, like computer science, engineering, economics, and mathematics, etc. that identify a target or aim to reduce (e.g., cost, time, risk) or maximize (e.g., profit, efficiency, performance).

According to Ahmed and Garcia (2003), the primary objective of optimization is to methodically investigate and assess potential solutions to choose the best one based on predetermined standards. Whether uncertainty or randomness is present in the objective function and constraints then optimization problems are used to characterize. When there are a large number of potential solutions, these strategies aid in the efficient discovery of the best ones. That means optimization is the process of making something as efficient or useful as feasible. It entails coming up with the greatest answer or producing the best result within the limitations at hand. That is optimization is a basic idea in several fields, including operations research, computer science, mathematics, engineering, and economics.

The following are the main steps of optimization:

- Setting an aim or goal to maximize (such as profit, efficiency, or performance) or minimize (such as cost, time, or error) is the first step in the optimization process.
- Identifying the decision factors that can be changed or managed to meet the restrictions and accomplish the goal.
- Taking into account restrictions or limitations (such as resource availability and operational or physical limitations) is constraints that limit the practical options. Throughout the optimization process, some frequent restrictions or limitations must be followed.
- Decision variables are those that can be changed or manipulated to affect the result in the optimization process. The optimization problem's solution is determined by the values of these variables.

The purpose of optimization is to identify the set of values for the decision variables when all constraints are met, either maximizing or minimizing the objective function. It is a fundamental component of decision-making in theoretical and applied contexts, assisting in the simplification of procedures, enhancement of productivity, and formulation of well-informed decisions grounded in quantitative analysis.

#### 1.4.2.1 Deterministic Optimization

In this type of optimization problem, there is no uncertainty, and the objective function and constraints are precisely understood; the results are predictable, and judgments are taken with a high degree of confidence. These situations frequently contain a large number of variables, and the deterministic structure of the problems makes it possible to predict the outcomes with confidence. In deterministic optimization, there is no uncertainty or unpredictability, and the objective function, or function being optimized, and the constraints are precisely known. Inputs of the optimization problems are considered to be fixed, predictable, and free of variables (Bazaraa et al., 2011). Deterministic optimization is widely used in many domains where decision-making depends on accurate models of the system under consideration, such as engineering (design optimization, scheduling), economics (resource allocation), operations research (logistics, supply chain management), and many more.

##### 1.4.2.1.1 Weakness of the Deterministic Model

- This strategy has a flaw in that decision $x$ is the best for the model rather than the circumstances.
- Assumes that all models and input data are 100% accurate, which is frequently unfeasible in practical situations. The type of choice made differs significantly depending on whether there is risk or ambiguity in the circumstance.
- When there is a measurement of unpredictability in the function, it is not appropriate.
- Real decision makers take precautions against a range of potential outcomes.

#### 1.4.2.2 Stochastic Optimization

When it becomes hard to forecast choice outcomes with confidence, stochastic optimization is used. This kind of optimization addresses issues with a restricted number of variables. The word "stochastic" denotes the existence of uncertainty or unpredictability, making it difficult to accurately predict how decisions will turn out. In summary, stochastic optimization works in where uncertainty is inherent and involves a relatively limited collection of variables, whereas deterministic optimization handles cases where outcomes can be safely predicted despite a high number of factors (Shapiro et al. 2021). Every environment has a unique set of difficulties that call for customized methods in the fields of decision analysis and operations research. When uncertainty is present, a method known as stochastic optimization is applied in order to maximize or minimize an objective function. Probability distributions are frequently used to describe the objective function or restrictions when there is uncertainty.

In recent decades, these techniques have grown in significance in a variety of sectors, including science, engineering, business, computer science, and statistics. By adding random factors to the optimization process, stochastic optimization offers a wide variety of implementation possibilities. As a result, several strategies in the field of stochastic optimization have been developed. Because uncertainty and randomness are ubiquitous in our everyday lives, ideas, and experiences, stochastic approaches are applicable in a broad variety of academic and practical contexts.

### 1.4.3 Two Main Types of Produced Goods in the Garment Industry

Weaved and knit goods are the two main kinds of garments in the RMG sector of Bangladesh. Each type has unique properties and production methods.

#### 1.4.3.1 Woven Wear

Threads are woven together in a perpendicular pattern to make woven textiles. This entails using a loom to weave together horizontal weft and vertical warp threads. In comparison to knit textiles, woven wear is usually less elastic and more rigid. They can keep their form better and frequently feel sharper. They are often more resilient to deterioration and wear and tear, which makes them

appropriate for structured clothing and formal attire (Alves, 2023). Examples of some products made using weave are shirts, blouses, pants like jeans and trousers (denim is a popular kind of woven cloth), jackets and blazers, dresses and skirts - fitted dresses and formal skirts etc.

### 1.4.3.2 Knit Wear

Yarn loops are interlocked to make knit textiles, which have an elastic, flexible texture. Both hands and knitting machines are used for this. Generally, more elastic, pliable, and softer than woven materials. They are frequently used for relaxed, informal attire as they can adjust to the motions of the body. Because of its flexibility and comfort, which promote ease of movement, it's frequently chosen for casual wear. Some examples of Knit Products are T-shirts, sweaters like cardigans and pullovers, Active wear like sports shirts, yoga pants, casual dresses and skirts. For comfort, they're frequently constructed of jersey or other knit materials.

Products that are knitted or woven each have benefits specific to the style, comfort level, and intended usage. These textiles are chosen in the RMG industry according to the final garment's required qualities (Murphy, 2010).

The statistics of woven and knit products' export from FY'18 to FY'23 are shown graphically in the Figure 1.3.

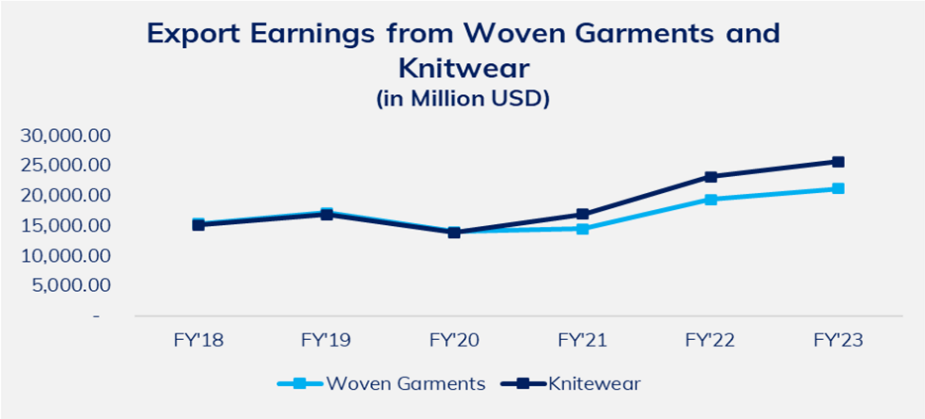


Figure 1.3: Export earnings of Knit and Woven. Source: (Calculated from ITC, Trade map database & Export Promotion Bureau, Compiled by BGMEA, 2024)

## 1.5 Computational Tools used in this Research

AMPL, MATLAB, and LINDO are the three computational programming languages that are used in this study to solve different DLP and SLP models are briefly discussed in this section. Every language's background, evolution, features, and potential uses are covered.

### 1.5.1 A Mathematical Programming Language or AMPL

Around 1985, David Gay, Robert Fourer, and Brian Kernighan developed it at Bell Labs' Computing Sciences Research Center. AMPL is an algebraic modeling language intended for use in mathematical programming. Because AMPL expressions are similar to normal algebraic notation, mathematical programming structures may be represented with ease. Supports common constructions used in mathematical programming, such as network flow restrictions and piecewise linearity. It uses subscripts enclosed in square brackets, just like in C, C++, and Awk. able to manage circumstances that are both linear and nonlinear. An interactive command environment called AMPL is available for configuring and resolving mathematical programming issues. It provides options to improve solver performance and supports a number of solvers. Capabilities include characteristics like presolve stage and computation of reduced costs, and they may represent a wide range of issues. It makes use of sparse data structures, which allows it to handle extremely large problem cases. Modelers receive instant feedback from the output on the best solutions for examination and further research. In this study mathematical programming language AMPL is applied for solving two-stage and multi-stage stochastic models.

### 1.5.2 Excel Solver

Excel Solver is a potent optimization tool in Microsoft Excel. Subject to restrictions on other cells, it assists anyone in determining the optimal solution (maximum, minimum, or particular value) for a formula in one cell, known as the goal cell. The graphical presentation of various comparative data is arranged in this study by Excel.

### 1.5.3 LINDO

A software tool called LINDO (Linear, Interactive, and Discrete Optimizer) is used for global optimization, linear programming, integer programming, nonlinear programming, and stochastic programming. Linus E. Schrage, a well-known expert in operations research and optimization, created the LINDO software suite. He is credited with developing LINDO, a fundamental tool for resolving a wide range of optimization issues, such as stochastic, linear, integer, and nonlinear programming. Linus Schrage was a co-founder of LINDO Systems, Inc., which is in charge of creating and marketing LINDO software, and is intended to address a variety of optimization issues. Applications in the fields of business, industry, research, and government are especially well-suited for it. Since its founding in 1981, LINDO has led the optimization software industry and established the benchmark for mathematical optimization tools. The deterministic model of this study was solved by the LINDO tool.

## 1.6 Research Objectives

The main objective of this research is to offer mathematical models for production planning of garment industries in Bangladesh under different deterministic and stochastic environments.

The specific objectives are

To assist the garment sector in making optimal profit by offering the best production planning under various deterministic and uncertain environments, along with the ideal utilization of available resources.

- To develop a deterministic linear programming model for offering an optimal production planning along with the profit optimization, while the best use of available resources is ensured as well.

- To propose different general stochastic models considering various uncertain atmospheres while raw material costs, labor costs, operating costs, prices, etc., related to every product vary.
- To offer a two-stage stochastic model with fixed recourse while the demand of specific products, raw materials' costs, labor/ operating costs, prices, etc., associated with those specific products fluctuate. AMPL language is used here to solve these SLP models
- To develop two-stage stochastic models considering fluctuating seasonal demand under two different cases while the actual selling amount is considered, and to extend it to multistage stochastic models while also the demand of specific products, raw materials costs, labor/ operating costs, prices, etc. are uncertain. AMPL is used here to solve these SP and MSLP models.
- To analyze EVPI (Expected value of Perfect Information) and VSS (Value of Stochastic Solution) of TSLP.

## 1.7 Outline of the Thesis

This thesis is comprised of seven self-contained chapters which are organized as follows:
Chapter 1 presents the introduction of this study - history and background of the RMG industry of Bangladesh, some basics of LP and SLP models, uncertainties, optimizations, methodologies, objectives, and outline of the thesis. In Chapter 2, the literatures on the works related to the garment industry, both from Bangladesh and global perspectives, are reviewed. Also, there are discussions on the works of various authors regarding the mathematical models - two-stage and multi-stage stochastic models, the literature related to the production planning in various domains by applying stochastic LP - two-stage and multistage models, and then the chapter concludes by finding a research gap. In Chapter 3, the basics of stochastic programming, fundamentals of two-stage and multistage stochastic models, some mathematical preliminaries, terms, and theorems on both the two-stage and multistage stochastic models are analyzed. Chapter 4 presents the development of a deterministic LP model for offering an optimal production planning with profit optimization along

with the best utilization of available resources. Next in Chapter 5, several stochastic models are formulated by considering various uncertain circumstances, such as due to any political or economic unrest, costing-pricing-demand-supply etc., may fluctuate, sometimes all variables' entities may vary, or from time to time $2/3/4$ variables might fluctuate. By considering all the conditions separately and also combining all situations, some general stochastic programming models and a two-stage stochastic model are formulated and analyzed here with numerical, mathematical and graphical presentation through wait-and-see, EEV calculation. EVPI and VSS are also shown to clarify the outcomes. In Chapter 6, two models of two-stage and two models of multistage stochastic programming are presented for profit optimization of the garment industry under two different cases, where the demand uncertainty and actual sold amount are rigorously taken into consideration. Numerical results and tests are then carried out by the AMPL solution with a graphical presentation to evaluate and compare these two models. Chapter 7 summarizes the research presented in this thesis with a conclusion and recommendations. The potential future research based on this thesis is also described in this chapter.

A detailed literature review relevant to this research will be presented in the next chapter.

# Chapter 02
# Literature Review

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# Literature Review

## 2.1 Introduction

A literature review is an overview and evaluation of previous studies on a certain subject. It illustrates existing knowledge, points out gaps, and sets the stage for future studies. A competent literature review organizes and assesses sources rather than merely listing them to provide a solid basis for one's work and also includes an outline of the subject, what is being researched, and why it is important. Trends and themes of literature review are how research has changed over time, gaps and debates, what's missing or still being argued. The goal of it is to comprehend the actions taken, to support the own study or endeavor to guide one's strategy or tactics. In this section, some works will be analyzed those are already been done on the production planning and profit optimization of the worldwide's (RMG) sector, and then from Bangladesh perspective. Then various studies are discussed on the application of Linear Programming (LP), Stochastic Linear Programming (SLP), Two-stage stochastic (TSLP) and multistage stochastic models (MSLP). Assessments of the prior writings of several domains are tried to be focused in this part, along with the concern area of this research, 'RMG industry'. The structure of various relevant writings is shaped in this section, inquiries about gaps are perceived and approval is given. The investigation thoughts are set up then. At last, this part completes with an outline of the whole discussion.

As the concerned area of this study is the RMG industry of Bangladesh but some works has already been done on the global textile or clothing sector. Some of them are listed below. The following parts of the chapter show a review of the literature on several aspects related to this study.

## 2.2 Discussion on the Global Garment Sector

Many countries' manufacturing and production bases are heavily reliant on the textile sector (Jatuphatwarodom et al., 2018). For example, there are several subprocesses involved in the manufacture of textiles and garments, including knitting, embroidery, and fabric making.

network participants coordinate with one another. According to Felfel et al. (2018), textile and clothing firms are under intense pressure to manage their supply chains (SC) due to the intense competition in today's industries. Making thorough supply chain planning (SCP) choices about everything from the procurement of raw materials to the delivery of finished goods to the client is essential to achieving this aim. A SC production planning challenge in textile and clothing was examined by Toni and Meneghetti (2000). A simulation model was used as a genuine example from the Italian textile and garment industry. The lack of manufacturing inputs in the garment industry is a persistent problem that lowers capacity utilization and ultimately production. The growth of the industry puts pressure on management to figure out how best to organize, direct, and regulate output at the various levels of the economy (Ghuge et al., 2025). A quantitative method of modeling and evaluating decision-making difficulties has been created in response to these obstacles. The most well-known scientific discovery to date is linear programming, which has applications in the textile sector as well (Delia, 2007).

Based on the case study carried out by Harianto (2018) in a textile industry named PT. Argo Pantes of indonesia. The Simplex method is applied for solving LP model in various papers like Komputer et al. (2023), Maringan et al. (2022), and Linier et al. (2023). Woubante (2017) took into consideration the data gathered from an Ethiopian clothing manufacturing unit. Using LINGO 16.0 software, he solved the model and demonstrated that satisfying client requests could increase the company's profit by 59.84%. Using a linear programming approach, Tesfaye et al. (2016), examined data from the Ethiopian clothing sector and found that the company's resource utilization could be raised by 46.41% above its current level. They also demonstrated how applying their strategy might raise the company's earnings by 145.5%. Goal programming was used by Leung et al. (2003) to solve a multi-objective, multi-site medium-term aggregate production planning issue. The objective functions that are taken into consideration include the maximizing profit, using import quotas, and minimizing hiring and firing expenses. In the textile and clothing business, Safra (2013) created an integrated strategy for production and distribution planning at the tactical and operational levels.

Numerous attempts to analyze and improve planning challenges to smooth production planning in SCM (supply chain management) challenge in clothing have been documented in the literature of

(Felfel et al., 2016a,b; Leung et al., 2003; Mok et al., 2013; Ren et al., 2017; Safra, 2013; Toni and Meneghetti, 2000) by applying SP (stochastic programming). Hutauruk et al. (2023) used this method for optimizing profit in different organizations, including Najwa Sewing House, Textile Business in Salatiga. Felfel et al. (2015) offered a two-stage stochastic model to maximize the predicted profit on apparel sector. Unpredictable client demand is the stochastic parameter, was the subject of several publications in the textile and apparel industries. In their studies Ait-alla et al. 2014; Felfel et al. 2016b; Karabuk 2008; Kong 2008, they developed a stochastic programming model that explicitly includes uncertainty in the form of discrete demand scenarios. Unpredictable client demand was the main subject of their research as ours. Many other studies (Leung et al., 2005, 2007) considered multiple random parameters, including labor costs, shortage costs, unit production and inventory costs, and consumer demand. As a viewpoint on their study, Leung et al. (2005) pointed out that taking selling prices into account can provide an opportunity to make production planning a more advantageous foundation for decision-making. Shafiee et al. (2024) applied a two-stage stochastic for designing a multi-objective closed-loop supply chain to the garment industry in Montreal, Canada. Integrated fabric procurement and multi-site apparel production planning with cross-docking was done by Darvishi et al. (2020) with a hybrid fuzzy-robust stochastic programming approach. The goal of Ghosh et al. (2020)'s study is to minimize production costs or maximize profit by developing a linear programming model that will simplify the scheduling challenge of the Composite Textile Industry. The model is solved using an Excel solver and is created with lead time in mind.

A seamless supply chain management strategy was suggested by Alam et al. (2016) for a fruitful RMG industry. Lead time and random demand were taken into consideration. Economic Order Quantity, the first inventory model created by Harris (1915), was expanded upon by Wilson (1934), who provided a formula for achieving economic order quantity following the development of calculus. The textile and garment industries have effectively used the stochastic programming technique (Karabuk 2008; Kong 2008; Leung et al. 2005). In an uncertain environment, Leung et al. (2005) created a two-stage stochastic programming approach to optimize the overall cost of a multi-site aggregate medium-term production planning issue. Ghare and Schrader (1963) created model for an inventory of apparel sector that reduces exponentially in situations where

consumption is high. When shortages are prohibited, Dave and Patel (1981) were the first to examine a declining inventory with a linearly growing demand. To enhance the business's operations, an LPP model has been developed by Ghuge et al. (2025) and the solver add-in in Microsoft Excel 2019 is being used to determine the best outcomes. According to an analysis of the data gathered, Sunshel Textiles Industry, India could increase their profits by 5.5% by using the optimal model while making managerial choices. Next, the work on BD clothing sector is displayed, which will help to find the specific research area by analyzing the research gap.

## 2.3 Study on Textile Sector: Bangladesh Perspective

So many works are done on the textile or clothing sector from a Bangladeshi perspective also. Some of them are analyzed here. Alam et. al. (2016); Hasan et al. (2016); Hossian et al. (2019); Hussain et al. (2020); Jadhav et al. (2017); Jia et al. (2020); Kashem et al. (2021); Yunus & Yamagata (2012) did qualitative researches on Garment sector of Bangladesh. These studies have been employed to investigate the tactics utilized by Bangladeshi RMG manufacturers to enhance their logistics operations, the general scenarios of the entire RMG sector. All these works are to manage different challenges, managing inventory and lead time, prospects, and the future of the BD garment industry etc. According to Rahman et al. (2018), future projections will be made in order to identify the actual issues and obstacles and provide viable solutions for them. Chowdhury et al. (2014) showed that the primary goal of supply chain management is to develop the best possible layout and design to produce high-quality clothing products while minimizing the waste of time, money, and human labor. This allows for efficient production and maximum profit with appropriate quality at a reasonable cost in the BD apparel sector. The management of lead time analysis covered in these studies, Arnob et al. (2020) and Samad et al. (2023) to ensure the smooth operation of the clothing industry's production process. They talked about the application of techniques and plans to reduce lead times and management procedures of the supply chain of a productive and sustainable clothing industry of BD. Many works are also conducted on women's empowerment in the BD garment sector. Work by Mamun & Hoque (2022) is such a work in which it is shown that this sector helps to increase employment opportunities, including for women. Women are now being independent by this sector directly or indirectly. Now, some other

writings are to be analyzed regarding profit and cost optimization, and various mathematical languages are applied there to solve these quantitative cases. Khan et al. (2014) applied the LP model to reduce bottleneck operation, which is an approach to minimize labor cost of the Ready-Made Garments Sector in Bangladesh. In their work, Shakirullah et al. (2020) show that for lead time management, apparel processes, including cutting, sewing, washing, dyeing, trimming, and finishing, must be optimized. A linear programming approach was developed in their study to optimize the garment industry's profit and reduce expenses. Additionally, the model maximizes resource use. According to this study, when there is enough demand, the case company's profit can rise by 22%, and when customers' wishes are fulfilled, it can rise by 12.33%. On the other hand, the model resulted in a 37% reduction in cost. Suraiya & Hasan (2023) worked on production planning and profit maximization of the textile industry, Bangladesh by applying the DLP model. They showed if the production could be rescheduled by excluding some products, then the profit would be optimized. Imran et al. (2023) optimized the total cost of the RMG industry of Bangladesh for a mixed make-to-order (MTO) and make-to-stock (MTS) production system. Reza et al. (2020) worked on combining forecasting methods and linear programming models for managing the inventory of a business organization in Bangladesh. Habib (2017) worked on inventory optimization for stochastic and stock-dependent demand combinations. Considering lead time and lead time demand uncertainty, this work develops an optimal inventory policy for stock-dependent demand items. Using the Cobb-Douglas production function to compare Khatun & Afroze (2016) have attempted to investigate the relationship between real GDP, labor, and capital in the case of some Asian countries, such as Bangladesh, India, China, Malaysia, and Thailand. The Cobb-Douglas production function is used in Mohajan's (2021) study to forecast the cost-minimization strategies of a Bangladeshi garment industry that is currently in operation. There are also several works after the COVID-19 pandemic. The main theme of these studies is "How has the COVID-19 pandemic drastically affected Bangladesh's ready-made clothing industry?" Some writings on the clothing sector of Bangladesh after the COVID-19 pandemic are (Aktar et al. 2021; Gazi et al. 2024; Hossain 2021; Islam & Halim 2022; Kabir et al. 2021).

So many works have ended on the profit optimization of the BD garment industry. Most of them are qualitative research on challenge meetups, managing supply chain, etc. Some works are done

with the optimization of profit and loss by applying a deterministic LP model. But no such works have been found yet so far where the stochastic model is applied by considering uncertain variable/s. This is the motivation of this study. In this study various uncertain situations are considered and several stochastic models for both two-stage and multistage are formulated, considering different cases.

## 2.4 Applications of the Linear Programming Model

Linear optimization is a widely used technique in many domains, such as operations research, economics, engineering, and finance, to name a few. It can be applied to a variety of problems, such as resource allocation, production planning, transportation, and network optimization (Aliu et al. 2023; Shamsuzzaman et al. 2023; Syifa et al. 2023). One such quantitative strategy is the application of LPP models, which determine the best course of action by mathematical methods. In a sentence, the mathematical model used to tackle an optimization problem is a linear program, which aims to maximize or minimize the function of objectives that depend on several choice factors. According to Eiselt & Sandblom (2007), all variables must be nonnegative to optimize a linear objective function while adhering to linear equality and linear inequality restrictions in LP. Expressing the problem as a system of linear equations and inequalities, then figuring out the best values for the decision variables that simultaneously satisfy all imposed constraints, Dantzig (1940) first proposed the basic method for solving linear programming problems, and Stigler (1944) later used this technique to solve problems about food as well as Lin et al. (2020) and Render et al. (2023) also applied LP. (Mallick et al. 2020).

Numerous studies have been conducted using the Deterministic LP model, dating back many years. Jain et al. (2019) used a linear programming model to maximize the net return for investors of the Central Bank of India, as well as the net return of interest on loans such as personal, car, home, agricultural, commercial, and education loans. Scott (1957) demonstrated the practicality of the management tool by using a linear programming model to find that the feed company profit that would be maximized. In their study, Yahya et al. (2012) showed how to apply a linear programming model in a soap manufacturing firm and discovered that focusing on unit output will optimize monthly profit and they utilized the "lpSolve" library with the R statistical package. Ben-

Horim and Sibler (1977) used the linear programming paradigm at National City Bank, a financial institution. Nasir et al. (2014) discussed how CSR (corporate social responsibility) initiatives might maximize a company's profits while also enhancing employees' happiness. Given that workers constitute a company's primary customer base, a relationship between CSR and profitability was demonstrated. Woubante (2017) suggested using a linear programming model to produce a certain pair of items in order to optimize the profit of the Golden Plastic Industry in Nigeria. To do this, he first projected future demand using forecasting techniques like weighted average and exponential smoothing. Jadhav et al. (2017) included a temporal analysis of the manufacturing process in their description. They discovered that a timely delivery of parts and order sheets may significantly increase productivity and provided details on how to shorten lead times and boost output. Suraiya & Hasan (2023) proposed a deterministic LP model for the production planning of the RMG sector of Bangladesh.

The advantages and significance of a supply chain with a coordination mechanism were covered by Uddin and Sano (2012). They proved that even if the demand function is non-linear, a coordinating mechanism might raise both the individual and joint profits. A Mixed Integer Programming (MIP) problem-based vendor-buyer multiple products-consumers facility selection problem with a price-sensitive linear demand function was created by Uddin and Sano (2011). They believed that the best solution and warehouse site might be found through a coordinated method among supply chain participants. The linear programming approach with the simplex model was used to improve woven fabric production to optimize company profit. A Mixed Integer Program for Bangladeshi poultry firms' manufacturer and retailer system was developed by Islam et al. (2014). They demonstrated that while there is no discernible link between fixed costs and profit selling price, there is a very good association between production cost and raw material cost. More precisely, Ezema and Amakom (2012) suggested that to improve an organization's development and competitiveness, one of the two strategies is required: either cost reduction or output maximization. Linear Fractional Programs and Mixed Integer models were also created by Islam et al. (2015). This model introduces a price-sensitive non-linear demand function in place of a price-sensitive linear or deterministic demand function. Momaya (2008) talked on the effects of globalization, technical advancement, and the fiercely competitive global corporate landscape.

Kumar (2010) talked on how a manufacturing company's capacity to provide the best products at the lowest feasible cost is a key factor in its ability to survive in an increasingly competitive market. According to Yalçinso et al.'s (2014) analysis, every manufacturing business needs to make the best use of its resources to boost profits. An efficient technique for allocating resources is called linear programming. It is one of the most used methods in operations research for figuring out how best to use resources. A linear programming methodology called LIPROTEX was suggested by Campo et al. (2018) to minimize a textile company's overall labor and inventory expenditures.

It is a productive technique for selecting, among many options, the best option to satisfy a given objective function while taking into account a variety of limitations and constraints. In this research, a Bangladeshi garment manufacturing facility located in the Gazipur district has been examined. Information was gathered from the case industry's monthly used resources, product volumes, raw materials' costs, operating, labor and inventory costs and profit per unit for several products. The case industry manufactures a variety of knit and woven clothing designs and sorts. The parameters of the suggested linear programming model (which is shown in Chapter 4) were derived from the gathered data. And using this study a deterministic LP model is offered for optimal production planning with optimal profit, while Tesfaye et al. (2016) and Woubante (2017) both used data gathered from the industries. They apply LPP and the mathematical language LINGO 16.0. They determine the best, or most efficient, way to use the available resources to meet the situation's goal by applying a linear programming technique. Here, two scenarios have been examined. Initially, they looked at the scenario in which there is a large enough market demand for all eight products produced by the case company on a global scale.

To model situations requiring decision-making in the face of uncertainty, stochastic programming considers the uncertainty of one or more parameters in terms of probability distribution functions (Louveaux and Birge, 1997) as uncertainty is everywhere. Without considering it, the target won't be accomplished. One may consider a stochastic program (SP) to be the ultimate operations research model in many aspects. A stochastic linear program (SLP) is formed when a portion of the data components in a linear program (LP) are best characterized by random variables. In this way, stochastic programming is a clever fusion of stochastic models and conventional

(deterministic) mathematical algorithms. Stochastic programming, to put it briefly, makes use of a broad range of conventional operations research techniques (Higle 2005).

## 2.5 Applications of Stochastic Programming

In real-world problem solving, optimization under uncertainty is a significant challenge. Any system's operation or design will inevitably lead to some degree of uncertainty. Many authors worked with different models to minimize this indistinctness. Plenty of literature is available on the topic of optimization under uncertain conditions, with various applications across different fields. Optimization under uncertainty is applied in the context of production planning, as seen in the works of Clay and Grossmann (1997) and Cheng et al. (2003). In production planning, it's essential to make decisions about production quantities, scheduling, and resource allocation while considering uncertain factors. Optimization under uncertainty is also utilized in scheduling problems. Balasubramanian and Grossmann (2002) and Birge and Dempster (1996) are cited as examples of optimization under uncertain environment.

Dantzig (1955) introduced the stochastic optimization model. Since then, several authors have developed optimization models that take uncertainty into account, including Beraldi et al. (2000); Birge & Louveaux (1997); Falasca & Zobel (2011); Haneveld & Vlerk (1999); Hoffman & Schniederjans (1994); Jayaraman (1998); Oliveira & Hamacher (2012); Riis & Andersen (2005); and Zhang et al. (2012). The recourse model and probabilistically restricted models are initially presented by Dantzig (1955) and Charnes and Cooper (1959), respectively, and these works provide the foundation for stochastic programming. To improve the biodiesel supply chains, Zhang et al. (2003) worked next to Lee (2014) and Xie et al. (2014) introduced two and multistage Mixed-Integer Linear Program (MILP) formulations.

In Stochastic Programming problems, Jonsbraten (1998) divided uncertainty into two main categories, endogenous uncertainty (like yields), where decisions have an impact on stochastic processes, and exogenous uncertainty (like demands), where stochastic processes are independent of decisions. Clay and Grossmann (1997), Ierapetritou and Pistikopoulos (1994) and Iyer and Grossmann (1998) solved a variety of production planning issues in the field of process systems

by taking exogenous uncertainty into account and formulating the solutions as two-stage stochastic programming. Furthermore, Schultz & Mankin (2019) and Sahinidis (2004) provide thorough assessments of earlier research on exogenous uncertainty concerns. Considering uncertainty, Li & Grossman (2021) reviewed the stochastic programming method for optimizing process systems. The cost of the decisions and the expected cost of the remedial actions are optimized in the two- and multi-stage cases (Gupta & Grossmann, 2011). According to Felfel et al. (2018), Stochastic programming (SP) is an optimization modeling scheme, which has been applied to numerous domains such as health care by Denton et al. (2010) and Guo et al. (2021), in finance by Bertocchi et al. (2006), energy by Zheng (2010), disaster management by Grass and Fischer (2016) and production planning (Hu and Hu, 2018) yielding significant benefits (Wang, 2021). In the garment business, Kong (2008) tackled a multi-period aggregate production planning issue under seasonal demand uncertainty. Karabuk (2008) suggested a stochastic programming approach for a multi-period yarn production planning issue under demand uncertainty in the textile sector.

According to Birge (1997), stochastic programming review, it is feasible to produce adaptable and reliable "near optimal" resource allocation judgments while dealing with unknown futures by applying stochastic programming models. Ariyawansa & Felt (2004) discussed a number of new SLP test questions for eleven distinct context groups in her work. Sodhi & Tang (2009) provide another recent analysis on the application of stochastic programming in supply chains to simulate demand uncertainty. In this research uncertain situation is considered in various cases. Clients' demand, costing-pricing, economic condition, weather condition, seasonal demand etc. are considered and some general stochastic models are developed for managing uncertainties in RMG sector of Bangladesh in different cases

## 2.6 Study on the Two-Stage Stochastic Model

Many works are done by formulating a two-stage stochastic model in different fields. As, Gupta et al. (2014) and Summerfield & Dror (2013)'s application shows, a generic, two-stage, stochastic programming-based Decision Support System (DSS) in a North American integrated steel company. The stochastic programming approach, as proposed by Birge and Louveaux (1997) and Dantzig (1955), is a widely used technique in supply chain planning literature for problems

involving uncertainty. The optimization problem's choice variables are split into two sets in their method. Before the awareness of uncertainty, the "here and now" decision variables of the first stage must be determined. In order to address the infeasibilities brought on by the realization of uncertainty, the second-stage decision variables are selected after the existence of uncertain parameters (also known as "wait and see" decisions). First-stage choices include how much manufacturing will be done on a regular and overtime basis, how many goods will be subcontracted, and how many employees will be employed, laid off, and how many workers will be needed. The number of inventories and the quantity of items that are under-filled are the topics of the second round of decisions. A number of scholarly papers have explored the use of stochastic programming models for supply chain planning issues. A two-stage stochastic programming approach was used by Gupta and Maranas (2000) to suggest a multi-site midterm supply-chain planning problem under demand uncertainty. They show that planning in a probabilistic demand scenario may be accomplished with a general, user-friendly DSS based on stochastic optimization. The algebraic formulation of a single-period deterministic model for process industry planning is described by Fourer (1983). Awudu and Zhang (2013) created a two-stage stochastic programming model in which the quantity of goods to be manufactured and the quantity of raw materials to be bought and used are regarded as the initial decisions. Second-stage decisions include backlog, missed sales, and the number of sold products. A stochastic programming model was presented by Lin et al. (2011) for the thin-film transistor-liquid crystal display industry's strategic capacity planning. To maximize the predicted profit, a two-stage stochastic linear programming approach is employed in Felfel et al. (2015)’s work. Abundant research is carried out using the two-stage and multi-stage stochastic programming models in various contexts. The objective of the study Shekarabi et al. (2024) is to provide a mathematical programming model for building a robust food supply chain, Ariningsih et al. (2024) examined a two-stage stochastic programming for a multi-period perishable vaccination network design issue. In their research, Guericke et al. (2012) provide a two-stage stochastic mixed integer linear programming model that enables decision making for particular postponement implementations in uncertain contexts in the clothing sector. Two stochastic mixed integer linear programming (SMILP) models are developed in their study Hamdi & Masmoudi (2022) to pick suppliers under disruption risk, taking into account variables such as capacity, failure probability, uncertain demand, and quantity discounts. The goal

is to minimize the total cost, which comprises both the costs associated with the initial decisions (1st stage) and the expected costs in the second stage, factoring in the expected value of these costs, while considering the uncertainty in the parameters. A deterministic equivalent big linear programming model is used by Chung & Yang (2009) to solve a two-stage stochastic programming model that is constructed based on scenarios. The goal of minimizing the overall cost of C/D wafers is expressly taken into account by their model.

After analyzing all these works a two-stage stochastic model is formulated in this study after considering three different scenarios on the basis of fluctuating demand under uncertain atmosphere.

## 2.7 Applications of Multi-Stage (Three) Stochastic Model

The three-stage remanufacturing system scheduling issue (3T-RSSP), which reduces the maximum completion time or makes span, is the subject of Wang et al. (2024)'s study. These studies Gómez-Rocha & Hernandez (2022); Mirzaee et al. (2024) and Zhong et al. (2024) present a comprehensive multi-stage, multi-period green supply chain design model that includes six resilience techniques and develops a mixed integer, linear, multi-stage, stochastic programming model for aggregate production planning (APP) and Darvishi et al. (2020), examined multi-site aggregate production planning (APP) as well as in their research while a two-stage stochastic programming model was created by Leung et al. (2006) to optimize a multi-site aggregate medium-term production planning problem in an unpredictable setting.

A monolithic model of a multi-stage, multi-site, multi-item production planning problem was created by Lin and Chen (2007). Leung et al. (2003) investigated a multi-site aggregate production planning problem of a multinational lingerie manufacturer based in Hong Kong. A multi-objective, multi-stage, multi-product, multi-period model for production and transportation planning in a multi-site manufacturing network was presented by Felfel et al. (2014).

Multistage stochastic programming-robust model for maximizing supply chain in a biomass power plant was developed by Shabani & Sowlati (2016), while Sen & Zhou (2014) showed a bridge

between stochastic programming and approximate dynamic programming. Spinelli et al. (2024) applied it to the waste collection problem. Multistage stochastic programming also applied for asset liability management in civil servants' pension funds (Khanlou et al., 2023). Dutta & Fourer (2004 and 2008) offer a multi-period deterministic model as an additional extension. Using a set of real data for a single period, Dutta & Fourer (2004) also assessed the optimization-based DSS and the multi-period model. Using multi-period data, Dutta & Fourer (2008) and Dutta et al. (2011) showed a substantial impact when applying a multi-period planning model in an aluminum and pharmaceutical firm, respectively. In order to handle multi-stage stochastic linear optimization problems with uncertain distributions, Bertsimas et al. (2023) provide a novel data-driven method. Nouri et al. (2023) and Nazare & Street (2023) recommended using a novel multi-stage stochastic portfolio optimization model and scenario-based stochastic linear program, respectively, in their investigation.

This research develops another multistage stochastic model for managing unstable demand in two different cases: firstly, it is considered that the sold amount or demand would be from the produced amount demand would be less than the production, and no extra demand more than the production is considered in this case. Secondly, the client's demand should be met. If the production is less than the demand, then products should be collected from external sources to meet the extra demand.

## 2.8 Applications of Stochastic Models in Various Other Domains

Numerous domains have seen the development of stochastic models, such as reservoir water management, electrical power generation, telecommunication network planning, financial planning, and air fleet management (Ferguson & Dantzig 1955; Sen et al. 1994; Sherali et al. 1984) etc. Everyone can check a wealth of literature on stochastic programming and probabilistic modeling from various studies such as Birge (1997), Frauendorfer (1992), Marti & Kall (1997), and Raghunathan (1992).

The use of two-stage stochastic programming in an integrated steel plant is the main topic of the research covered in their article, Summerfield & Dror (2013). A linear multi-stage stochastic

program has a linear objective function and constraints that make it a stochastic sequential optimization (Casey and Sen, 2005), and authors have applied the multistage stochastic program technique in a variety of ways. For instance, Nagar & Jain (2008) claimed that applying a multistage method can result in significant savings in problems like supply chain planning, which they approached using a scenario-based approach. According to Sahinidis (2004), even though the stochastic two-stage model solution has made significant progress, multistage still presents a significant computational challenge when compared to deterministic models; as a result, a deeper comprehension of the issue is required for the effective application of solution algorithms.

The topic of gas field development under uncertainty on the quantity and quality of reserves was addressed by Goel & Grossmann (2004) and El-Bakry & Mulkay (2006). Decisions regarding the timing of field drilling provide an instantaneous resolution of the uncertainty. Later, Tarhan et al. (2009) included nonlinearities to the reservoir model and expanded their method for gradually resolving endogenous uncertainty. Tarhan and Grossmann (2008) employed incremental uncertainty resolution in the model and implemented endogenous uncertainty in the synthesis of process networks problem with uncertain yields. Colvin and Maravelias (2008, 2010) developed a branch and cut framework to solve these MSSP problems with endogenous uncertainty under the supposition that only a small number of non-anticipativity constraints be active at the optimal solution. Specifically, Solak et al. (2007) presented several theoretical properties for the problem of scheduling clinical trials with uncertain outcomes in the pharmaceutical R&D pipeline by applying portfolio optimization while taking into consideration decision-dependent uncertainty that gradually resolves.

Deterministic methods are the main emphasis of the majority of publications on multi-site production planning problems. A monolithic model of a multi-stage, multi-site, multi-item production planning problem was created by Lin and Chen (2007). The suggested approach integrated the monthly and daily time buckets—two distinct time scales—at the same time. Using a goal programming approach, Leung et al. (2003) investigated a multi-site aggregate production planning problem of a multinational lingerie manufacturer based in Hong Kong. Three primary goal functions were taken into account in their research: minimizing the cost of hiring and firing

employees, maximizing profit, and minimizing the overuse or underuse of import limits for various goods. A multi-objective, multi-stage, multi-product, multi-period model for production and transportation planning in a multi-site manufacturing network was presented by Felfel et al. (2014). The created approach seeks to maximize the quality level of the products while simultaneously minimizing the total cost. Numerous strategies for dealing with uncertainty have been put out in the literature. According to Sahinidis (2004), these approaches fall into four main groups: stochastic dynamic programming approach, fuzzy programming approach, resilient optimization approach, and stochastic programming approach. A two-stage stochastic programming model was created by Leung et al. (2006) to optimize a multi-site aggregate medium-term production planning problem in an unpredictable setting, and a real-world case study from a global lingerie company based in Hong Kong demonstrated the efficacy of the suggested model. The rover configuration, frame configuration, and production quantity are the first-stage decisions in the author's stochastic programming approach.

In their research, Darvishi et al. (2020) examined multi-site aggregate production planning (APP) and integrated inbound logistics decisions within the textile sector throughout the tactical planning horizon. A mixed-integer nonlinear mathematical programming model is developed in a hybrid fuzzy-stochastic environment to make decisions about production and purchase at the same time, while Ko (2021) presents a two-stage methodology that maximizes the profit on each particular product by combining simulation and optimization. They use the notion of consumer margin, or benefits less price, to help customers choose which product (if any) to buy from a competitive set. Wang & Gao (2023) provide a robust technique for identifying the worst probabilistic connection of information update within the two-stage SP. Nazare & Street (2023) recommended using a novel multi-stage scenario-based stochastic linear program in this investigation, together with a recourse model that takes the meteorological weather into account forecast data for coordinated. Weskamp et al. (2019) provide a two-stage stochastic mixed-integer linear programming model that takes postponement principles into account and incorporates an integrated production and distribution planning method.

This application (scenario-based stochastic model) involves designing and optimizing chemical processes, considering various uncertainties. Researchers like Acevedo and Pistikopoulos (1998), Liu and Sahinidis (1996), and Rooney and Biegler (2003) have contributed to this field. Electricity generation and distribution involve complex systems with uncertain factors. Researchers, including Takriti et al. (1996) and Nowak et al. (2005), have explored optimization under uncertainty in the context of electricity production. Researchers in these fields aim to develop methods and models that can help make decisions and plans more robust and adaptable in the face of uncertain conditions and variables. In their paper, Bounitsis et al. (2022) proposed a new data-driven model for tackling stochastic programming problems. a mixed-integer stochastic programming model is proposed in this paper, Nayeem & Alam (2022), to minimize unmet demand at relief collection centers, considering uncertainty in demand and capacity. Kang & Bansal (2024) applied multistage stochastic integer programs and two-player interdiction games with (without) decision-dependent uncertainty that are robust and distributional risk-responsive.
A combined approach for the best possible planning of municipal plug-in electric vehicle (PEV) charging stations in metropolitan environments is presented in Kim et al.'s (2022) study. The two primary parts of the framework are as follows: They used a two-stage stochastic mixed-integer programming (TSMIP) model that gauge's insecurity using a limited number of situations. This work, Chung (2009), examines a practical planning environment for wafer manufacture in the context of the control or dummy (C/D) wafers dilemma, which involves uncertain demand.

This study, Bernardini & Bemporad (2009) offers a formulation of stochastic model predictive control (MPC) for linear systems impacted by discrete multiplicative disturbances, based on scenario generation. The problem of solving two-stage stochastic programming's deterministic equivalents, multi-scenario optimisation models is used by Karuppiah et al. (2010). By reducing the number of scenarios, they may approximate the original multi-scenario optimization problem using the heuristic approximation technique that they provide. This work, García-Bertrand & Minguez (2014), presents an iterative technique for solving stochastic issues that uses a set of scenarios and/or finite sampling to depict stochastic processes. Modeling risk is done with conditional value-at-risk.

A mid-term multi-product, multi-period, multi-site production-distribution planning problem under cost and demand uncertainty was put out by Al-e-hashem et al. (2011a). Using the uncertain parameters, a two-stage stochastic programming model was created. An aggregate production planning problem including many sites, multiple products, and several periods was created by Al-e-Hashem et al. (2011b). A method to measure scheduling robustness in the presence of uncertainty was devised by Vin and Ierapetritou (2001). Several robustness criteria, including the level of violation, the deterministic standard deviation, and the corrected standard deviation, were employed to achieve this. A stochastic programming model was presented by Lin et al. (2011) for the thin-film transistor-liquid crystal display industry's strategic capacity.

Thus, it is shown that various research is ended by applying the Stochastic Model (both two-stage and multi-stage). In this research, it is made to apply both two-stage and multi-stage stochastic programming models to choose the optimal production planning along with profit maximization for managing various uncertain circumstance of the garment industry in Bangladesh.

## 2.9 Research Gap

The literatures which have already been addressed in previous sections focuses mostly on increasing efficiency, reducing costs, and optimizing profit in industries such as pharmaceuticals, poultry farming, closed-loop supply chains, integrated electricity and heat systems, integrated steel companies, biodiesel, complex asset-liability management, biofuel supply chains, and timber production etc. The textile industry was also in focus like several other industries. There were various studies on many areas, fields applied by the models: multi-stage stochastic programming, stochastic integer optimization, two-stage stochastic framework, and a multi-objective, multi-stage, multi-product, multi-period model for production and transportation planning in a multi-site manufacturing network etc. The profit-cost optimization for the garment sector is another much research theme. LPP, SLP (stochastic linear programming) models have been developed globally in the industry like RMG industry (Felfel et al., 2015; Felfel et al., 2018). But from Bangladesh's perspective, the apparel sector has not yet completely embraced the potential advantages of SLP, which entails taking uncertainty into account while making decisions. According to review of the literatures, SLP models have not yet been used by the BD

apparel sector to determine profit margins. The literature also shows that the sector, where competitive pricing strategies and market research are prevalent, relies heavily on traditional decision-making techniques. Lots of deterministic LP models are used for cost/profit optimization in Bangladesh's textile industry such as Suraiya & Hasan (2023), Shakirullah et al. (2020), etc. Since uncertainty permeates every aspect of our everyday lives, the industry is unable to fully realize the spectrum due to the lack of mathematical tools, especially stochastic programming models, in decision-making processes. To provide the optimum production planning, profit maximization, and resource usage under various uncertain conditions, this study is working on general stochastic models, two - and multi-stage stochastic approaches.

## 2.10 Conclusion

In this chapter, several literatures are discussed on RMG industry for both global and Bangladesh perspective, and on various works related to LP, SLP, TSLP and MSLP. Connecting all of these works a research gap is found to improve and enhance this work. The next chapter will cover the fundamentals and preliminaries of stochastic programming-Two-stage and Multistage programming, mathematical forms, key concepts and solution methods of all these models with applications. The basic theorems and propositions will also be studied there.

# Chapter 03
# Basics of Stochastic Programming

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## 3.1 Introduction

The term "Stochasticity" describes how systems or processes are inherently unpredictable or random. It characterizes circumstances in which probabilistic elements impact outcomes, rendering them variable even in the presence of comparable initial conditions. A key idea in many fields, such as physics, mathematics and economics, which contrasts with deterministic systems (in which the results are exactly dictated by the original conditions) is stochasticity. Dantzig (1940) first gave the idea of uncertainty. Later, Charnes and Cooper (1959) worked on it. Probability distributions or scenarios can be used to represent uncertainty. The creation of a stochastic model entails abstracting a decision scenario from the real world, a process that is frequently regarded as more of an artistic endeavor rather than a scientific one. Stochastic programming allows decision-makers to successfully manage risk by optimizing decisions in situations where there is hesitation. Uncertainty is a common feature in several decision-making situations, where data can be unpredictable due to several factors like fluctuating demand, varying prices, or disruptions in the supply chain.

In this chapter the basics and preliminaries of stochastic programming - TSLP and MSLP, mathematical form of both of the models. key concepts and solution methods of all these models with applications are discussed. The fundamental theorems and propositions are also studied here.

## 3.2 Stochastic Programming (SP)

An approach for expressing and resolving uncertain optimization issues is called stochastic programming. Stochastic programming addresses decision-making under uncertainty by including probabilistic information about unknown parameters, in distinction to deterministic programming, which assumes that all parameters are known with confidence (Shapiro et al., 2021).
The main thoughts and elements of stochastic programming are as follows:

- **Uncertainty:** To deal with uncertainty in decision-making, stochastic programming explicitly represents the uncertain factors that influence decision outcomes. These uncertainties may result from variables like changes in the market, variations in demand, the state of the weather, etc.
- **Decision variables:** Factors, such as production levels, resource allocations, investment decisions, etc., are the decision variables that the decision-maker may select or control.
- **Constraints:** In stochastic programming problems, uncertainty must be met by constraints. Usually, linkages between unknown parameters and decision variables are involved in these limitations.
- **Stochastic Processes:** Stochastic programming frequently uses probability distributions or stochastic processes to characterize the behavior of unknown parameters to model uncertainty. Poisson, uniform, and normal distributions are a few examples of frequently used distributions.
- **Solution Techniques:** Since uncertainty adds complexity, solving stochastic programming issues is a bit difficult. Scenario-based approaches are a smart method to solve it.
- **Scenario-Based Approaches:** Defining the issue in terms of a limited number of possible outcomes or realizations of uncertain parameters and then doing optimization across these possibilities is the way how this approach works.

### 3.2.1 Mathematical form of Stochastic Program (SP)

Mathematical programs that contain some unknown data in the objective or in the restrictions are known as stochastic programming SP which can be linear, integer, or nonlinear in type.
The results are expressed in terms of items $(x)$ within a set $(X)$. Where $X$ represents the range of potential monthly requests.

A general form of the Deterministic LP Model is

$$z = c^T x$$
$$subject\ to\ Ax = b,$$
$$x \geq 0$$

If the uncertainty occurs in any of the following three cases:

(i) Uncertainty about the demand or available resources $b$

(ii) Uncertainty about the input prices $c$, and

(iii) Uncertainty about the technical coefficient matrix $A$ can be treated in SP.

Then these three types of uncertainties are related to the parameters in SP. A stochastic program is a mathematical program that has random parameters.

### 3.3 Stochastic Linear Programming (SLP)

SLP is based on a linear programming approach, which involves optimizing a linear objective function under linear equality and inequality constraints. One essential prerequisite for linear programming is that all variables must be non-negative, in line with the pragmatic demands of real-world situations. An array of areas, including economics, finance, chemistry, materials science, astronomy, physics, structural and molecular biology, engineering, computer science, and medicine, use stochastic linear programming (SLP), as a mathematical optimization approach. These unrelated fields are all united by the existence of optimization problems, which SLP proves to be an invaluable resource for addressing and resolving (Birge & Louveaux, 2011).

An objective function of SLP can be optimized for profit, minimization, or maximization depending on the particular objectives and nature of the problem. It is important to remember that these limitations may be expressed slightly differently in different sources.

The SLP issue is expressed in matrix vector notation as follows:

$$Optimize: Z = E[c^T x]$$
$$Subject\ to: \quad Ax\ (\leq\ ,\ =\ ,\ \geq)\ b$$
$$Additional\ Constraint: \quad x_j \geq 0\ (j = 1, 2, \ldots, n)$$

### 3.3.1 Formulation of SLP Model

One of the most crucial steps in problem-solving is formulation, which is the act of converting a real-world issue into a mathematical representation. This process involves linear optimization under uncertainty and involves the creation of a mathematical representation, consisting of an objective function and a set of constraints organized into an equality or inequality system. It deals with issues where certain parameters are modeled as random variables and are unknown, such as costs, resource availability, demands etc.

### 3.3.2 General Form of an SLP Model

$$Minimize\ (or\ Maximize)\ E[c^T x]$$

Subject to:

$$Ax \leq b, \quad x \geq 0$$

where

$x \in R^n$ : Decision variables

$c \in R^n$ : Coefficients of the objective function (random or deterministic)

$A \in R^{m \times n}$: Coefficient matrix for constraints

$b \in R^m$: Right-hand side constraints (random or deterministic)

$E[\cdot]$: Expected value operator to account for randomness

## 3.4 Stochastic Integer Programming (SIP)

Different from classical linear programming, a new class of optimization problems called stochastic integer programming (SIP) which provides a special requirement that some or all of the variables must be integers (Ahmed & Garcia, 2003). This is in contrast to linear programming

problems, which allow variables to take on non-negative real values. The optimization issue in the context of SIP may be stated using matrix vector notation as follows.

$$Optimize\colon\ Z\ = E[c^T x]$$
$$Subject\ to:\quad Ax\ (\leq\ ,\ =\ ,\ \geq)\ b$$
$$Additional\ Constraint\colon\quad x_j\ \ integer\ (For\ some\ or\ all\ j\ =\ 1, 2,\ \ldots,\ n)$$

SIP issues are useful in situations when discrete, whole-number values of the decision variables are required by nature. This may be particularly relevant in issues with item quantities, staffing assignments, or other circumstances where fractional amounts are neither possible nor realistic. Since integer restrictions introduce another level of complexity to the optimization process, SIP is a useful technique for solving real-world issues that need integer solutions by nature.

## 3.5 Types of Stochastic Programming

Based on the kind of uncertainty and the way the optimization framework depicts it, there are some forms of stochastic programming. The primary categories are:

- **Two-stage Stochastic Fixed Recourse Model**
- **Probabilistically constrained Model or Chance-constrained Model:**
- **Multistage Stochastic Linear Programming Model**

## 3.6 Two-Stage Stochastic Linear Programming (TSLP)

In optimization under uncertainty, the "**two-stage stochastic fixed recourse model**" is a mathematical framework that is most frequently applied. There are two phases to the decision-making process. Firstly, has to make decision must be made before the realization of uncertainty, and it cannot be adjusted. And following the observation of uncertainty, a second-stage decision is made. It is receptive to change. The goal of this model is to balance the costs of first-stage decisions and second-stage modifications to maximize the expected result.

The **first stage**, known as "**Here-and-Now Decisions**," refers to choices taken before the precise results of unknown parameters are known. Based on the knowledge at hand and probabilistic projections of unknown outcomes, a decision-maker makes commitments to specific actions during this stage. A number of decisions need to be made prior to beginning any experimental activities and with limited understanding about specific random events. These decisions are usually **strategic** or **long-term** in nature.

In the **Second Stage** or in the **recourse judgments**, the decision-maker takes more "recourse" judgments to adjust to the observed reality after learning the true results of unknown parameters. The decisions taken in the second step are represented by the variable $'y'$. Once the experimental phase is over and one has a thorough grasp of the actual results of certain random events, new options may be taken. Known as second-stage choices, this step comes just after the first in terms of time. Any discrepancy between the initial choices and the final result is lessened by these choices. The structure of the recourse acts is set in a "fixed recourse" paradigm, which means that only their values vary regardless of the outcome (Birge & Louveaux 2011).

This two-stage model is frequently used in domains where future conditions are unpredictable yet amenable to probabilistic modeling, such as supply chain management, finance, energy, and production planning. The objective is to balance the expected costs of first-stage decisions with the expected costs of recourse measures in order to minimize overall expected costs or maximize expected benefits.

### 3.6.1 Mathematical Form of the Two-Stage HN Model

The mathematical form Two-stage here and now (HN) is

$$\begin{aligned} &Min\ c^T x + E_\xi[Q(x,\xi)] \\ &subject\ to\ \ Ax = b \\ &x \geq 0 \end{aligned} \tag{3.1}$$

Where $[Q(x,\ \xi)]$ is the optimal value of the second-stage problem

$$For\ a\ given\ scenario\ \xi,\ the\ second-stage\ problem\ is$$

$$Q(x,\xi) = Min\ q^T y$$
$$subject\ to \quad T(\xi)x + Wy = h(\xi), \tag{3.2}$$
$$y \geq 0$$

where

$q$: $Cost\ coeficients\ for\ second-stage\ decisions.$

$W$: $Technology\ matrix\ for\ second-stage\ decisions.$

$T(\xi), h(\xi)$: $Scenario-dependent\ parameters.$

### 3.6.2 A Recourse Problem with the Scenario Method

In stochastic programming, the scenario technique is frequently used to solve recourse issues, especially in two-stage optimization problems. The scenario technique helps to mitigate uncertainty by describing random variables with a discrete collection of potential outcomes or scenarios, each with a corresponding probability.

#### 3.6.2.1 Key Concepts of Scenario Representation of Recourse

There are several dynamic concepts of the scenario representation of recourse model

**First Stage**: Before the realization of uncertainty, decisions are taken.

**Second Stage**: Following the observation of the random results (scenarios), corrective measures or adjustments are implemented.

To model uncertainty, a finite number of scenarios ($s = 1, 2, \ldots, S$) are used.

Every situation has:

- A probability $p_s$.
- Scenario-specific data (e.g., cost coefficients, constraints etc.).

#### 3.6.2.2 Mathematical Formulation of the Recourse Model with Scenarios

The mathematical formulation of the recourse model with scenarios is presented below.

**a. Objective Function**

$$Minimize\ Z = c^T x + \sum_{s=1}^{S} p_s {q_s}^T y_s$$

Where:

$x$: $First - stage\ decision\ variables$ $(same\ across\ scenarios)$.

$y_s$: $Second - stage\ decision\ variables$ $(scenario - dependent)$.

$c^T x$: $First - stage\ cost$.

${q_s}^T y_s$: $Second - stage\ recourse\ cost\ in\ scenario\ s$.

$p_s$: $Probability\ of\ scenario\ s$.

**b. Constraints**

**i. First-Stage Constraints**

$Ax \leq b$ which is deterministic constraints.

**ii. Second-Stage Constraints (Scenario-Specific)**

$$T_s x + W_s y_s \leq h_s, \qquad \forall s = 1, 2, \dots, S$$

- $T_s$: $Contribution\ from\ first - stage\ decisions$.
- $W_s y_s$: $Recourse\ actions\ in\ scenario\ s$.
- $h_s$: $Scenario\ specific\ right\ hand\ side$.

**iii. Non-Negativity**

$$x \geq 0, \qquad y_s \geq 0, \qquad \forall s = 1, 2, \dots, S$$

### 3.6.2.3 Key Steps in Solving a Scenario-Based Recourse Problem

There are mainly three steps of solving the scenario-based recourse model. They are:

**a. Establishing Scenarios**

- Calculate the random variables' probability distributions.
- To generate a small number of situations, use expert estimation or sampling.

**b. Model Construction**

- Model the problem using parameters unique to the circumstance.
- Incorporate probabilities and costs into the objective function.

**c. Optimization**

Next, optimization of the model takes place whether it is maximization or minimization.

### 3.6.3 Applications of the Two-stage Stochastic Model (TSLP)

Two-stage stochastic models are a method that is frequently employed for optimizing decisions in the presence of unpredictable outcomes and in the case of making judgments in the face of uncertainty. There is an extra complication of deciding before uncertain parameters are revealed (Stage 1) and another decision (Stage 2) after the uncertainty is known. Several typical uses of two-stage stochastic models in various sectors are listed below:

- Inventory and Supply Chain Management
- Power Generation and Energy Systems
- Financial Portfolio Optimization
- Construction and Project Management
- Planning for Healthcare and Treatment
- Planning for Agriculture
- Network Design and Telecommunications
- Logistics and Transportation
- Production and Manufacturing
- Disaster and Environmental Management

Considering all the things, two-stage stochastic models are useful instruments in any situation where choices must be made with limited knowledge and uncertainty about the future. The model offers a methodical approach for making the best choices by weighing the costs of initial choices against the freedom to change course when new information becomes available. This type of flexibility is essential in fields like supply chain management, finance, healthcare, and more.

## 3.7 Distribution-Based Recourse

A stochastic optimization issue where uncertainty is described by probability distributions for the random parameters is a recourse problem with a distribution-based method. This approach computes expected values directly from the distributions rather than discretizing uncertainty into scenarios. It deals with the big linear or mixed-integer program that emerges as the deterministic equivalent problem.

### 3.7.1 Important Elements of the Distribution-Based Recourse

There are four main elements in the distribution-based recourse model. They are:

- **Two-Stage Problem Framework**
    - First Stage: Before viewing the random variables $\xi$, decisions $x$ is taken.
    - Second Stage: After $\xi$ is realized, decisions $y(\xi)$ (recourse actions) are modified.
- **Random Parameters**
    - Uncertain parameters are represented as random variables with established probability distributions, such as expenses, resource availability, or demands.
- **Objective**
    - To reduce the total amount of first-stage expenses and the anticipated amount of second-stage recourse expenses.
- **Constraints**
    - There is 1st stage, 2nd stage constraints and non-negative restrictions in a distribution based recourse model

### 3.7.2 Mathematical Form of Distribution-Based Recourse

The mathematical form of the distribution-based recourse model is shown below.

- **Objective Function**

$$Minimize\ Z = c^T x + E_\xi[q(\xi)^T y(\xi)]$$

where

$c^T$: $First - stage\ costs.$

$E_\xi[q(\xi)^T y(\xi)]$: $Expected\ value\ of\ second - stage\ costs.$

- **Constraints**
  - **First-Stage Constraints**: $Ax \leq b$
  - **Second-Stage Constraints** $(dependent\ on\ \xi)$: $Tx + W(\xi)y(\xi) \leq h(\xi)$
  - **Non-Negativity**: $x \geq 0, \quad y(\xi) \geq 0$

## 3.8 Multistage Stochastic Linear Programming (MSLP)

The term "multistage" refers to the fact that choices are taken in stages. **Multistage Stochastic Linear Programming (LP) Model** is a mathematical framework used to optimize decisions over multiple time stages under uncertainty, where the relationships between variables and constraints are linear. A sophisticated optimization method called multistage stochastic programming, or MSLP, is used to make judgments in situations where uncertainty is a major factor (Shapiro et al., 2021). It is especially helpful in situations when future results are unpredictable and subject to change over time, such as supply chain management, energy management, and financial planning. It is an extension of linear programming that incorporates uncertainty represented through a finite set of scenarios, each with associated probabilities.

### 3.8.1 General Form of a Multistage Stochastic Model

The goal of a multistage stochastic model is to minimize the total expected cost (or maximize the reward) over a sequence of decisions, while accounting for uncertainty at each stage.

**a. Objective Function**

$$\min_{x_1,\ x_2(\xi_1),\ldots,\ x_T(\xi_{1:T-1})} E\left[\sum_{t=1}^{T} c_t(x_t,\ \xi_t)\right]$$

Subject to

**b. Stage-Specific Constraints**

$$A_t x_t + B_t x_{t-1} = b_t(\xi_t), \quad \forall t = 1, \ldots, T$$

where

$x_t$: $Decision\ variables\ at\ stage\ t$

$\xi_t$ : $Random\ variables\ at\ stage\ t\ representing\ uncertainty.$

$A_t, B_t$ : $Matrices\ defining\ constraints\ at\ stage\ t.$

$b_t(\xi_t)$: $Constraints\ that\ depend\ on\ uncertain\ parameters.$

**c. Non-Negativity or Feasibility Constraints**

$$x_t \geq 0, \quad \forall t = 1, \dots, T$$

**d. Recourse Relationship**

Decisions $x_t$ must be feasible given past decisions $x_{t-1}$ and realizations of $\xi_{1:t-1}$

### 3.8.2 Key Components of MSLP

There are mainly five features in an MSLP. They are:

**a. Stages**

- The model runs in several stages, $t = 1, 2, \dots, T$.
- Every stage's decisions are influenced by ambiguity and have an impact on subsequent decisions.

**b. Situations**

- The discrete scenarios $S = \{s_1, s_2, \dots, s_k\}$
- Describe the problem's uncertainty (such as demands, costs, or resource availability).
- Every scenario has a chance that adds up to 1.

**c. Decision Variables**

- Variables are unique to each stage ${x_t}^s$. Decision factors for the scenario $s$ at stage $t$.
- While some variables rely on scenario realizations in subsequent phases, but others are "first-stage" (decisions made before uncertainty develops).

**d. Objective Function**

- The aim is to optimize the expected value of a linear function of decision factors in every scenario

$$\sum_{s\epsilon S} p_s \left( {c_1}^T x_1 + \sum_{t=2}^{T} {c_t}^{s^T} {x_t}^s \right)$$

where ${c_t}^s$ represents cost coefficients at the stage $t$ for scenario $s$.

**e. Constraints**

- **System Constraints**

  Resource limits and other linear constraints for every step and situation are called system constraints.

$$A_t{}^s x_t{}^s + B_t{}^s x_{t-1}{}^s = b^s \qquad \forall\ t,\ s$$

- **Non-anticipativity Constraints**:

  As uncertainty develops, make sure choices made in earlier phases are consistent across scenarios, for scenarios $s$ and $s'$, they are indistinguishable at stage $t$. ${x_t}^s = {x_t}^{s'}$, for scenarios $s\ and\ s'$ at stage $t$.

### 3.8.3 Solution Techniques of MSLP

The solution techniques of multistage stochastic models are stated here.

- **Deterministic Equivalent Formulation (DEP)**
  - By clearly listing every scenario, the Deterministic Equivalent Formulation (DEP) transforms the stochastic issue into a huge deterministic LP.
  - Computationally costly, although for small to medium-sized problems, it can be solved using ordinary LP solvers.

- **Scenario Decomposition**
  - In this method solve each scenario separately, then use dual variables or aggregation to reconcile the results.

- **Scenario Tree Method**

As we move down the scenario tree (for any scenario) we have the following order of events-

- Making a judgment about how much to create in the initial stage, realizing the demand, the stochastic element, and so on (Heitsch & Römisch, 2009).
- A choice on the values of the variables for recourse; thereafter, making a judgment on how much to create in the second stage, a manifestation of the stochastic element (demand) in the third stage; and ultimately, a choice about the relevant variables' values.

### 3.8.4 Scenario Trees Formulation of MSLP

Let's $s = \{1, \dots, S\}$ represents the scenario index and $p_s$ represent the likelihood that $s = \{1, \dots, S\}$ will occur. For scenario $s$ in period $t = \{2, \dots, T\}$, assume $({A_t}^s,\ {B_t}^s,\ {x_t}^s,\ {b_t}^s)$ represent the choice variables and LHS/RHS coefficients. For scenario $s$ in time $t$, let ${c_t}^s$ be the objective function coefficient for the decision vector ${x_t}^s$.

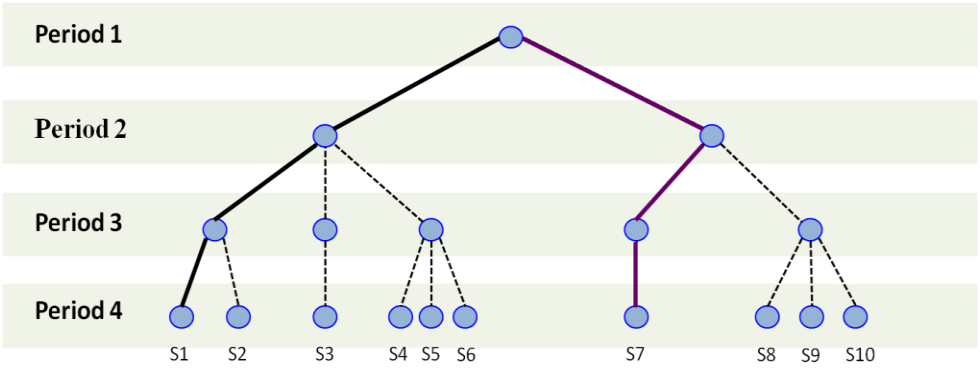
So the objective function is

$$\sum_{s \epsilon S} p_s \left( {c_1}^T x_1 + \sum_{t=2}^{T} {c_t}^{sT} {x_t}^s \right)$$

Subject to

$${A_t}^s {x_t}^s + {B_t}^s {x_{t-1}}^s = b^s \qquad \forall\ t,\ s$$

Scenario trees are used to represent the history of decision-making.




*A path from the root to a leaf is called a "scenario".*

Figure 3.1: Scenario Generation of a MSLP model

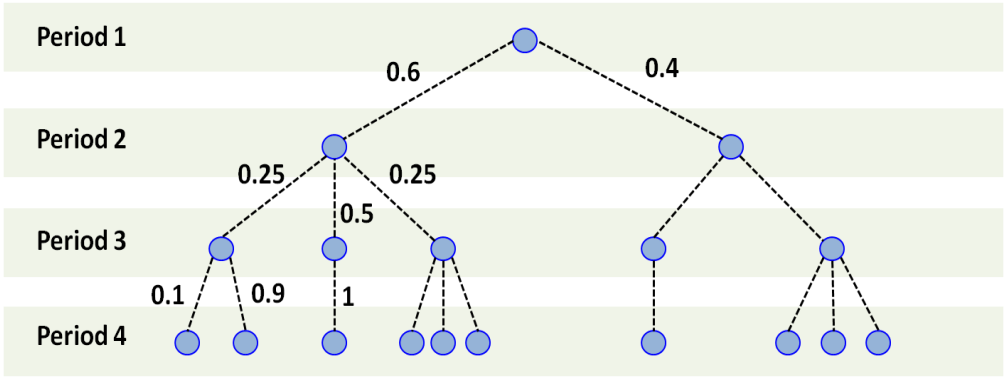
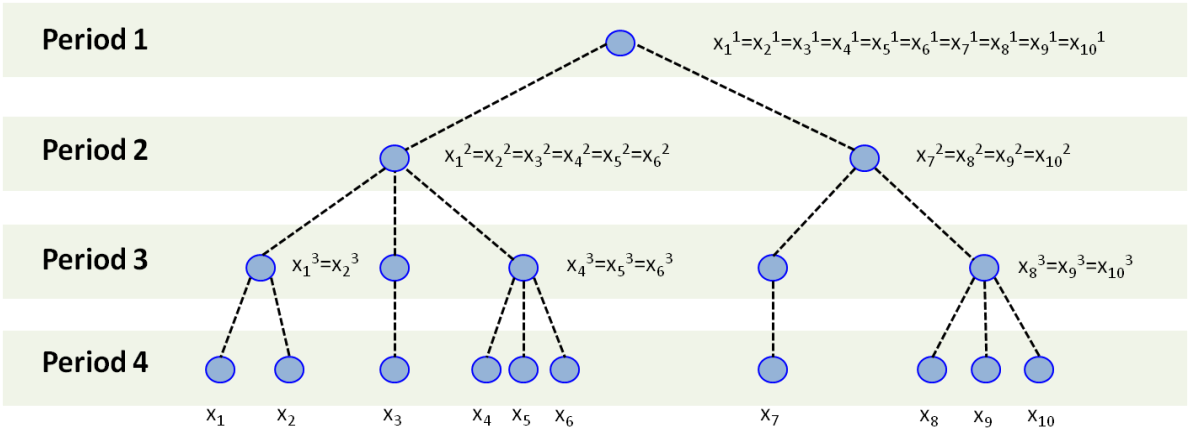



Figure 3.2: Scenarios with Probabilities (The probability of scenario 1 is $0.6 \times 0.25 \times 0.1 = 0.015$)

Scenario trees are used to define non-anticipativity constraints. For the moment, let $x_s^t$ represent the decisions made for scenario $s$ and time $t$. Let $x_s$ represent the set of decisions $(x_s, \ldots, x_s^T)$.

Scenario trees are used to define non-anticipativity constraints.




We can group $x_i^t$ variables according to the scenario tree.

Figure 3.3: Non Anticaptivity Variables

### 3.8.5 Applications of Multi-Stage SPs (MSLP)

In many real-world situations where sequential decisions must be made under uncertainty, multistage stochastic linear programming (MSLP) models are so effective tool in these cases. The following are a few important application areas of MSLP.

**Agriculture and Water Resources**

- **Reservoir operations:** MSLP with real-time inflow scenarios from weather forecasts is used for daily coordinated control of multi-reservoir systems, which improves performance over deterministic models.
- **Agricultural irrigation:** Compared to static or two-stage models, multistage models that adjust decisions over time under rainfall unpredictability provide higher revenues in farmland management. Decisions about the allocation of precious water in agriculture are aided by these models, which balance crop yields under a number of uncertain conditions, according to literature assessments.

**Planning for Energy and Production**

- **Hydrothermal dispatch:** Price and inflow uncertainties are addressed by MSLP for long-term hydrothermal energy planning.
- **Microgrid management:** Real-time management of renewable output, demand, and pricing is guided by multistage stochastic or stochastic dual dynamic programming frameworks.

**Supply Chains and Logistic**s

- **Closed-loop supply chains:** MSLP is used to plan with movable/modular facilities under demand unpredictability. Complexity is managed using specialized decomposition techniques.
- **Disaster logistics & evacuation:** To reduce anticipated costs for storm relief, MSLP streamlines facility activation, evacuation, and supply flows across several phase.

**Finance and Portfolio Management**

- **Issuance of corporate bonds:** Using a multi-phase framework, companies minimize risk and expense while dealing with cash-flow volatility.
- **Asset-liability planning:** Multistage stochastic programs are used by pension funds (such as those in Brazil) to manage assets and liabilities over several uncertain future periods.
- **Portfolio optimization:** Using the concepts of dynamic programming, investors adjust their holdings over time in the face of stochastic returns.

**Production and Planning**

- **Scheduling of production:** Outperforming deterministic heuristics, MSLP helps maximize predicted profitability in master production scheduling with uncertain demand and restricted capacity.
- **Nanomaterial capacity planning:** Taking demand, regulations, and technological risks into consideration, multistage models maximize when and how much to invest in the manufacturing of carbon nanotubes.

**Infrastructure and Facility Location**

Multistage integer stochastic programming outperforms static two-stage approaches for optimizing the dynamic staged opening of facilities under demand and budget uncertainty.

## 3.9 Theorems and Propositions

In this section, the fundamental theorems and propositions on two-stage and multistage stochastic models are stated with proofs.

**Proposition 3.9.1**

The function $Q(.,\xi)$ is convex for every given $\xi$. Furthermore, the function $Q(.,\xi)$ is polyhedral if the $\{\pi: W^T\pi \leq q\}$ is non-empty, and the problem where $Q(x;\ \xi)$ is the optimal value of the second-stage problem

$$Q^T x\ ;\ s.t.\ |\ Tx + Wy = h,\quad y \geq 0 \text{ is feasible for at least one } x.$$

*Proof.*

The features of $Q(x,\xi)$ that following are derived from the equivalent properties of the function $s_q(.)$ since $Q(x,\xi) = s_q(h-Tx)$,

□

**Proposition 3.9.2**

Assume that $Q(x,\xi)$ has a finite value for $x = x_0$ and $\xi \epsilon\, \Xi$. Once $x_0$ is reached, $Q(.,\xi)$ becomes subdifferentiable and

$$\delta Q(x,\xi) = -T^T H(x_0,\xi), \tag{3.3}$$

Where $H(x_0,\xi) \coloneqq arg \max_{\pi\in\Pi(q)} : \pi^T(h-Tx)$

is the collection of optimal answers to the dual problem $\pi^T(h-Tx)$

$$s.t.\ \ W^T\pi \le q$$

*Proof.*

Since the set $\Pi(q)$ specified in $\Pi(q) \coloneqq \{\pi: W^T\pi \le q\}$ is nonempty and $Q(x_0,\xi)$ is finite, $s_q(x)$ is a support function. The definitions make it clear that the support function $s_q(.)$ is the indicator function $I_q(\pi) \coloneqq \{0, if\ \pi\epsilon\Pi(q)$

$+\infty, otherwise$ 's

The function $I_q(.)$ is convex and lower semicontinuous since the set $\Pi(q)$ is convex and closed. Therefore implies that is $I_q(.)$ is the conjugate of $s_q(.)$. Consequently, it is obtained

$$\delta s_q(\chi_0) = arg \max_{\pi}: \{\pi^T\chi_0 - I_q(\pi)\} = arg \max_{\pi\in\Pi(q)} : \pi^T\chi_0 \tag{3.4}$$

$Hence\ \delta s_q(\chi_0)$ is nonempty since $s_q(\chi_0)$ is finite and the set $\Pi(q)$ is polyhedral. Additionally, the function $s_0(.)$ is piecewise linear, hence the chain rule of subdifferentiation leads to formula (3.3) from (3.4).

□

**Proposition 3.9.3**

Assume that the predicted recourse cost, $\varphi(\cdot)$, has a finite value in at least one point $x \in R^n$, and the probability distribution of $\xi$ has finite support, $\Xi = \{\xi_1, \ldots, \xi_k\}$. Then the function $\varphi(\cdot)$ is polyhedral for any $x_0 \in dom\ \varphi$,

$$\partial\varphi(x_0) = \sum_{k=1}^{K} p_k \partial Q(x_0, \xi_k) \tag{3.5}$$

*Proof.*

All values $Q(x, \xi_k)$, $k = 1, \ldots, K$, are finite since $\varphi(x)$ is finite. Therefore, each function $Q(\cdot, \xi_k)$ is polyhedral by Proposition 3.6.2. The fact that a linear combination of polyhedral functions with positive weights is also polyhedral is easily understood. Consequently, $\varphi(\cdot)$ must be polyhedral. Additionally, we have that for each $h \in R^n$ the directional derivatives

$Q_k'(x_0, h) > -\infty$ and $dom\ \varphi \cap_{k=1}^{K} dom\ Q_k$. Where $Q_k(\cdot) := Q(\cdot, \xi_k)$ *and*

$$\varphi'(x_0, h) = \sum_{k=1}^{K} p_k Q_k'(x_0, h) \tag{3.6}$$

By duality arguments, formula (3.5) then follows from (3.6). Keep in mind that the Moreau–Rockafellar Theorem is used specifically in equation (3.5). Here, an extra regularity criterion is not necessary for (3.5) to hold …since the functions $Q_k$ are polyhedral.

□

**Proposition 3.6.4**

Assume that there is a fixed recourse.
E [||q|| ||h||] < +∞ and E [||q|| ||T||] < +∞. (3.7)

Let $x \in R^n$ . then $E[Q(x,\xi)_+]$ is finite if and only if the following condition is met $w.p.1$,

$$h - T\,x \in pos\,W \tag{3.8}$$

*Proof.*

$Q(x,\xi) < +\infty\ iff$ condition (3.7) is satisfied. Consequently, $Q(x,\xi) = +\infty$ with positive probability if condition (3.8) does not hold $w.p.1$, and thus $E[Q(x,\xi)_+] = +\infty$.

On the other hand, assume that $w.p.1$ is held by condition (3.8). Hence, $Q(x,\xi) = s_q(h - T\,x)$, where $s_q(\cdot)$ is the support function of the set $\Pi(q)$. According to

$s_q(\cdot) \leq s_{q_0}(\cdot) + \kappa\,||q - q_0||\,||.||$, $\kappa$ is a constant that,

for each $\chi$, $s_q(\chi) \leq s_0(\chi) + \kappa\,||q||\,||\chi||$.

Furthermore, it is known that $s_0(\chi) = 0$ for any $\chi \in pos\,W$. As a result, $w.p.1$,

$s_q(h - T\,x) \leq \kappa\,||q||\,||h - T\,x|| \leq \kappa\,||q||\,(||h|| + ||T||\,||x||)$.

then by (3.5) it follows that $E[s_q(h - T\,x)_+] \leq \infty$.

□

***Note:*** As $\Pi(q)$ is polyhedral, if it is nonempty then $s_q(\cdot)$ is piecewise linear on its domain, which coincides with $posW$, and

$$|s_q(\chi_1) - s_q(\chi_2)| \leq \kappa\,||q||\,||\chi_1 - \chi_2||, \qquad \forall\ \chi_1, \chi_2 \in pos\,W \tag{3.9}$$

**Proposition 3.9.5**

Assume that: (i) the recourse is fixed; (ii) the set $\Pi(q)$ is nonempty for a.e. q; and (iii) condition (3.7) is satisfied.

Then, for any $x \in R^n$, the expectation function $\varphi(x)$ is well defined and $\varphi(x) > -\infty$. Additionally, on dom φ, φ is convex, lower semicontinuous, and Lipschitz continuous. Its domain is a convex closed subset of $R^n$ provided by $R^n$given by

$$dom\,\varphi = \{x \in R^n : h - T x \in pos\, W\ w.p.1\} \tag{3.10}$$

*Proof.*

The dual problem's feasible set $\Pi(q)$ is nonempty $w.p.1$ according to assumption (ii). Consequently, for each $x, Q(x,\xi) = s_q(h - T x)$ where $s_q(\cdot)$ is the support function of the set $\Pi(q)$. Let $\pi(q)$ be the nearest member to 0 in the set $\Pi(q)$. Since $\Pi(q)$. is closed, it exists. There is a constant κ such that $||\|\pi(q)|| \leq ||\kappa(q)||$, according to Hoffman's lemma. Then, the following holds true for any $x$ in $w.p.1$:

$$s_q(h - T x) \geq \pi(q)^T (h - T x) \geq -\kappa\big|\,|q|\,\big|\big(\big||h|\big| + \big||T|\big|\big||x|\big|\big) \tag{3.11}$$

Because of condition (3.7), (3.11) implies that $\varphi(\cdot)\ is$ properly defined and that, for any

$$x \in R^n \varphi(x) > -\infty$$

Furthermore, Fatou's lemma implies that $\varphi(\cdot)$has lower semicontinuity since $s_q(\cdot)$ is lower semicontinuous. From the convexity and lower semicontinuity of $\varphi$, convexity and closedness of $dom\ \varphi$ follow. According to Proposition 3.9.4 if condition (3.8) is met, $\varphi(x) < +\infty$ and $w.p.1$ is valid. This suggests (3.10).

Let $x,\ x'$ be two points in dom φ. Then, by (3.9), *w.p.1* is valid as follows:

$$h - Tx \in pos\, W \quad and \quad h - T\,x' \in pos\, W. \tag{3.12}$$

By (3.9), if (3.12) holds and the set $\Pi(q)$ is nonempty, then

$$|s_q(h - T\,x) - s_q(h - T\,x')| \leq \kappa||q||\ ||T||\ ||x - x'||.$$

Consequently,

$$|\varphi(x) - \varphi(x')| \leq \kappa\, E[||q||\ ||T||\,]\,||x - x'||.$$

This implies the Lipschitz continuity of $\varphi$ on its domain, together with condition (3.7). The support of the probability distribution (measure) of $(h, T)$ is shown by Σ. According to formula (3.10), if the probability of the occurrence $\{h - T\,x \in pos\,W\}$ is one, then a point x belongs to dom φ. It should be noted that the set $\{(h,T) : h - T\,x \in pos\,W\}$ is closed since it is polyhedral and convex. Thus, x is a member of $dom\varphi\ iff\ h - T\,x \in pos\,W$ follows for each $(h,T) \in \Sigma$. Thus, formula (3.10) may be written in the following format.

$$dom\,\varphi = \setminus \bigcup_{(h,T)\in\,\Sigma} \Sigma\{x : h - T\,x \in pos\,W\} \tag{3.13}$$

It is noticeable that fixed recourse is assumed, set $\bigcup_{h\in H}(-h + pos\,W)$ is convex and polyhedral for every set H of vectors h. Pos W may be expressed as the intersection of a finite number of half spaces, $A_i = \{\chi : a_i^T\,\chi \leq 0\}$, for each $i = 1,\ldots,l\}$, since it is a convex polyhedral cone. The set $\bigcup_{h\in H}(-h + pos\,W)$ can be represented as the intersection of half spaces of the form $b_i + A_i$, $\{i = 1,\ldots,l\}$, and is therefore polyhedral because any number of half spaces of the form b + $A_i$with b ∈ B, is still a half space of the same form (as long as this intersection is nonempty). Consequently, the set at the right side of (3.13) is convex and polyhedral if T and W are fixed. Let us now talk about the expectation function $\varphi(x)$. and its differentiability features. We arrive to the following conclusion using Proposition 3.9.2

□

**Proposition 3.9.6**

Assume that the interior of the domain of the expectation function $\varphi(\cdot)$ is nonempty and that it is appropriate. Following that, for every $x_0 \in dom\ \varphi$,,

$$\partial\varphi(x_0) = -E[T^T D(x_0, \xi)] + N_{dom\ \varphi}(x_0), \tag{3.14}$$

Whereas

$D(x, \xi) := arg\ max_{\pi \in \Pi(q)} \pi^T (h - T\ x)$. Furthermore, the set

$D(x_0, \xi)$ is a singleton $w.p.1$, and $\varphi$ is differentiable at $x_0$ if and only if $x_0$ is inside $dom\ \varphi$. The expectation function $\varphi$ is piecewise linear on its domain and is differentiable everywhere,

only in the trivial case if it is linear, as we previously mentioned, when the distribution of $\xi$ has a finite support (i.e., there are a limited number of possibilities). The expectation operator smoothes out the piecewise linear function $Q(\cdot, \xi)$ in the case of a continuous distribution of $\xi$.

□

**Theorem 3.9.7**

Let $x$ be a workable solution to problems $(3.1) - (3.2)$, meaning that $\varphi(x)$ is finite and $x \in X$. If there exists $\pi_k \in D(x, \xi_k),\ k = 1, \ldots, K$, and $\mu \in R^m$,

then $x$ is the best solution for problems $(3.1) - (3.2)$

$$\text{For } \sum_{k=1}^{K} p_k T_k^T \pi_k + A^T \mu \leq c, \tag{3.15}$$

$x^T (c - \sum_{k=1}^{K} p_k T_k^T \pi_k - A^T \mu) = 0$..

*Proof.*

We may write $0 \in c + \partial\varphi(\bar{x}) + N_\chi(\bar{x})$ (3.16)

as necessary and sufficient optimality requirements for minimizing $c^T x + \varphi(x)$ over $x \in \chi$ where the normal cone to the feasible set φ is denoted by $N_\chi(x)$. Keep in mind that condition

(3.16) suggests that $x \in \chi$ and $\varphi(x)$ is finite as it implies that the sets $N_\chi(x)$and $\partial\varphi(x)$ are nonempty. Note that since $\varphi(\cdot)$ and $\chi$ are convex and polyhedral, no extra regularity criteria are required in this case. We conclude that (3.16) is equivalent to the presence of $\pi_k \in D(x,\ \xi_k)$ such that using the characterization of the subgradients of $\varphi(\cdot)$, provided in

$$\partial\varphi(x_0) = -\sum_{k=1}^{K} p_k T_k D(x_0, \xi_k).$$

Such that

$$0 \in c - \sum_{k=1}^{K} p_k T_k^T \pi_k + N_\chi(x).$$

observing that

$$N_\chi(x) = \{A^T\mu - h : h \geq 0, h^T x = 0\} \tag{3.17}$$

Conditions (3.15) are equal to the final two relations.

□

**Theorem 3.9.8**

Assume that x is a workable solution to problems $(3.1) - (3.2)$. Let us assume the following: $N_{dom\,\varphi}(x) \subset N_\chi(x)$, $int\,(dom\,\varphi) \cap X$ is nonempty, and the anticipated recourse cost function $\varphi(\cdot)$ is valid. then $x$ is the best solution for the problems $(3.1) - (3.2)$

$Iff$ there is a vector $\mu \in R^m$ and a measurable function $\pi(\omega) \in D\big(x, \xi(\omega)\big),\ \omega \in \Omega$,

Such that

$x^T\,(c - E[T^T\pi] - A^T\mu) = 0,$

$E[T^T\pi] + A^T\mu \leq c,$

*Proof.*

Given that $int(dom\ \varphi)\ \cap\ X$ is nonempty, the Moreau-Rockafellar Theorem gives us the following:

$$\partial(c^T x\ +\ \varphi(x)\ + I_\chi\ (x) =\ c\ +\ \partial\varphi(x)\ +\ \partial I_\chi\ (x).\ Moreover, \partial I_\chi\ (x)\ =\ N_\chi(x)$$

Thus, for minimizing $c^T x + \varphi(x)$ over $x\ \in\ \chi$, we have here that (3.16) are necessary and sufficient optimality criteria. Based on the description of the subdifferential of $\varphi(\cdot)$ provided in (3.12), we deduce that (3.16) corresponds to the presence of a measurable function $\pi(\omega)\ \in\ D(x, \xi(\omega))$, such that

$0\ \in\ c\ -\ E[T^T\pi] +\ N_{dom\ \varphi}(x)\ + N_\chi(x)$. (3.18)

Furthermore, the word $N_{dom\ \varphi}(x)$ can be deleted due to the constraint $N_{dom\ \varphi}(x)\ \subset N_\chi(x)$. Using (3.18) in conjunction with formula (3.17) for the normal cone $N_\chi(x)$. completes the proof.

□

**Theorem 3.9.9**

Assume that $x$ is a workable solution to problems $(3.1) - (3.2)$. Assume that the matrix $T$ is deterministic, $int(dom\ \varphi)\ \cap\ \chi$ is nonempty, and the conditions of Proposition 3.9.5 are met. Then, the best solution to problems $(3.1) - (3.2)$ is $x$ if a vector

$\mu\ \in\ R^m$ and a measurable function $\pi(\omega)\ \in\ D(\ x, \xi(\omega)), \omega\ \in\ \Omega$ exist such that

$T^T E[\pi] +\ A^T\mu\ \leq\ c,$

$x^T\ (c\ -\ T^T E[\pi] -\ A^T\mu) =\ 0.$

*Proof.*

Given that $T$ is deterministic, we may write the optimality criteria (3.18) as $E[T^T\pi] = T^T E[\pi]$

$$T^T E[\pi]\ +\ N_{dom\ \varphi}(\bar{x})\ + N_\chi(\bar{x}) = 0 \in c$$

It is now necessary to compute the cone $N_{dom\,\varphi}(\bar{x})$. Remember that $\varphi(\cdot) > -\infty$ and formula (3.10) is obtained under the assumptions of Proposition 3.9.5 (namely, that the recourse is fixed and $\Pi(q)$ i is nonempty w.p.1). Here, we have that both matrices $W\ and\ T$ are deterministic, but only $q\ and\ h$ are random, and (3.10) simplifies to

$$dom\ \varphi\ = \{x : -T\,x\ \in \bigcap_{h\epsilon\Sigma}(-h\ +\ pos\ W)\},$$

where Σ is the random vector $h's$ distribution support.

$$T_{dom\,\varphi}(x)\ =\ \{d : -T\,d\ \in \bigcap_{h\epsilon\Sigma}(\ pos\ W\ +\ lin(-h\ +\ Tx\ ))\}$$

is the shape of the tangent cone to $dom\ \varphi$ at $x$ .

$$\left\{d : -T\,d\ \in pos\ W + \bigcap_{h\epsilon\Sigma} lin(-h\ +\ Tx)\right\}$$

Defining the linear subspace

$$L := \bigcap_{h\epsilon\Sigma} lin(-h\ +\ Tx),$$

we can write the tangent cone as

$$T_{dom\,\varphi}(x)\ =\ \{d : -T\,d\ \in\ pos\ W\ +\ L\}.$$

Consequently, the normal cone is equal to

$N_{dom\,\varphi}(x)\ =\ \{-T^T\,v : v\ \in (pos\ W\ +\ L)^*\ \}\ =\ -T^T[(pos\ W\ )^*\ \cap L^{\perp}]$.

In order to calculate $(pos\ W\ +\ L)^*$ We took use of the fact that pos W is polyhedral and does not require an interior condition. Recalling equation $\Pi_0^*\ =\ pos\ W$ we infer that

$$N_{dom\,\varphi}(x) = -T^T(\Pi_0\ \cap\ L^{\perp})$$

Note that for every $\xi\ \in\ \Xi$. Thus $v$ is an element of the recession cone of the set $D(\bar{x},\xi)$,

if $\nu \in \Pi_0 \cap L^{\perp}$. For practically any $\omega \in \Omega, \pi(\omega) + \nu$ is therefore also a member of the collection $D\ (x, \xi(\omega))$.

Thus, $-T^T E[D(x,\xi)] + N_{dom\,\varphi}(x) = -T^T E[D(x,\xi)] - T^T(\Pi_0 \cap L^{\perp})$

$$= -T^T E[D(x,\xi)],$$

and the result follows.

□

## 3.10 Conclusion

This chapter focused on the basics of stochastic programming, mathematical models of TSLP, MSLP with scenario generations, applications of different stochastic models, and basic propositions with proofs. In the next chapter, the development of a linear programming model to offer an optimal production planning for the RMG industry is presented with the necessary calculations and output.

# Chapter 04

# Deterministic Optimal Production Planning Model for RMG Industry

# Chapter 04

# Deterministic Optimal Production Planning Model for RMG Industry

## 4.1 Introduction

In this chapter, the formulation of a linear programming model will be presented to offer a better production planning for the RMG industry of Bangladesh in a deterministic environment in order to optimize profit and cost along with the proper utilization of available resources. The model is solved by LINDO (a mathematical tool). The necessary calculations for formulating and the output of the model are presented here in tabular form.

## 4.2 Linear Programming Model (LP)

Deterministic linear programming (DLP) is used to maximize or minimize a linear objective function while taking into account a set of linear restrictions. In this type of model, all parameters, including restrictions and objective function coefficients, are known with certainty and do not vary over time, and the solutions are exact. The applications of deterministic LP models are optimization of manufacturing, diet planning, transportation logistics etc. The mathematical formulation of a deterministic LP model is

$$Maximize\ (or\ Minimize)\ Z = C^T x$$

$$or\ equivalently,$$

$$Maximize\ (or\ Minimize)\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

*where*

$Z\ is\ the\ objective\ function\ value,$

$c = [c_1,\ c_2, \ldots,\ c_n]^T\ \ is\ the\ vector\ of\ coefficients,$

$x = [\ x_1,\ x_2, \ldots,\ x_n]^T\ is\ the\ vector\ of\ decision\ variables.$

$$Subject\ to$$

$$Ax \leq b$$

$$or\ equivalently,$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

*where*

$A\ is\ the\ m \times n\ matrix\ of\ coefficients,$

$b = [b_1,\ b_2, \ldots,\ b_m]^T\ is\ the\ vector\ of\ right - hand\ side\ constants.$

***Non − negativity Constraints***

$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$
$$or\ equivalently$$
$$x \geq 0$$

## 4.3 Background of Model Formulation

To formulate an LP or SLP model, some parameters like cost or profit are necessary for formulating the objective function and constraints as well. Several information are mandatorily required though there may be some uncertainty. In this study, different data were collected from an established RMG factory in Gazipur, Dhaka, which is considered as the case industry of this research, produces different types of woven and knit products such as T-Shirts, vests, sweatshirts, leggings, formal shirts, hoodie jackets, joggers, polo-shirts, casual-shirts, girls' tops etc. To produce these types of products various raw materials and trims (trims are all materials other than fabric used in a garment) like fabrics, thread, zippers, buttons, hand tags, elastics, rivets lace, tapes/velcro, hangers, polybags, cartons, etc are used. In that case industry, the monthly average production of T-shirt is 28, 45, 902 pcs and for this 4, 93,793 kgs of fabrics 71, 448 cones of thread, 3834461 pc of labels, 63628 pcs of button, 750185 pcs of polybags, 26961 pcs of cartons, 514310 pcs of hand tags, 516257 pcs of hanger, 12550 yds of tapes/Velcro, and 52000yds of elastics are needed. Besides producing one unit of T-shirt, labor takes 1.5 minutes, and the machine takes 6

minutes. Labor cost is $0.28976, operating cost including machinery overhead is $0.153 for producing per piece of T-shirt. Similarly, the rest of the 9 products need the raw materials, labor cost, and operating cost in a certain quantity, which are given in Table 4.1.

| Outputs / Input | Unit | Product-1 (T Shirt) | Product-2 (Vest) | Product-3 (Sweat Shirt) | Product-4 (Legging) | Product-5 (Shirt) | Product- 6 (Hoodie Jacket) | Product-7 (Jogger) | Product-8 (Polo- shirt) | Product- 9 (Casual shirt) | Product-10 (Girls' Tops) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Monthly Production** | Pc s | 2845902 | 684752 | 538560 | 388708 | 2950900 | 280200 | 450049 | 950854 | 2548725 | 845265 |
| **Fabrics** | Kg | 493791 | 57071 | 101433 | 87407 | 801200 | 920323 | 180020 | 223564 | 714253 | 213578 |
| **Thread** | Co ne | 71448 | 13493 | 34332 | 16770 | 100200 | 32262 | 36004 | 23675 | 84520 | 75846 |
| **Labels** | pc s | 3834461 | 1238152 | 1259324 | 1002062 | 2434461 | 575103 | 472551 | 1057000 | 2124523 | 548216 |
| **Zipper** | PC S | | | 4404 | | | 294565 | | | | |
| **Button** | PC S | 63628 | 1326 | 324480 | | 75888 | | | 50365 | 69254 | 49875 |
| **Polybag** | PC S | 750186 | 237435 | 125021 | 220371 | 70853 | 94000 | 42551 | 29547 | 65824 | 254731 |
| **Carton** | PC S | 26961 | 5427 | 9167 | 13284 | 28411 | 58913 | 59069 | 8452 | 23541 | 6584 |
| **Hand tags** | PC S | 514310 | 30015 | 162052 | 135949 | 617000 | 24000 | 471000 | 17563 | 502485 | 14523 |
| **Rivets lace** | YD S | | 98395 | | | | 53385 | 94103 | | | 169053 |
| **Hanger** | PC S | 516257 | 207300 | 391517 | 131009 | 626245 | | | 158568 | 514658 | 14520 |
| **Tapes/Velcro** | YD S | 12550 | 4000 | 32400 | | | | | 4587 | | 4200 |
| **Elastics** | YD S | 52000 | 153000 | 137950 | 392606 | | 210150 | 22522 | 17520 | | 16542 |
| **Labor cost (Include OT)** | B D T | 70,758,075 | 17,025,088 | 13,390,296 | 9,664,503 | 70821600 | 6724800 | 10801176 | 22820496 | 61169400 | 2028000 |
| **Operating cost (Include Machinery Overhead)** | B D T | 35,029,143 | 8,428,356 | 6,628,934 | 4,784,461 | 35410800 | 17866148 | 5513100.25 | 11885675 | 31859062.5 | 1056250 |
| **Inventory Cost** | B D T | 3394550.217 | 631278.429 | 860823.5459 | 721600.8116 | 3565547 | 2977691 | 2467485 | 2675210 | 3065547 | 2368470 |
| **Total Demand Monthly** | pc s | 30,00,000 | 750,000 | 650,000 | 400,000 | 31,25,000 | 3,00,000 | 4,60,000 | 9,00,000 | 25,00,000 | 10,00,000 |
| **Total Supply yearly** | | As per Monthly Production | | | | | | | | | |
| **Selling Price** | $ | 5294974 | 1112414 | 1495089 | 1080608.24 | 10151096 | 1134810 | 2019369.86 | 2576789.95 | 7773611.25 | 2121615.15 |

Table 4.1: Monthly requirement of raw materials of the case industry

In Table 4.1, the monthly production of each 10 products (T-shirt, vest, sweat shirt, leggings, shirt, hoodie jacket, jogger, polo-shirt, casual shirt, girl's top) and the required monthly raw materials (fabrics and trim) for respective product/s (those are collected from the case industry of this study) including monthly selling prices are displayed. Some entries of some rows are missing because not all raw materials are required for every product. e.g. zipper is not necessary for making a T-shirt, vest, shirt etc., but it is needed for a hoodie jacket, sweat shirt etc.

### 4.3.1 Requirement of Raw Materials for One Unit

Now the monthly required materials are divided for manufacturing one unit of each product and shown in the Table 4.2, from where it can be shown that to produce 1 piece of T-shirt it requires 0.173509kg of fabric, 0.0251 cone of threads, 1.373 pcs of label, and 0.022 pcs of button and so on. Similarly, for the other products like shirts, sweatshirts, casual shirts, leggings, joggers, etc., the required raw materials are also shown in Table 4.2

| Outputs / Input | Unit | Product-1 (T Shirt) | Product-2 (Vest) | Product3 (Sweat Shirt) | Product-4 (Legging) | Product-5 (Shirt) | Product- 6 (Hoodie Jacket) | Product-7 (Jogger) | Product-8 (Polo- shirt) | Product- 9 (Casual shirt) | Product-10 (Girls' Tops) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Monthly Production** | Pcs | 2845902 | 684752 | 538560 | 388708 | 2950900 | 280200 | 450049 | 950854 | 2548725 | 845265 |
| **Fabrics** | Kg | 0.173509 kg/pc | .0833 | 0.188341 | 0.22486 | 0.2715 | .329 | .40 | .23511 | .280 | .2526 |
| **Thread** | Cone | .0251con e/pc | .01970 | .06374 | .04314 | .03 | .1151 | .080 | .02 | .0331 | .08 |
| **Labels** | Pcs | 1.3473 | 1.80817 | 2.338 | 2.577 | .8249 | 2.05 | 1.0499 | 1.11 | .8335 | .648 |
| **Zipper** | PCS | | | .008177 | | | 1.051 | | | | |
| **Button** | PCS | .02235 | .001936 | .6024 | | .0257 | | | .0522 | .02717 | .059 |
| **Polybag** | PCS | .2636 | 0.34674 | .23213 | 0.56693 | .02 | .3354 | .094 | .031 | .0258 | .301 |
| **Carton** | PCS | 0.0094736 | 0.00792 | 0.01702 | 0.034174 | .00962 | .2102 | .1312 | .0088 | .009236 | .007789 |
| **Hand tags** | PCS | 0.180719 | 0.04383 | 0.30189 | 0.34974 | .02 | .0856 | .1044 | .084 | .19715 | .01713 |
| **Rivets lace** | YDS | | 0.14369 | | | | .1905 | .2090 | | | 0.2 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Hanger** | PCS | 0.181403 | 0.3027 | 0.72697 | 0.337037 | .0212 | | | .166 | .201 | .01717 |
| **Tapes/Velcro** | YDS | .004409 | .005841 | .06016 | | | | | .0048 | | .0046 |
| **Elastics** | YDS | 0.018271 | 0.22343 | 0.25614 | 1.010028 | | 0.75 | .05001 | .0184 | | .0195 |
| **Labor cost (Include OT)** | $ | $0.28976/pc | $0.28976/pc | $0.28976/pc | $0.28976/pc | $0.28976/pc | $0.28976/pc | $0.28976/pc | $0.28976/pc | $0.28976/pc | $0.28976/pc |
| **Operating cost (Include Machinery Overhead)** | $ | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc | $0.153857 /pc |
| **Inventory Cost** | dollar | $0.014/p c | $0.0108 4 | 0.0188 | 0.02184 | 10.06 | | $.035 | $.015 | | |
| **Total Demand Monthly** | Pcs | 30,00,000 | 750,000 | 650,000 | 400,000 | 3000000 | 250000 | 450000 | 250000 | 2300000 | 850000 |
| **Total Supply Yearly** | | As per Monthly Production | | | | | | | | | |
| **Selling Price** | $ | $2.11386/pc | 1.628 | 2.7560 | 2.75108 | 3.44 | 4.05 | 4.48 | 2.71 | 2.918 | 2.55 |

Table 4.2: Monthly requirement of raw materials for producing one unit of product

### 4.3.2 Cost per kg or per Piece of Resources/Raw Materials

In Table 4.3, there is the per unit cost of each raw material (fabrics and trim) which are needed for mentioned 10 products mentioned to manufacture. Suppose the cost of 1 kg of fabric (the fabric that is used for making a T-shirt) is $6.09 or the cost of 1 piece of zipper (the zipper that is used for making a hoodie jacket) is $0.70, and so on. The per-unit cost of each of the raw materials corresponding to producing each of the products is shown in Table 4.3 below.

| Outputs / Input | Unit | Product-1 (T Shirt) | Product-2 (Vest) | Product-3 (Sweat Shirt) | Product-4 (Legging) | Product-5 (Shirt) | Product-6 (Hoodie Jacket) | Product-7 (Jogger) | Product-8 (Polo-shirt) | Product- 9 (Casual shirt) | Product- 10 (Girls' Tops) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Fabrics** | Cost$/kg | 6.09 | 7.58 | 7.08 | 6.68 | 7.01 | 7.50 | 7.20 | 6.09 | 7.00 | 6.05 |
| **Thread** | Cost$/pcs | 0.56 | 0.50 | 0.52 | 1.34 | 0.56 | 1.50 | 1.50 | .56 | .56 | .565 |
| **Labels** | Cost $/pc | 0.01 | 0.02 | 0.01 | 0.01 | .01 | 0.02 | 0.02 | .01 | .01 | .01 |
| **Zipper** | Cost $/pc | | | 0.21 | | | 0.70 | | | | |
| **Button** | Cost $/pc | 0.02 | 0.003 | 0.02 | | .021 | | | .02 | .021 | .02 |
| **Polybag** | Cost$/pcs | 0.02 | 0.04 | 0.04 | 0.02 | .02 | 0.05 | 0.05 | .02 | .02 | .02 |
| **Carton** | Cost$/pcs | 1.15 | 0.80 | 1.06 | 0.65 | 1.05 | 1.20 | 1.02 | 1.05 | 1.10 | 1.15 |
| **Hand tags** | Cost$/pcs | 0.01 | 0.03 | 0.01 | 0.01 | .01 | 0.03 | 0.04 | .01 | .01 | .01 |
| **Rivetslace** | Cost$/yds | | 0.99 | | | | 0.60 | 0.60 | | | .80 |
| **Hanger** | Cost$/pcs | 0.08 | 0.13 | 0.07 | | .07 | 0.15 | | .075 | .067 | .08 |
| **Tapes/Velcro** | Cost $/yds | 0.03 | 0.03 | 0.03 | | | | | .03 | | .032 |
| **Elastics** | Cost$/yds | 0.05 | 0.05 | 0.09 | 0.08 | | 0.05 | 0.08 | .048 | | .045 |
| **Labor hour** | Minutes/pcs | 1.5 | 2 | 3 | 1.5 | 2.00 | 3.00 | 2.50 | 1.5 | 2.00 | 1.5 |
| **Machinery hour** | Minutes/pcs | 6 | 8 | 12 | 8 | 6.5 | 12.00 | 9.00 | 6.0 | 6.25 | 6 |

Table 4.3: Per unit cost of each raw material

### 4.3.3 Cost for Producing One Unit of Product

Suppose 0.17 kg of fabrics, 0.02 cones of thread (from Table 4.2), etc. are needed for producing one piece of a T-shirt. Then the cost of each raw material that is needed for each of the products is shown in Table 4.4, i.e., the unit production costs are shown for each of the 10 products. The last three rows of the table are presenting the per-unit total cost, selling price, and profit, which are used as the parameters for formulating the deterministic model. The rest of the entries of the table be used as the cost coefficients of the constraints.

| Outputs / Input | Unit of cost | Product-1 (T Shirt) | Product-2 (Vest) | Product3 (Sweat Shirt) | Product-4 (Legging) | Product-5 (Shirt) | Product- 6 (Hoodie Jacket) | Product-7 (Jogger) | Product-8 (Polo-shirt) | Product-9 (Casual shirt) | Product-10 (Girls' Tops) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Fabrics | Cost$/pc T-Shirt | $1.05667 | 0.6317 | 1.334 | 1.5021 | 1.903 | 2.465 | 2.88 | 1.4317 | 1.86 | 1.5282 |
| Thread | Cost$/pcs | 0.01405 | 0.00985 | 0.0331 | 0.05781 | .0168 | .1726 | .12 | .448 | .01853 | .0452 |
| Labels | Cost$/pcs | 0.01347 | 0.03616 | 0.02338 | 0.02577 | .00829 | .041 | .02098 | .0111 | .00833 | .00648 |
| Zipper | Cost$/pcs | | | 0.01717 | | | .0735 | | | | |
| Button | Cost$/pcs | 0.000447 | 0.00000580 | 0.01204 | | .0053 | | | .002099 | .000569 | .00118 |
| Polybag | Cost$/pcs | 0.00527 | 0.01386 | 0.00928 | 0.011338 | .0004 | .016777 | .0047 | .00662 | .00516 | .00602 |
| Carton | Cost$/pcs | 0.01089 | .0063 | 0.01804 | 0.02221 | .0101 | .25224 | .133824 | .00924 | .01015 | .00907 |
| Hand tags | Cost$/pcs | 0.001807 | 0.00135 | 0.003008 | 0.003497 | .0002 | .002568 | .004176 | .00084 | .00197 | .00017 |
| Rivets lace | Cost$/yds | | 0.142257 | | | | .1143 | .1254 | | | .11 |
| Hanger | Cost$/pcs | 0.01451 | 0.0393 | 0.05088 | 0.05055 | .00148 | | | .01245 | .013467 | .00137 |
| Tapes/Velcro | Cost$/pcs | 0.0001322 | 0.0017503 | 0.0018048 | | | | | .000144 | | .0001472 |
| Elastics | Cost$/yds | 0.0009135 | 0.0117 | 0.02305 | 0.080802 | | .070 | .05 | .00071 | | .008771 |
| **Total cost per unit ($)** | | 1.113489 | 0.892659 | 1.5102998 | 1.754095 | 1.9408 | 3.064 | 3.2767 | 1.922 | 1.913 | 1.6066 |
| **Selling Price ($)** | | $2.11386/pc | 1.628 | 2.7560 | 2.75108 | 3.44 | 4.05 | 4.48 | 2.71 | 2.918 | 2.55 |
| **Profit per unit** | | 1 | 0.8 | 1.246902 | 1.001 | 1.5 | 0.99 | 1.21 | 0.79 | 1.005 | 0.95 |

Table 4.4: Cost for manufacturing one unit of each product

### 4.3.4 Monthly Cost of Raw Materials for 10 Products

The monthly total cost for manufacturing each of the 10 products is shown in Table 4.5 below. Each column of the 1st row shows the total monthly cost of fabrics for producing the respective products $(x_1, x_2, \dots, x_{10})$ and the last column shows the total monthly cost of only fabrics for

making all the products. Similarly, the rest of the rows indicate all the raw materials' monthly costs one by one. And the last columns of each row show the monthly total cost of all raw materials that are used to manufacture the 10 products. These values are considered as the monthly supply or the available resources of the developed model that is the righthand sides of the constraints are assumed from these values.

| Outputs / Input | Unit of cost | Product-1 (T Shirt) | Product-2 (Vest) | Product3 (Sweat Shirt) | Product-4 (Legging) | Product-5 (Shirt) | Product- 6 (Hoodie Jacket) | Product-7 (Jogger) | Product-8 (Polo- shirt) | Product-9 (Casual shirt) | Product- 10 (Girls' Tops) | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Fabrics | Cost in dollar | 3000000 | 400000 | 718000 | 583878 | 5518700 | 690200 | 1296000 | 1361337 | 4740628 | 1291564 | 19555476.97 |
| Thread | | 4000 | 6744 | 178269 | 22471 | 48720 | 48420 | 54000 | 13311.95 | 47151.41 | 38205 | 629812.5 |
| Labels | | 38135 | 24760 | 12591 | 10017 | 24462 | 11488 | 9442 | 10554 | 21230 | 5477 | 168156 |
| Zipper | | | | 9532 | | | 19637 | | | | | 29169 |
| Button | | 1337 | 9039 | 6484 | | 2060.15 | | | 1995 | 14502 | 997 | 36414 |
| Polybag | | 14997 | 9490 | 4997 | 4407 | 11803 | 4698 | 18001 | 6294.65 | 13151 | 5088 | 88926.65 |
| Carton | | 30991 | 4331 | 9715 | 8633 | 29801 | 70677 | 60216 | 8785.89 | 25869.55 | 7666.55 | 256685.9 |
| Hand tags | | 5142.54 | 924.41 | 1619.98 | 1359.31 | 5901.8 | 719.55 | 1876.70 | 798.71 | 5020.98 | 143.69 | 23507.67 |
| Rivets lace | | | 97410.76 | | | | 32026.86 | 56436.14 | | | .92979.15 | 278852.15 |
| Hanger | | 41294 | 26910.75 | 27401.93 | 19649.18 | 161709 | | | 11838.13 | 34323.07 | 1158.013 | 324284.07 |
| Tapes/ Velcro | | 396.6 | 1198.22 | 1295.84 | | | | | 196.0325 | | 190.11822 | 3320.69 |
| Elastics | | 2599.73 | 8011.59 | 12413.83 | 31408.38 | | 19614 | 22502 | 675.106 | | 7413.819 | 104638.455 |
| Labor hour | in minutes | 4268853 | 1369504 | 1615680 | 583062 | 5901800 | 840600 | 11251.22 | 1426281 | 5097450 | 1267897.5 | 22382387.12 |
| Machine hour | in minutes | 17075412 | 5478016 | 6462720 | 3109664 | 19180850 | 3362400 | 4050441 | 5705124 | 15929531.25 | 5071590 | 108761317.3 |

Table 4.5: Monthly total cost of each of the raw materials

## 4.4 Development of the Model

There is a restriction on producing three ($x_4, x_5, and\ x_{10}$) of the products in this case industry. So, there would be offered fixed production for those three variables. By using the data (per unit cost of each raw material, inventory, labor and operating cost) associated to specific variable/s, which

are found from the above 5 tables, a deterministic linear programming model is developed, which is published in a Scopus-indexed Q4-ranked Journal (Suraiya and Hasan, 2023)

The variables and parameters those are used in formulating the model are:

***decision variables*:**

$x_j$ = *monthly production of jth product* $(j = 1, 2, 3, \dots, 10)$

$x_1$ = *amount of T − shirt should be produced monthly*

$x_2$ = *amount of vest should be produced monthly*

$x_3$ = *amount of sweat − shirt should be produced monthly*

$x_4$ = *amount of legging should be produced monthly*

$x_5$ = *amount of shirt should be produced monthly*

$x_6$ = *amount of hoodie − jacket should be produced monthly*

$x_7$ = *amount of jogger should be produced monthly*

$x_8$ = *amount of polo − shirt should be produced monthly*

$x_9$ = *amount of casual shirt should be produced monthly*

$x_{10}$ = *amount of girls' top should be produced monthly*

***parameters*:**

$f_j$ = *monthly required fabrics to produce one unit of jth product* $(j = 1, 2, 3, \dots, 10)$

$t_j$ = *monthly required threads to produce one unit of jth product* $(j = 1, 2, 3, \dots, 10)$

$l_j$ = *monthly required labels to produce one unit of jth product* $(j = 1, 2, 3, \dots, 10)$

$z_j$ = *monthly required zipper to produce one unit of jth product* $(j = 1, 2, 3, \dots, 10)$

$b_j$ = *monthly required button to produce one unit of jth product* $(j = 1, 2, 3, \dots, 10)$

$p_j$ = *monthly required polybag to produce one unit of jth product* $(j = 1, 2, 3, \dots, 10)$

$cr_j$ = $monthly\ required\ carton\ to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

$ht_j$ = $monthly\ required\ hand\ tags\ to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

$r_j$ = $monthly\ required\ rivets\ lace\ to\ produce\ one\ unit\ of\ jth\ product$ ($j$ = 1,2,3, … ,10)

$h_j$ = $monthly\ required\ hangers\ to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

$t_j$ = $monthly\ required\ tapes\ to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

$e_j$ = $monthly\ required\ elastics\ to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

$lh_j$ = $monthly\ required\ labor\ hour\ to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

$mh = monthly\ required\ machine\ hour\ to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

lc = $monthly\ required$ labor cost $to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

oc = $monthly\ required$ operating cost $to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

ic = $monthly\ required$ inventory cost $to\ produce\ one\ unit\ of\ jth\ product\ (j = 1, 2, 3, \ldots, 10)$

Thus, the general form of the LP model is

$$Max\ z = \sum_{j=1}^{10} C_j^{\ T} x_j$$

*Subject to*

$$\sum_{j=1}^{10} A_{ij} x_j \leq b_i\ ,\ \ where\ i = 1,\ 2,\ 3, \ldots, 20$$

$$x_j \geq 0$$

Also, the details of all the constraints are as follows

$$\sum_{j=1}^{10} f_j x_j \leq b_1\ \ total\ fabrics$$

This constraint describes the total monthly requirement of fabrics for producing all 10 types of products and the right side indicates the monthly available fabrics or the monthly budget of fabrics.

$$\sum_{j=1}^{10} t_j x_j \leq b_2 \quad total\ thread$$

This constraint describes the total monthly requirement of threads for producing all 10 types of products and the right side indicates the monthly available threads or the monthly budget of threads.

$$\sum_{j=1}^{10} i_j x_j \leq b_3 \quad total\ labels$$

This constraint describes the total monthly requirement of labels for producing all 10 types of products and the right side indicates the monthly available labels or the monthly budget of labels.

$$\sum_{j=1}^{10} z_j x_j \leq b_4 \quad total\ zipper$$

This constraint describes the total monthly requirement of zippers of producing all 10 types of products, and the right side indicates the monthly available zippers or the monthly budget of zippers.

$$\sum_{j=1}^{10} b_j x_j \leq b_5 \quad total\ button$$

This constraint describes the total monthly requirement of buttons for producing all 10 types of products, and the right side indicates the monthly available buttons or the monthly budget of buttons.

$$\sum_{j=1}^{10} p_j x_j \leq b_6 \quad total\ polybag$$

This constraint describes the total monthly requirement of polybags for producing all 10 types of products, and the right side indicates the monthly available polybags or the monthly budget of polybags.

$$\sum_{j=1}^{10} cr_j x_j \leq b_7 \quad total\ carton$$

This constraint describes the total monthly requirement of cartons of producing all 10 types of products, and the right side indicates the monthly available cartons or the monthly budget of cartons.

$$\sum_{j=1}^{10} ht_j x_j \leq b_8 \quad total\ handtags$$

This constraint describes the total monthly requirement of hand tags of producing all 10 types of products, and the right side indicates the monthly available hand tags or the monthly budget of hand tags.

$$\sum_{j=1}^{10} rl_j x_j \leq b_9 \quad total\ rivet\ lace$$

This constraint describes the total monthly requirement of rivet lace of producing all 10 types of products and the right side indicates the monthly availability of rivet lace or the monthly budget of rivet lace.

$$\sum_{j=1}^{10} h_j x_j \leq b_{10} \quad total\ hanger$$

This constraint describes the total monthly requirement of hangers of producing all 10 types of products and the right side indicates the monthly availability of hanger or the monthly budget of hangers

$$\sum_{j=1}^{10} t_j x_j \leq b_{11} \quad total\ tapes/velcro$$

This constraint describes the total monthly requirement of tapes/velcro of producing all 10 types of products and the right side indicates the monthly availability of tapes/velcro or the monthly budget of tapes/Velcro.

$$\sum_{j=1}^{10} e_j x_j \leq b_{12} \quad total\ elastic$$

This constraint describes the total monthly requirement of elastic of producing all 10 types of products and the right side indicates the monthly available elastic or the monthly budget of elastic.

$$\sum_{j=1}^{10} lh_j x_j \leq b_{13} \quad total\ labour\ hour$$

This constraint describes the total monthly requirement of labor hour of producing all 10 types of products and the right side indicates the monthly availability of labor hour.

$$\sum_{j=1}^{10} mh_j x_j \leq b_{14} \quad total\ machine\ hour$$

This constraint describes the total monthly requirement of machine hour of producing all 10 types of products and the right side indicates the monthly available of machine hour.

$$\sum_{j=1}^{10} lc_j x_j \leq b_{15} \quad total\ labor\ cost$$

This constraint describes the total monthly requirement of labor cost of producing all 10 types of products and the right side indicates the monthly available budget of labor cost.

$$\sum_{j=1}^{10} oc_j x_j \leq b_{16} \quad total\ operating\ cost\ (including\ machinery)$$

This constraint describes the total monthly requirement of operating costs (including machinery) of producing all 10 types of products and the right side indicates the monthly available budget of total operating cost (including machinery).

$$\sum_{j=1}^{10} i_j x_j \leq b_{17} \quad total\ inventory\ cost$$

This constraint describes the total monthly requirement of inventory cost of producing all 10 types of products and the right side indicates the monthly available budget of inventory cost.

The collected data obtained from the case industry are presented in matrix-vector notation as follows.

$$c = \begin{pmatrix} 1 \\ 0.8 \\ 1.2469 \\ 1.001 \\ 1.5 \\ 0.99 \\ 1.21 \\ 0.79 \\ 1.005 \\ 0.95 \end{pmatrix}$$

$$A = \begin{pmatrix} 1.056 & .63 & 1.334 & 1.5 & 1.9 & 2.46 & 2.88 & 1.43 & 1.96 & 1.52 \\ .014 & .009 & .033 & .057 & .016 & .172 & .12 & .49 & .018 & .045 \\ .013 & .036 & .023 & .025 & .0082 & .04 & .02 & .011 & .008 & .006 \\ 0 & 0 & .0017 & 0 & 0 & .00182 & 0 & 0 & 0 & 0 \\ .0004 & .000005 & .012 & 0 & .005 & 0 & 0 & .002 & .0056 & .001 \\ .005 & .013 & .009 & .011 & .0004 & .0167 & .004 & .0008 & .0019 & .0001 \\ .01 & .006 & .018 & .022 & .01 & .25 & .133 & .009 & .01 & .009 \\ .001 & .0013 & .003 & .003 & .0002 & .0025 & .0041 & .0008 & .0019 & .00017 \\ 0 & .14 & 0 & 0 & 0 & .11 & .125 & 0 & 0 & 0 \\ .0145 & .039 & .05 & .05 & .001 & 0 & 0 & .012 & .013 & .0013 \\ .0001 & .0001 & .0018 & 0 & 0 & 0 & 0 & .0001 & 0 & .008 \\ .0009 & .011 & .02 & .08 & .0007 & .07 & .05 & .0007 & 0 & .008 \\ 1.5 & 2 & 3 & 1.5 & 3.5 & 3 & 3 & 1.5 & 2.5 & 1.6 \\ 6 & 8 & 12 & 8 & 10 & 12 & 9 & 6 & 6 & 5.5 \\ .153 & .153 & .153 & .153 & .153 & .153 & .153 & .153 & .153 & .153 \\ .289 & .289 & .289 & .289 & .289 & .289 & .289 & .289 & .289 & .289 \\ .014 & .01 & .01 & .018 & .02 & .2 & .1 & .02 & .03 & .015 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{pmatrix} \text{ and } b = \begin{pmatrix} 20000000 \\ 300000 \\ 220000 \\ 29000 \\ 35000 \\ 100000 \\ 90000 \\ 23000 \\ 50000 \\ 237000 \\ 35000 \\ 100000 \\ 25000000 \\ 6000500 \\ 950000 \\ 1700000 \\ 5500000 \\ 5000 \\ 8500 \\ 9000 \end{pmatrix}$$

…… (4.1)

Thus, the algebraic form of the deterministic model is

$$\textbf{\textit{Maximize}}\ z = x_1 + 0.8x_2 + 1.2469x_3 + 1.001x_4 + 1.5x_5 + 0.99x_6 + 1.21x_7 + 0.79x_8 + 1.005x_9 + 0.95x_{10}$$

***Subject to***

$$1.05667x_1 + 0.6317x_2 + 1.334x_3 + 1.5021x_4 + 1.903x_5 + 2.465x_6 + 2.88x_7 + 1.4317x_8 + 1.96x_9 + 1.528x_{10} \leq 20000000$$

$$0.01405x_1 + .00985x_2 + .0331x_3 + .05781x_4 + 0.0168x_5 + .1726x_6 + .12x_7 + .448x_8 + 0.01853x_9 + 0.0452x_{10} \leq 300000$$

$$0.013470x_1 + .03616x_2 + 0.02338x_3 + .02577x_4 + 0.00829x_5 + 0.041x_6 + .02098x_7 + 0.0111x_8 + 0.00833x_9 + 0.00648x_{10} \leq 220000$$

$$.001717x_3 + .00182x6 \leq 29000$$

$$.00044x_1 + .00000580x_2 + .01204x_3 + 0.0053x_5 + 0.00209x_8 + .00569x_9 + .00118x_{10} \leq 35000$$

$$.00527x_1 + .01386x_2 + .00928x_3 + .011338x_4 + 0.0004x_5 + .01677x_6 + .0047x_7 + .00662x_8 + .00516x_9 + .006x_{10} \leq 100000$$

$$.01089x_1 + .0063x_2 + .01804x_3 + .02221x_4 + 0.0101x_5 + .25224x_6 + .133824x_7 + 0.00924x_8 + .01015x_9 + .009077x_{10} \leq 90000$$

$$.001807x_1 + .00135x_2 + .003008x_3 + .003497x_4 + 0.0002x_5 + .002568x_6 + .004176x_7 + .00084x_8 + .00197x_9 + .000117x_{10} \leq 23000$$

$$0.142257x2 + .1143x6 + 0.1254x_7 + .11x_{10} \leq 50000$$

$$.0145x_1 + .0393x_2 + .05088x_3 + .05055x_4 + 0.00148x_5 + .01245x_8 + .013467x_9 + .00137x_{10} \leq 237000$$

$$.0001322x_1 + .000175x_2 + .0018048x_3 + .000144x_8 + .0001472x_{10} \leq 35000$$

$$.0009135x_1 + .01117x_2 + .02305x_3 + .080802x_4 + 0.0007x_5 + .07x_6 + 0.05x_7 + .0007x_8 + .008771x_{10} \leq 100000$$

$$1.5x_1 + 2x_2 + 3x_3 + 1.5x_4 + 3.5x_5 + 3x_6 + 3x_7 + 1.5x_8 + 2.5x_9 + 1.6x_{10} \leq 25000000$$

$$6x_1 + 8x_2 + 12x_3 + 8x_4 + 10x_5 + 12x_6 + 9x_7 + 6x_8 + 6x_9 + 5.5x_{10} \leq 60000500$$

$$.1538x_1 + .1538x_2 + .1538x_3 + .1538x_4 + .1538x_5 + .1538x_6 + .1538x_7 + .1538x_8 + .1538x_9 + .1538x_{10} \leq 950000$$

$$.28976x_1 + .28976x_2 + .28976x_3 + .28976x_4 + 0.5x_5 + 0.3x_6 + 0.2x_7 + 0.3x_8 + 0.27x_9 + .28x_{10} \leq 1700000$$

$$.014x_1 + 0.0108x_2 + .0188x_3 + .0218x_4 + 0.2x_5 + .1x_6 + .02x_7 + .03x_8 + .015x_9 + .011x_{10} \leq 5500000$$

$$x_4 \geq 5000$$
$$x_5 \geq 8500$$
$$x_{10} \geq 9000$$

…… (4.2)

## 4.5 Output of the Model after Solving by LINDO

The output obtained by LINDO is displayed in Table 4.6

| **Objective function's value (optimal profit in dollar)** | **Variables (Products are represented by these variables)** | **Solution (production in pieces)** |
|---|---|---|
| **6539290** | $x_1(T-shirt)$ | 3014303 |
| | $x_2\ (Vest)$ | 000 |
| | $x_3\ (Sweat\ shirt)$ | 2790443 |
| | $x_4\ (Leggings)$ | 5000 |
| | $x_5(Shirt)$ | 8500 |

| | | |
|---|---|---|
| | $x_6(Hoodie\ Jacket)$ | 000 |
| | $x_7\ (Jogger)$ | 48990 |
| | $x_8\ (Polo\ shirt)$ | 000 |
| | $x_9\ (Casual\ shirt)$ | 000 |
| | $x_{10}\ (Girl's\ top)$ | 9000 |

Table 4.6: Solution of DLP model

Table 4.6 describes that after solving the LP model by LINDO (a mathematical tool), it is found that if the considered garment industry produces 6 types of products instead of 10, it would be more optimal for them. After the data collection, it was found that they had a monthly production schedule of T-shirts 2845902 pcs, vests 684752 pcs, sweatshirts 538560 pcs, leggings 388708 pcs, shirts 2950900 pcs, hoodie jackets 280200 pcs, joggers 450049 pcs, polo shirts 950854 pcs, casual shirts 2548725 and girls' tops 845265 pcs. But by analyzing their available resources it is found that if the factory produces 3014303 pcs of T-shirt, 279044 3pcs of sweat shirts, 5000 pcs of leggings, 8500 pcs of shirts, 48990 pcs of jogger, and 9000 pcs of girls' tops (since a fixed schedule is offered for three products due to customer's need) it would be more optimal in terms of their profit and the total profit found by solving this model is $6539290. In this optimal solution, there is no production of vests, jackets, polo shirts, or casual shirts. But this is not feasible in the sense of survival. Every type of product has some demand in the market and has to be produced in any such industry; its quantity may be less or more. To overcome this issue, a model will be offered where the lower and upper bound is set as per the market demand at the end of the Chapter 5. Besides any kind of uncertainty, is not considered here.

## 4.6 Conclusion

This chapter concludes with an LP model for offering optimal production planning of the RMG industry in deterministic circumstances. Now, by considering uncertainty in costing, pricing, demand, supply, etc., under different economic, weather, and seasonal conditions, some stochastic programming models are formulated in the next chapter of this study.

# Chapter 05

# Stochastic Optimal Production Planning Models for RMG Industry

# Chapter 05

# Stochastic Optimal Production Planning Models for RMG Industry

## 5.1 Introduction

The economic, political, and natural situations of any country are not always steady. Any unrest or disaster may occur at any time. For some political unrest or any natural calamities, the economic condition is disrupted first. 5 years ago, we were affected by COVID-19 globally. Worldwide uncertainty resulted from this pandemic's disruption of supply lines, economies, and healthcare systems. Businesses shut down, governments enforced lockdowns, and millions of people experienced job loss or a radical disruption in their daily routines. The entire world literally got stuck for 6 to 7 months. It takes about 2 years to become normal. The next issue was the war of Russia-Ukraine. International instability has been made worse by this war, which started in 2022. Energy shortages, heightened geopolitical tensions, and a humanitarian disaster have resulted from it. The consequences of sanctions, changes in energy prices, and population dislocation are being felt in many nations. Collectively, these occurrences have produced an unpredictable environment that affects everything, from personal lives to the state of the world economy.

In previous chapter, the developed LP model considers the deterministic situation only, but there is a natural degree of uncertainty in many real-world systems, including financial markets, biological processes, and weather patterns that deterministic models are unable to capture. Assuming perfect knowledge and no randomness, a deterministic model produces a fixed output for a given set of initial conditions. But in practice, things like weather disruptions, genetic mutations, and market swings are frequently erratic and unpredictable. Arises from the necessity of taking into consideration unpredictability, variability, and randomness in real-world systems in uncertain situations. This study's primary motivation is to take these uncertainties into account.

The need to take into consideration supply fluctuations, demand uncertainty, and other uncontrollable factors that affect the RMG industry shows the necessity of practicing a stochastic model rather than a deterministic one in the context of production planning for the ready-made

garment (RMG) sector. Seasonal changes in demand, market volatility, consumer preferences, and global supply chain disruptions make the apparel industry especially vulnerable to these uncertainties.

In this chapter, five circumstances are studied by converting the LP model to the general stochastic models under various economic situations. In these situations, fluctuations in the cost of different raw materials, labor, operating, and inventory expenses, as well as changes of selling prices, are taken into account. Since it is impossible to assume which scenario will occur, a different model is developed by merging all of the scenarios in the next section. In next a two-stage stochastic (TSLP) is also formulated with WS, EEV, RP and EVPI-VSS calculations. Finally a stochastic model that can be aligned with real life will also be presented with AMPL solution.

## 5.2 Uncertainties in the RMG Industry and Motivation for the Stochastic Model

Uncertainty can arise in various states. In the garments sector, uncertainty can also be seen in different ways. A few of them are discussed here. And also, there is a discussion on how elegantly a stochastic model minimizes the uncertainties in comparison to a deterministic model.

### 5.2.1 Demand Uncertainty

The demand for products in the apparel industry is frequently very erratic and fluctuates because of things like:

- **Trends in fashion:** Unexpected increases or decreases in demand may result from shifts in customer preferences.
- **Seasonal variations:** Demand trends vary from one season to the next (e.g., winter vs. summer collections).
- **Market conditions:** Demand volatility may be brought on by shifting consumer behavior, geopolitical factors, or uncertain economic conditions.

Overproduction or underproduction would result from a deterministic model's assumption that demand is steady or predictable. A stochastic model, on the other hand, can better plan for a variety of outcomes by modeling various potential demand scenarios to capture the randomness and variability in demand.

### 5.2.2 Uncertainty in Inventory Management

Effective inventory management is essential in the RMG sector since stockouts result in lost sales and excess inventory raises storage expenses. By taking into account the possibilities of demand spikes and declines, stochastic models can optimize the reordering procedure and safety stock levels.

### 5.2.3 Uncertainty in Adaptive and Flexible Decision-Making

The capacity to modify production plans dynamically is essential in an RMG production setting. Deterministic models are less adaptable to changing conditions because they only offer a fixed solution. On the other hand, stochastic models enable decision-makers to make real-time adjustments to production schedules, resource allocation, and inventory levels while investigating a variety of possible outcomes. This flexibility is essential for maintaining competitiveness in a market where conditions and demand trends are ever-evolving. A stochastic model can assist the production planner in determining the probability of a sudden spike in demand for a particular style of clothing as a result of a viral fashion trend. The planner can then modify orders, production volumes, and labor shifts appropriately.

### 5.2.4 Uncertainty in Risk Management and Cost Minimization

Stochastic models enable the RMG industry to better model and manage risk by introducing randomness. Under ideal circumstances, a deterministic model might reduce costs, but it ignores the possibility of either underproduction (resulting in stockouts) or overproduction (resulting in excess inventory). However, a stochastic model considers the chance of various demand scenarios, allowing the business to make choices that reduce overall risk and expenses. Even in situations where demand is uncertain, stochastic optimization can be used to determine the best production schedules and quantities to reduce the cost of inventory holding (from overproduction) and lost sales (from stockouts).

### 5.2.5 Uncertainty in Forecasting and Planning

Accurate forecasting is crucial for efficient production planning in the RMG sector, but demand projections are inherently erratic because of the previously listed factors. Stochastic models account for forecast errors and present a range of possible future outcomes, whereas deterministic

models assume a fixed or predicted demand. Stochastic models are therefore perfect for scenario planning.

### 5.2.6 Uncertainty in Optimization of the Product Mix

The RMG sector frequently works with several product lines, each of which has distinct demand trends. Although demand for various products varies independently in practice, a deterministic model might assume that demand for each product is known with certainty. By taking into account the variation and correlation in demand across various product lines, stochastic models enable the product mix to be optimized. For example, to minimize the possibility of overproducing low-demand items and underproducing high-demand ones, stochastic models can assist in determining the best way to allocate resources (labor, machinery, and raw materials) to various product lines based on their anticipated demand distributions.

Thus, the stochastic model is a potent tool for managing the uncertainty and variability inherent in demand forecasting, supply chain management, inventory control, and risk management in the context of production planning for the RMG industry. Stochastic approaches allow for more robust and adaptive decision-making by modeling probabilities and randomness. This ensures that production plans are more in line with real-world fluctuations, which lowers costs, increases customer satisfaction, and boosts overall efficiency. On the other hand, in a highly unpredictable market, a deterministic model, with its assumptions of certainty, would probably result in less-than-ideal choices.

## 5.3 Development of Stochastic Models with all Uncertain Parameters

In this section, the deterministic LP (developed in Chapter 4) model is converted to general stochastic models where all the parameters associated to all ten variables are considered as uncertain.

### 5.3.1 Background of Formulation

The deterministic model presented in Chapter 4 is

$$Max\ z = \sum_{j=1}^{10} C_j x_j$$

$$Subject\ to \quad \sum_{j=1}^{10} A_{ij} x_j \leq b_i\ ,\ \ where\ i = 1,\ 2,\ 3, \ldots .20$$

$$x_j \geq 0$$

The objective function of this model was on profit maximization, and the coefficient matrix of the objective function is $C_j's$ (per unit profit) and the constraints were on the cost of various raw materials, labor costs, operating costs, inventory costs, etc., which are used for producing each type of product. A is the coefficient matrix of the constraints. The right sides $(b_i)$ of the constraints were on the available resources of the cost of raw materials, labor, operating, and inventory costs, labor hour, machine hour etc. Now, considering all of these discussed parameters as uncertain under different economic conditions, five separate stochastic models are formulated in the next section. And the rest of the sections consist of formulating the model of combining all the scenarios, another two-stage stochastic model with selected uncertain variables (not all variables), and WS, RP, EEV calculations are also discussed as well as the EVPI and VSS solutions.

### 5.3.2 Scenario Representation with all Uncertain $Cj's\ and\ A$

The cost of each raw material may fluctuate at any time due to unforeseen circumstances. Considering these uncertain conditions of the economy, five different scenarios are measured in this study, and all $C_j's$ (the coefficient matrix of the objective function) $and\ A$ (the coefficient matrix of the constraints) are considered to be unstable here, as all of these are cost-related parameters, which are very much uncertain in this world.

Taking fluctuated economic conditions under consideration, five different scenarios can arise:

**Scenario 1: In a Favorable Economic Conditions**

The cost of each raw material associated with all products, including labor, operating and inventory costs, could be decreased by 20%, and the selling price is fixed.

**Scenario 2: In Unfavorable Economic Conditions**

The cost of each raw material related to every product, including labor, operating and inventory costs, could be increased by 20% when the price is fixed.

**Scenario 3: In a Stable Economic condition**

The cost of each raw material, including labor, operating, and inventory costs, would remain the same, and the price is fixed.

**Scenario 4: Selling Prices can be increased by 20%**

Under fluctuated economic conditions, if any such situation arises that the respective company would increase the selling prices of each product by 20% despite of the stable cost of all raw materials.

**Scenario 5: Selling Prices can be decreased by 20%**

Under fluctuated economic conditions if any such situation arises that the respective company would decrease the selling prices of each product by 20% and the costs of all items remain stable.

### 5.3.3 Models through the Five Scenarios with Outcomes

**Scenario 1: In a favorable economic condition**

The cost of each raw material associated with all products, including labor, operating, and inventory costs, could be decreased by 20% and the selling prices are fixed. So the general form of the model is,

$$\boldsymbol{maximize\ Z} = \sum_{j=1}^{10}(C_{1_j} - 0.80 * C_{2_j}) * x_j$$

*Subject to*

$$\sum_{j=1}^{10}\left(0.80 * A_{ij}\right) * x_j \leq b_i\ ,\ \ where\ i = 1,\ 2,\ 3, \ldots .20$$

$$x_j \geq 0$$

where

$C_{1_j}$= Selling price of $jth$ product

$C_{2_j}$ = Costing of $jth$ product

$x_j$ = Monthly production of $jth$ product

$A_{ij}$ = Cost coefficients of $ith$ raw materials of making $jth$ product

$b_i$ = Available resources of $ith$ raw materials

And the values of the $C_{1j}$ *and* $C_{2j}$, collected from case industry and used in formulating deterministic model (Chapter 4), are is as follows:

| $C1$: | $C2:=$ |
|---|---|
| 1 2.41 | 1 1.4079 |
| 2 1.928 | 2 1.186 |
| 3 2.956 | 3 1.81 |
| 4 2.97108 | 4 2.015 |
| 5 3.64 | 5 2.24669 |
| 6 5.15 | 6 4.1409 |
| 7 4.48 | 7 3.6178 |
| 8 2.71 | 8 1.784 |
| 9 3.018 | 9 2.3025 |
| 10 2.67; | 10 1.997; |

Table 5.1: The costing and selling prices of each of the 10 products

When the costs of each raw material, including labor, operating, and inventory costs, are decreased by 20% under better economic conditions, then the optimal profit is certainly increased, and it is $10245100. And the production schedule for 10 different products is:

$x_1 = 3292610$
$x_2 = 0$
$x_3 = 3508770$
$x_4 = 497996$
$x_5 = 8500$
$x_6 = 0$
$x_7 = 15821.4$
$x_8 = 0$
$x_9 = 0$
$x_{10} = 9000$

**Scenario 2: In an Unfavorable Economic Condition**

The cost of each raw material related to every product, including labor, operating and inventory costs could be increased by 20%, and there is no change of the prices. Thus, the model is:

$$\boldsymbol{maximize\ Z} = \sum_{j=1}^{10} (C_{1_j} - 1.20 * C_{2_j}) * x_j$$

*Subject to*

$$\sum_{j=1}^{10} (1.20 * A_{ij}) * x_j \leq b_i \ , \ \ where\ i = 1,\ 2,\ 3, \ldots .20$$

$$x_j \geq 0$$

$C_{1_j}, C_{2_j}, A_{ij}\ etc.$ are defined as before

In the case of costs of each raw material, including labor, operating and inventory costs are increased by 20% under the worst economic condition, then the optimal profit is undoubtedly decreased with $z = \$\ 3663110$. And the production schedule for 10 different products is:

$x_1 = 2535540$
$x_2 = 0$
$x_3 = 2325200$
$x_4 = 5000$
$x_5 = 8500$
$x_6 = 0$
$x_7 = 0$
$x_8 = 0$
$x_9 = 0$
$x_{10} = 9000$

**Scenario 3: In a stable economic condition**

The cost of each raw material, including labor, operating, and inventory costs, would remain the same and the price is fixed.

$$\boldsymbol{maximize\ Z} = \sum_{j=1}^{10} (C_{1_j} - C_{2_j}) * x_j$$

*Subject to*

$$\sum_{j=1}^{10} (A_{ij}) x_j \leq b_i \ , \ \ where\ i = 1,\ 2,\ 3, \ldots, 20 \qquad\qquad x_j \geq 0$$

At the time when the costs of each raw material, including labor, operating and inventory costs, remain the same as it is found at the time of survey under steady economic conditions, then the optimal profit remains the same as it was found by the deterministic LP model with $z = \$\ 6539640$. And the production schedule for 10 different products is:

$x_1 = 3012560$
$x_2 = 0$
$x_3 = 2792260$
$x_4 = 5000$
$x_5 = 8500$
$x_6 = 0$
$x_7 = 48887.3$
$x_8 = 0$
$x_9 = 0$
$x_{10} = 9000$

**Scenario 4: The Selling Price of each Product could be increased by 20%**

If such a situation arises that the organization declares the increase of the selling prices by 20% while the costs of each raw material, including labor, operating and inventory costs remain the same, the offered stochastic model in this circumstance is:

$$\boldsymbol{maximize\ Z} = \sum_{j=1}^{10} (1.20 * C_{1j} - C_{2j}) * x_j$$

*Subject to*

$$\sum_{j=1}^{10} (A_{ij}) * x_j \leq b_i\ ,\ \ where\ i = 1,\ 2,\ 3, \ldots .20$$

$$x_j \geq 0$$

$C_{1j}$, $C_{2j}$, $A_{ij}$ etc. are defined as before

Then the optimal profit will be in this case with $z = \$\,9447370$. And the production schedule for all the 10 different products is:

$x_1 = 2630120$
$x_2 = 0$
$x_3 = 2806240$
$x_4 = 398339$
$x_5 = 8500$
$x_6 = 0$
$x_7 = 12844.1$
$x_8 = 0$
$x_9 = 0$
$x_{10} = 9000$

**Scenario 5: The Selling Price of each Product could be decreased by 20%**

If such a situation arises that the organization declares a decrease of selling prices by 20% while the costs of each raw material, including labor, operating and inventory costs remain the same, the offered stochastic model in this situation is,

$$\boldsymbol{maximize\ Z} = \sum_{j=1}^{10} (0.80 * C_{1_j} - C_{2_j}) * x_j$$

*Subject to*

$$\sum_{j=1}^{10} (A_{ij}) * x_j \leq b_i \ , \ \ where\ i = 1,\ 2,\ 3, \ldots .20$$

$$x_j \geq 0$$

In case of costs of each raw material, including labor, operating and inventory costs remain stable and the selling prices of all products would be decreased by 20%, then the optimal profit is decreased with $z = \$\,3142200$. And the production schedule for 10 different products is:

$x_1 = 3047580$
$x_2 = 0$
$x_3 = 2790980$
$x_4 = 5000$
$x_5 = 8500$
$x_6 = 0$
$x_7 = 0$
$x_8 = 0$
$x_9 = 0$
$x_{10} = 9000$

Thus, the five distinct stochastic models on five different scenarios are developed and the outcomes obvious with respect to in favorable, unfavorable, and stable economic conditions. These types of models can be grabbed by any RMG factory when any such situation arises. And by applying these models they would be benefitted by getting an optimal production planning in every distinct situation. Next the model combining all the scenarios will be formulated in the coming section.

## 5.4 General Model of the Stochastic Program: Combining all Scenarios

It is realized that it's quite difficult to make a perfect decision for an RMG factory that would be best in all circumstances; it would be better then if all the scenarios were combined. So, a

combination of all five scenarios can be measured, which are equally probable. Here, the three scenarios are considered based on only the fluctuation of costs. Variation in selling price is not considered here in the combined case because selecting the selling price/s is in the hands of any organization. It depends on the organization's individual decision, not on actual uncertainty. That's why the distinction of selling prices of any particular product is not considered in this combined case, while the costs of all the raw materials, labor, operating cost etc. connected to all of the 10 products are considered as fluctuating across the three different scenarios under various uncertain conditions like weather demand, seasonal demand, and of course, fluctuated economic conditions with equal probability of $1/3$.

Assuming the organization wants to maximize the long-run profit, it is reasonable for them to seek a solution that maximizes their expected profit. This assumption means that they are neutral about risk. It is useful to index those decisions by a scenario index $s = 1, 2, 3$ corresponding to a favorable situation, stable/same, or an unfavorable situation, respectively. This creates a new set of variables of the form $x_{is}$, $i = 1, 2, 3, 4, \ldots, 10$ $and$ $s = 1, 2, 3$. And as an example, $x_{32}$ represents the amount of produced $(x_3)$ sweatshirts under the stable economic condition and so on. The model reads as follows. Here, all the parameters related to 10 variables are considered as uncertain.

$$\boldsymbol{maximize\ Z} = \frac{1}{3}\sum_{k=1}^{3}\sum_{j=1}^{10}(C_{1\,jk} - (D_{1k} * C_{2\,jk})) * x_{jk}$$

*Subject to*

$$\sum_{k=1}^{3}\sum_{j=1}^{10}\left(D_{1k} * A_{ij}\right) * x_{jk} \leq b_i, \ \ where\ i = 1,\ 2,\ 3, \ldots .60$$

$$\sum_{k=1}^{3}\sum_{j=1}^{10} x_{jk} \geq Demand_{jk} \quad (when\ demand\ of\ any\ product\ would\ be\ fixed)$$

$$x_{jk} \geq 0$$

where

$C_{1\,jk}$= Selling price of $jth$ product in the scenario $k$
$C_{2\,jk}$ = Costing of $jth$ product in the scenario $k$

$A_{ij}$ = Cost coefficients of $ith$ raw materials for making $jth$ product
$x_{jk}$ = Monthly production of $jth$ product at $kth$ scenario
$b_i$ = Available resources of $ith$ raw materials
$D_{1k}$ = Scenario indicating parameter
$D_{11} = 0.8; \quad D_{12} = 1; \quad D_{13} = 1.20$
$Demand_{jk}$ = Demand of $jth$ product at $kth$ scenario

From the collected data the costing and selling prices are presented in the Table 5.2,

| $C1$: | 1 | 2 | 3: = |
|---|---|---|---|
| 1 | 2.41 | 2.41 | 2.41 |
| 2 | 1.928 | 1.928 | 1.928 |
| 3 | 2.956 | 2.956 | 2.956 |
| 4 | 2.97108 | 2.97108 | 2.97108 |
| 5 | 3.64 | 3.64 | 3.64 |
| 6 | 5.15 | 5.15 | 5.15 |
| 7 | 4.48 | 4.48 | 4.48 |
| 8 | 2.71 | 2.71 | 2.71 |
| 9 | 3.018 | 3.018 | 3.018 |
| 10 | 2.67 | 2.67 | 2.67 |

| $C2$: | 1 | 2 | 3: |
|---|---|---|---|
| 1 | 1.1279 | 1.4079 | 1.6879 |
| 2 | 0.956 | 1.186 | 1.416 |
| 3 | 1.45 | 1.81 | 2.17 |
| 4 | 1.612 | 2.015 | 2.415 |
| 5 | 1.7973 | 2.24669 | 2.6959 |
| 6 | 3.3197 | 4.1409 | 4.962 |
| 7 | 2.8944 | 3.6178 | 4.3416 |
| 8 | 1.428 | 1.784 | 2.14 |
| 9 | 1.84 | 2.3025 | 2.765 |
| 10 | 1.598 | 1.997 | 2.396; |

Table 5.2: The costing and selling prices of each of the 10 products at different scenarios

Here $C2_{11}$, $C2_{12}$ $and$ $C2_{13}$ represent the per-unit total cost of $x_1(T-shirts)$ at scenarios 1, 2 and 3 respectively, and $C2_{21}$, $C2_{22}$ $and$ $C2_{23}$ represent the per-unit total cost of $x_2$ at scenarios 1, 2 and 3, and so on. And, $C1_{11}$, $C1_{12}$ $and$ $C1_{13}$ are representing the selling prices of $x_1$ at the three scenarios.

The value of $A_{ij}$ and $b_i$ are used from the matrix notation of (4.1)

### 5.4.1 Solution of the Combined Model

By solving this model by AMPL the obtained solution is presented in Table 5.3.

| $X$ [*,*] | | | |
|---|---|---|---|
| | 1 | 2 | 3 := |
| 1 | 0 | 3012450 | 4856810 |
| 2 | 0 | 300 | 0 |
| 3 | 1111900 | 2792830 | 0 |
| 4 | 1172480 | 6000 | 7000 |
| 5 | 8500 | 7000 | 8500 |
| 6 | 0 | 0 | 0 |
| 7 | 90943.1 | 48684.8 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 5326370 | 0 | 0 |
| 10 | 9000 | 10000 | 11000 |

Table 5.3: The solution of the stochastic model of combining three scenarios

And the optimal profit **is $z_{max} = 6861380$**

In Table 5.3, it is observed that the output of the combined case is better than the developed deterministic model (Chapter 4) in terms of profit. Here, Table 5.3 indicates that $x_{11}$(monthly production of $x_1$ in scenario 1) has no production by AMPL solution, i.e. in favorable conditions, to make optimal profit no production of $x_{11}$ is found. *A*nd there is a production $3012450$ piece of $x_{12}$ ($monthly\ production\ of\ x_1\ in\ scenario\ 2$) and so on. The optimum profit for the RMG industry is found $ 6861380 which is more optimal then DLP or any other general stochastic models. Thus, the readymade garment industry of Bangladesh can apply this model and get benefits as it supports to combine all the possible scenarios rather than a particular one.

## 5.5 Development of Stochastic Models with Selected Uncertain Parameters

In previous models, the parameters of all variables (10) were uncertain. But if the demand for particular 3/4 items suddenly increases rather than all products, then the selling price/s of those particular items would be increased generally, and also the cost of each raw material associated with those specific items could be increased, so that the profit would definitely change. The objective function's parameters of those particular products are changed along with the constraints' coefficients of those particular variables. In addition, labor cost and operating cost might definitely increase or decrease according to the change in demand. Here, it is considered that the three products' demands could be increased or decreased by 20% due to the fluctuated seasonal conditions in each scenario/s, as there is fluctuation of demand for the 3 products. Actually, the demand of all products varies in different situations but from the beginning of this study, it is seen that the demand for three products, leggings ($x_4$), shirts ($x_5$) and girls' tops ($x_{10}$) had to be fixed and assigned certain amount of lower bound (as per the company's requirement) in the deterministic case (Chapter 4), and now it fluctuates under different scenarios. The fluctuation of these three products is taken into consideration in this model; thus, only the parameters related to these 3 products are varying in this model instead of all parameters. The two-stage stochastic (TSLP) model is going to be formulated now.

### 5.5.1 Background of the Modeling

In this study, there is a restriction of the demand of leggings ($x_4$), shirts ($x_5$) and girls' tops ($x_{10}$) and would be changed in different scenarios and should have an lower bound varying scenario to scenarios because of the fluctuation of weather as well as economic conditions that becomes good, average and bad. The parameters related to these 3 products are also varying in this model.

It is faced that with a series of decisions that must be made without complete information regarding certain random events. These initial decisions, referred to as first-stage decisions, are typically represented by a vector $x$. In this context, the decisions pertain to the quantity of items such as T-shirts ($x_1$) and vests ($x_2$), sweatshirts ($x_3$), hoodie jackets ($x_6$), joggers ($x_7$), polo shirts ($x_8$), casual shirts ($x_9$), etc., to be produced. Since in this situation the demand of 3 products ($x_4$, $x_5, and\ x_{10}$) are fluctuating over time and have to be set a certain lower bound value, but the rest of the 7 products' demand remains the same and certain (there is at least no restriction/s) so the production of these products is considered stable for all months (certain in this sense that any production of these seven is acceptable and there would be no change of the raw materials cost, labor cost, operating and inventory cost associated with these seven products). This is the deterministic part of the stochastic model. And also consider as first stage variables. Though all the raw materials associated with the seven products are considered stable, but due to the variation of the output of three products, the solution of these seven will also vary from situation to situation. And the production of these three products in different scenarios is consider as 2nd stage variables.

## 5.6 Introduction of Two-Stage Stochastic (TSLP) Model

In this case, the random vector represents the collection of corrective measures that consist of the production of leggings ($x_4$), shirts ($x_5$), girls' tops($x_{10}$). From the perspective of mathematical programming, this scenario illustrates a two-stage stochastic program with recourse, expressed as follows

$$min\ \ C^T x + E_\xi Q(x, \xi)$$

$$subject\ to$$

$$Ax = b, x \geq 0$$

In this formulation, $Q(x,\xi)$ is defined $as\ max\{q^T y \mid Wy = h - T\,x, y \geq 0\}$, where $\xi$ is the vector comprising the elements of $q^T, h^T, and\ T, and\ E_\xi$ signifies the mathematical expectation concerning $\xi$. Complete information is obtained regarding the realization of a random vector $\xi$. At this point, the second-stage or corrective actions, usually denoted as $y$, are implemented. Additionally, a functional representation may be utilized, such as $\xi(\omega)$ or $y(s)$ to explicitly illustrate dependence on a specific element, either $\omega$ or $s$. It is important to note that it is assumed $W$ is constant (fixed recourse). Initially, it is established the fundamental two-stage stochastic linear program with fixed recourse, which is reiterated here for the sake of clarity.

Thus, the basic two-stage stochastic linear program with recourse is formed and repeated here for clarity.

$$max\ z = C^T x + E_\xi[minq(\omega)^T y(\omega)] \tag{5.1}$$

$$s.t.\ Ax = b$$

$$T(\omega)x + Wy(\omega) = h(\omega)$$

$$x \geq 0\,, y(\omega) \geq 0$$

Where $C$ is a known vector in $R^{n1}$, $b$ a known vector in $R^{m1}$, $A\ and\ W$ are known matrices of size $m1 \times n1$ and $m2 \times n2$ , respectively, and $W$ is called the recourse matrix. This allows us to characterize the feasibility region in a convenient manner for computation.

Problem (5.1) is equivalent to the so-called deterministic equivalent program (DEP):

$$max\ z = C^T x + Q(x)$$

$$s.t. Ax = b$$

$$x \geq 0$$

$$where\ Q(x) = E_\xi Q\big(x, \xi\ (\omega)\big)$$

$$and$$

$$Q\big(x, \xi\ (\omega)\big) = max\ \{q(\omega)^T y \mid Wy = h(\omega) - T(\omega)\,x\,, y \geq 0\}\,.$$

### 5.6.1 Scenario Representation of Two-Stage Stochastic Model

The three (3) scenarios which are going to be analyzed in this section, are generated from fluctuated seasonal demand, which are:

**S1: Increased Seasonal Demand**

Demand of 3 products increased due to seasonal circumstances; as a result, per unit profit, cost coefficients related to those particular products, labor, and operating costs related to those individual products are increased. As the costing of each item/s associated to these 3 products are also increased (since there is an old trend, when demand increases of any product(s) or service(s), then costs of all raw materials associated with that specific product/service also increase) so profit will not be increased that much what supposed to be, despite of the increased selling price.

**S2: No seasonal impact on demand**

Demand will remain as stable as the whole year, along with the costing of all items related to these three products and prices.

**S3: Decreased seasonal demand**

Demand of 3 products decreased with seasonal state so that profit coefficient, cost coefficient of constraints (of those particular products), labor and operations cost related to those particular products are decreased slightly.

### 5.6.2 Conversion of the Scenarios to the Mathematical Form

By relating to equation 5.1, now the scenario-based stochastic model is represented as:

$$maximize\ z = c^T x + \sum_{s=1}^{3} p_s\, Qs(x)$$

$$Subject\ to$$

$$\sum_{j=1}^{10} A_{ij} x_j \le b_i\ ,\ T(\xi)x + Wy \le h(\xi);$$

$$i = 1,\ 2,\ 3, \ldots, 60;\quad x_j \ge 0;\ y \ge 0$$

Where $c^T x$ is the first stage profit and

$Q_s(x)$ is the expected 2nd stage profit with scenario $s$.

with $\sum_{s=1}^{3} Qs(x) = Q_1(x) + Q_2(x) + Q_3(x)$ and

$p_s$= probability of scenario $s$ (here equal probabilities are considered in all of the three cases)

### 5.6.3 Conversion to Deterministic Equivalent Model

$$maximize\ z = c^T x + \frac{1}{3}\sum_{s=1}^{3} Qs(x)$$

$$Subject\ to$$

$$\sum_{j=1}^{10} A_{ij}x_j \le b_i\ ,\ T(\xi)x + Wy \le h(\xi);$$

$$i = 1,\ 2,\ 3, \dots, 60;\quad x_j \ge 0;\ y \ge 0$$

$$maximize\ z = c^T x + \tfrac{1}{3}(Q_1(x) + Q_2(x) + Q_3(x))$$

$$where$$

Expected profit of scenario 1: $Q_1(\mathrm{x}) = (1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101})$

Expected profit of scenario 2: $Q_2(x) = (1.001 * x_{42} + 1.5 * x_{52} + 0.95 * x_{102})$

Expected profit of scenario 3: $Q_3(x) = (0.7 * x_{43} + 1 * x_{53} + 0.6 * x_{103})$

The constraints of this model would be taken from the DLP model (Chapter 4). Firstly, for the increased demand of three products, cost co efficient associated with specific those three products are also be increased slightly for 17 constraints. Then the 2nd 17 constraints remain same and next 3rd 17 constraints for decreasing demand and reduced co efficient of the constraints are added.

Thus, the algebraic form of the model is

$$\textbf{\textit{maximize z}}: x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + 1/3(1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101}) + 1/3(1.001 * x_{42} + 1.5 * x_{52} + 0.95 * x_{102}) + 1/3\ (0.7 * x_{43} + 1 * x_{53} + 0.6\ * x_{103});$$

***Subject to***

The 1st set of constraints with the increased demand and raw materials cost with $i = 1, 2, 3, \dots, 17;$ and for the increased demand of $x_4, x_5, and\ x_{10}$ the lower bound is set as:

$$x_{41} \geq 8500$$
$$x_{51} \geq 12000$$
$$x_{101} \geq 13500$$

The 2nd set of constraints with the stable demand and the raw materials' cost with $i = 21, 22, , \dots, 37;$ and for the stable demand of $x_4, x_5, and\ x_{10}$ the lower bound is set as it is in DLP model

$$x_{42} \geq 5000$$
$$x_{52} \geq 9000$$
$$x_{102} \geq 10000$$

The 3rd set of constraints with the decreased demand and raw materials cost with $i = 41, 42, 3, \dots, 57;$ and for the decreased demand of $x_4, x_5, and\ x_{10}$ the lower bound is set.

$$x_{43} \geq 4000$$
$$x_{53} \geq 7000$$
$$x_{103} \geq 7500$$

…… (5.2)

The above constraints $(5.2)$ are found by modifying the $(4.2)$ (constraints of the DLP model)

Solving this model by AMPL, the optimal solution is obtained and presented in Table 5.4

<table>
<tr><th>Scenarios</th><th colspan="2">Solution of the 2nd stage Stochastic model</th><th>The objective function's value</th></tr>
<tr><td>Demand of a specific product could be increased due to seasonal conditions</td><td rowspan="3">$x_1 = 3008890$<br>$x_2 = 000$<br>$x_3 = 2791740$<br>$x_6 = 000$<br>$x_7 = 48837.3$<br>$x_8 = 000$<br>$x_9 = 000$</td><td>$x_{41} = 33345.8$<br>$x_{51} = 12000$<br>$x_{101} = 13500$</td><td rowspan="3">$z = 6594711.306$</td></tr>
<tr><td>Demand of a specific product could remain the same</td><td>$x_{42} = 5000$<br>$x_{52} = 9000$<br>$x_{102} = 16823.3$</td></tr>
<tr><td>Demand of a specific product could be decreased due to seasonal conditions</td><td>$x_{43} = 6728.61$<br>$x_{53} = 14001.4$<br>$x_{103} = 7500$</td></tr>
</table>

Table 5.4: Solution of Stochastic Model (RP)

Through the solution solved by AMPL here is the optimum profit of two-stage stochastic model:

$$\boldsymbol{Z} = 3008890 + 0.8 * 0 + 1.2469 * 2791740 + 0.99 * 0 + 1.21 * 48837.3$$
$$+ 0.79 * 0 + 1.005 * 0 + .33333 * (1.3 * 33345.8 + 1.6 * 12000 + 1.25 * 13500)$$
$$+ .33333 * (1.001 * 5000 + 1.5 * 9000 + 0.95 * 16823.3)$$
$$+ .33333 * (0.7 * 6728.61 + 1 * 14001.4 + 0.6 * 7500)$$

(Using the value from Table 5.4)

$$Max\, Z = 6549003.739 + 1/3(Q_1 + Q_2 + Q_3)$$
$$= 6549003.739 + 1/3\,(79424.54 + 34487.135 + 23211.027)$$
$$= 6549003.739 + \frac{1}{3}(137122.702)$$
$$= 6549003.739 + 45707.56733$$
$$= \mathbf{6594711.306}$$

Therefore, the $RP$ solution obtained from the stochastic solution shown in Table 5.4 is 6594711.306

This type of stochastic decision program model is called the extensive form of the stochastic program because it explicitly describes the second-stage decision variables for all scenarios. The optimal solution of this model is shown in Table 5.4. The 2nd column provides the range of production of 7 products; this decision is referred to as the deterministic part. The 3rd column shows the values of 3 products in the three scenarios whose demand was fluctuating in various scenarios; these are referred to as the second-stage variables. The last column of Table 5.4 displays the total expected profit. Thus, the two-stage stochastic model is offered to the RMG industry so that they can manage their uncertainty while some of the products' demand is volatile, rather than all of it.

### 5.6.4 Wait and See (WS) Solution of Three Situations

The manufacturing firm may want to determine whether the product fluctuations will affect the best course of action and what will happen in the case of a better environment as well as a worse. Table 5.5 is formed based on the outputs that are above, average and below average. The middle row of Table 5.5 indicates the average solution. Variations in the solution have a significant impact on the ideal solution. The best answers found in these situations are shown in Table 5.5 below.

| Scenario/es | Mod File | Model | Output (value of objective function) | Output (decision variables) |
|---|---|---|---|---|
| **In increased seasonal demand** | | | | |
| S1: In case of i**ncreased seasonal demand** the cost of raw materials, labor cost and operating cost (related to those 3 products whose demand is increased with seasonal change) could be more by 20% with increased seasonal demand | | $\boldsymbol{maximize} = P(x) + Q_1(x)$<br>Where $P(x) = x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9$<br>And<br>$Q_1(x) = (1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101})$<br>*Subject to*<br>$\sum_{j=1}^{10} A_{ij}x_j \leq b_i,\ i = 1, 2, 3, \ldots, 20; x_j \geq 0$ | $x_1 = 0$<br>$x_2 = 0$<br>$x_3 = 74876.5$<br>$x_6 = 0$<br>$x_7 = 258137$<br>$x_8 = 0$<br>$x_9 = 0$<br>$x_{41} = 8500$<br>$x_{51} = 12000$<br>$x_{101} = 5779330$ | $z = 7660130$ |
| **In average or stable situation** | | | | |
| S2: **In stable situation** the cost of each raw material would remain same (If the weather situation remains same) | | $\boldsymbol{maximize} = P(x) + Q_2(x)$<br>Where<br>$P(x) = x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9$<br>And<br>$Q_2(x) = (1.001 * x_{42} + 1.* x_{52} + 0.95 * x_{102})$<br>*Subject to*<br>$\sum_{j=1}^{10} A_{ij}x_j \leq b_i,\ i = 1, 2, 3, \ldots, 20; x_j \geq 0$ | $x_1 = 3010950$<br>$x_2 = 0$<br>$x_3 = 2792000$<br>$x_6 = 0$<br>$x_7 = 48947.8$<br>$x_8 = 0$<br>$x_9 = 0$<br>$x_{42} = 5000$<br>$x_{52} = 9000$<br>$x_{102} = 10000$ | z = 6579520 |
| **In decreased seasonal demand** | | | | |
| S3: In case of **decreased seasonal demand** the cost of raw materials, labor cost and operating cost (related to those 3 products whose demand is decreased with seasonal change) could be lesser by 20% with decreased seasonal demand | | $\boldsymbol{maximize} = P(x) + Q_3(x)$<br>Where<br>$P(x) = x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9$<br>And<br>$Q_3(x) = (0.7 * x_{43} + 1 * x_{53} + 0.6 * x_{103})$<br>*Subject to*<br>$\sum_{j=1}^{10} A_{ij}x_j \leq b_i,\ i = 1, 2, 3, \ldots, 20; x_j \geq 0$ | $x_1$ = 1065260<br>$x_2$ = 0<br>$x_3$ = 1002620<br>$x_6$ = 0<br>$x_7$ = 150078<br>$x_8$ = 0<br>$x_9$ = 3938900<br>$x_{43}$ = 4500<br>$x_{53}$ = 7500<br>$x_{103}$ = 8000 | z = 6306420 |

Table 5.5: Separate solution of 3 scenarios (Wait and See-WS) Thus, the average of the results of

the three (3) scenarios are from Table 5.5, which represent the WS value is:

$$\frac{1}{3}(7660130 + 6579520 + 6306420) = \frac{1}{3}(20546070)$$
$$= \mathbf{6848690}$$

Therefore, the $WS$ solution obtained from the three scenarios shown in Table 5.5 is $6848690$.

### 5.6.5 Two-Stage Expected Value Problem (EEV)

The company may also choose to assume predicted output and always distribute the ideal production planning surface based on these outputs. The expected value solution is represented by this method. It is typical in optimization. A long-term monthly profit of $\$\ 6562484.53$ is obtained by using the expected value solution.

The first stage decision variables are replaced by the values obtained from the average scenario of Table 5.5.

$$\boldsymbol{maximize}\ z = c^T x + \sum_{s=1}^{3} p_s\, Qs(x)$$

$\boldsymbol{maximize\ z}$: $x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + \sum_{s=1}^{3} p_s\, Qs(x)$

$$= 3010950 + 0.8 * 0 + 1.2469 * 2792000 + 0.99 * 0 + 1.21 * 48947 + 0.79 * 0 + 1.005 * 0 + 1/3(1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101}) + 1/3(1.001 * x_{42} + 1.5 * x_{52} + 0.95 * x_{102}) + 1/3(0.7 * x_{43} + 1 * x_{53} + 0.6 * x_{103});$$

$$Subject\ to$$

$$\sum_{j=1}^{10} A_{ij} x_j \le b_i\ ,\ \ i = 1,\ 2,\ 3, \dots, 60;\quad x_j \ge 0$$

$$T(\xi)x + Wy = h(\xi),$$

$$y \ge 0$$

In this section, the total expected profit is calculated by adding 1st stage profit to the expected second stage profit. Values of the first-stage decision variables are replaced by the average scenario's (2nd row) of Table 5.5.

<table>
<tr><td rowspan="2">Scenario 1: EEV</td><td>$Q_1(x) = Max\ (1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101})$<br><br>$Subject\ to \sum_{j=1}^{10} A_{ij}x_j \leq b_i\ ,$<br>$i = 1,\ 2,\ 3, \ldots, 20;\quad x_j \geq 0$</td><td>$x_{41} = 10100$<br>$x_{51} = 9505$<br>$x_{101} = 13500$<br>$z = 45213$<br><br>$i.e.\ Q_1(x) =$ 45213</td></tr>
<tr><td colspan="2">$Q_1(x) = (1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101})$<br>$= 1.3 * 10100 + 1.5 * 9505 + 1.25 * 13500$<br>$= 13130 + 15208 + 16875$<br>$= 45213$<br><br>Objective function's value of $Q_1(x)$: 45213</td></tr>
<tr><td rowspan="2">Scenario 2: EEV</td><td>$Q_2(x) = Max\ (1.001 * x_{42} + 1.* x_{52} + 0.9 * x_{102})$<br>$Subject\ to$<br>$\sum_{j=1}^{10} A_{ij}x_j \leq b_i\ ,$<br>$i = 1,\ 2,\ 3, \ldots, 20;\quad x_j \geq 0$</td><td>$x_{42} = 5000$<br>$x_{52} = 8501.77$<br>$x_{102} = 9000$<br>$z = 26307$<br><br>$i.e.\ Q_2(x) =$ 26307</td></tr>
<tr><td colspan="2">$Q_2(x) = 1.001 * 5000 + 1.5 * 8501.77 + 0.95 * 9000$<br>$= 5005 + 12752.655 + 8550$<br>$= 26307$<br><br>Objective function's value of $Q_2(x)$: 26307</td></tr>
<tr><td rowspan="2">Scenario 3: EEV</td><td>$Q_3(x) = Max\ (0.7 * x_{43} + 1 * x_{53} + 0.6 * x_{103})$<br>$Subject\ to$<br>$\sum_{j=1}^{10} A_{ij}x_j \leq b_i\ ,$<br>$i = 1,\ 2,\ 3, \ldots, 20;\quad x_j \geq 0$</td><td>$x_{43} = 4000$<br>$x_{53} = 11599$<br>$x_{103} = 8000$<br>$z = 19199.5$<br><br>$i.e.\ Q_3(x)$: 19199.5</td></tr>
<tr><td colspan="2">$Q_3(x) = (0.7 * x_{43} + 1 * x_{53} + 0.6 * x_{103})$<br>$= 0.7 * 4000 + 1 * 11599 + 0.6 * 8000$<br>$= 2800 + 11599 + 4800$<br>$= 19199.5$<br><br>Objective function's value of $Q_3(x)$: 19199.5</td></tr>
</table>

Table 5.6: Solution of EEV

Therefore, 1st stage profit : $3012560 + 0.8 * 0 + 1.2469 * 2792260 + 0.99 * 0 + 1.21 * 48887.3 + 0.79 * 0 + 1.005 * 0 = \mathbf{6532255.87}$

Thus, the first stage profit: 6532255.87 .

Expected second stage solution is obtained by applying AMPL and the solution is presented in Table 5.6.

Expected second stage profit: $= 1/3(1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101}) + 1/3(1.001 * x_{42} + 1.5 * x_{52} + 0.95 * x_{102}) + 1/3(0.7 * x_{43} + 1 * x_{53} + 0.6 * x_{103}$

$= 1/3(45213 + 26307 + 19199) = 1/3(90686)$

$= \mathbf{30228.66}$

Thus, Total profit = 1st stage profit + Expected 2nd stage profit

$= 6532255.87 + 30228.66 = \mathbf{6562484.53}$

Thus, the optimal solution obtained by $EEV$ is 6562484.53.

Therefore, the relation $WS \geq RP \geq EEV$ is found in the case of a maximization problem. And this is shown as a pictorial presentation in Figure 5.1.

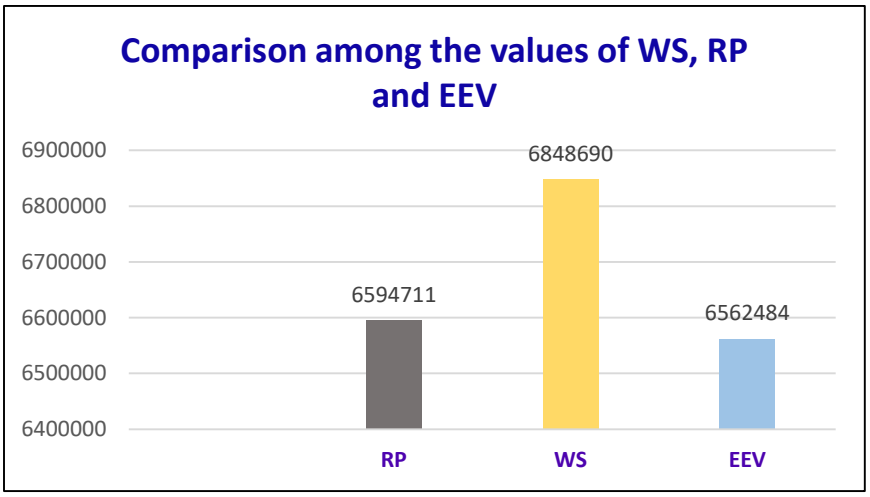



Figure 5.1: Comparison among WS, RP and EEV in terms of profit

## 5.7 Model that Aligned with Real Life

Now it is not feasible that the production of any specific product is nil of any manufacturing company i.e. ready-made garment industry. Every product has some minimum demand in real life. But all the stochastic models showed at least 2 or 3 products have zero production. In these circumstances, another stochastic model is developed here where the lower bound and upper bound is set for all the products as per the demand of any particular garment factory. The profit might be less in this model by it meets the reality.

$$\textbf{\textit{maximize}}\ z\colon\ x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + \sum_{s=1}^{3} p_s\, Qs(x)$$

$$\begin{aligned}\textbf{\textit{maximize}}\ z\colon\ & x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 \\ & * x_9 + .33333 * (1.3 * x_{41} + 1.6 * x_{51} + 1.25 * x_{101}) \\ & + .33333 * (1.001 * x_{42} + 1 * x_{52} + 0.9 * x_{102}) \\ & + .33333 * (0.7 * x_{43} + 1 * x_{53} + 0.6 * x_{103});\end{aligned}$$

***Subject to***

$$\sum_{j=1}^{10} A_{ij} x_j \le b_i\ ,\ \ i = 1,\ 2,\ 3, \dots, 60;\quad x_j \ge 0$$

$$T(\xi)x + Wy = h(\xi),$$

The constraints are used as it is from equation (5.2) and the following extra restrictions are also be added

$$\begin{aligned} x_1 &\ge 10000; \\ x_1 &\le 30000; \\ x_2 &\ge 8000; \\ x_2 &\le 20000; \\ x_3 &\ge 5000; \\ x_3 &\le 15000; \\ x_6 &\ge 12000; \\ x_6 &\le 25000; \\ x_7 &\ge 5000; \\ x_7 &\le 28000; \\ x_8 &\ge 15000; \\ x_8 &\le 24000; \\ x_9 &\ge 5000; \\ x_9 &\le 20000; \end{aligned}$$

### 5.7.1 Output of the Model

| **Output of 7 products that are in deterministic type** | | | | | | | **Output of $x_4$ in different scenarios** | | | **Output of $x_5$ in different scenarios** | | | **Output of $x_{10}$ in different scenarios** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{41}$ | $x_{42}$ | $x_{43}$ | $x_{51}$ | $x_{52}$ | $x_{53}$ | $x_{101}$ | $x_{102}$ | $x_{103}$ |
| 30000 | 8000 | 15000 | 12000 | 28000 | 15000 | 20000 | 5800 | 630755 | 4000 | 2353900 | 85000 | 2071560 | 1000 | 528045000 | 7749700 |
| **Optimal Profit** | | | **5523100** | | | | | | | | | | | | |

Table 5.7: Output as per the demand of any industry

When production of all products is mandatory, in fact in real life scenario it is true that every product has some demand in market then the organization can apply this model to find the best production planning. Thus Table 5.7 shows the output that any readymade garment industry wants to get as per their demand. And the optimal profit in this case $5523100 is which is less than any of the stochastic models but meets the real-life demand.

## 5.8 EVPI and VSS

It is well known that solving stochastic programming is computationally challenging. When presented with real-world challenges, many people have an innate tendency to tackle the simpler ones. For example, simplified variants are sometimes utilized to solve deterministic programs created by substituting the predicted values for each random variable, or to resolve several deterministic programs, each of which corresponds to a certain situation, and then using a heuristic algorithm to integrate these many answers. It is reasonable to inquire as to whether these methods can occasionally be almost ideal. Or if they are entirely wrong. The importance of having a clear understanding of the future is measured by $EVPI$(expexted value of perfect information). In some circumstances, where more data could be obtained by conducting more thorough sampling, forecasting, or exploration, $EVPI$ could be helpful then in determining whether to make further attempts or not.

The notion of $EVPI$ was initially formulated within the framework of decision analysis and is documented in traditional sources like Raiffa and Schlaifer (1961). In the context of stochastic programming, it may be defined as follows. Assume several scenarios can be used to model uncertainty. Let $\xi$ be the random variable whose values match up with the different situations. Defining

$$z(x,\xi) = c^T x + \{Wy = h - Tx, y \geq 0\}$$

$$s.t. \quad Ax = b, x \geq 0$$

As the optimization issue related to a certain scenario $\xi$, where $\xi(\omega)^T = (q(\omega)^T, h(\omega)^T, T_1(\omega), \dots, T_{m2}(\omega))$ is the same as previously.

Notation $K_1 = \{Ax = b, x \geq 0\}$ $and$ $K_2(\xi) = \{x | \exists y \geq 0 \; s.t. Wy = h - Tx\}$ is repeated to finish the definition.

The benefit of understanding and using distributions on future outcomes is evaluated by $VSS$ (value of stochastic solution). When there is no more knowledge available about the future, the $VSS$ becomes more practically significant.

For the maximization models in particular, the following inequalities are satisfied, Madansky (1960)

$$EEV \leq RP \leq WS$$

where the value $EEV$ represents the expected outcome of using the deterministic model $EV$ solution, which is the one that is produced by substituting the expected values for each random variable. The $RP$ value, sometimes referred to as the here and now solution, indicates the ideal solution value to the recursion problem, whereas the $WS$ value, also known as the wait and see solution value in the pertinent literature, indicates the expected value of applying the perfect solution for each scenario (WS models).

**Proposition 5.8.1**

$WS < RP \leq EEV$ (in case of minimization problem)

*Proof.*

There is a connection of $z(x(\xi), \xi) \leq z(x^*, \xi)$, for any realization $\xi$, where $x^*$ indicates an optimal solution to the recourse issue $RP = \min_x E_{\xi z(x,\xi)}$, The first disparity results from taking both parties' expectations into account. While $\underline{x}(\underline{\xi})$ is only one solution to $RP = \min_x E_{\xi z(x,\xi)}$, provides the second inequality, $x^*$ is an optimum solution to the recourse issue.

**Proposition 5.8.2**

In the case of stochastic programs with fixed $W$ and minimization problem and objective coefficients,

$EV \leq WS$.

*Proof.*

It is noted that the function $z(x, \xi = (h,T)) = c^T x + Q(x,h,T) + \delta(Ax = b, x \geq 0)$ is jointly convex in $x, h$, and $T$. The indicator function of the point $x$ for set $X$ is denoted by $\delta(x|X)$. In order to demonstrate that $f(\xi) = min_x z(x, \xi)$ is convex in $\xi$, let's look at $\xi_1 \; and \; \xi_2$, where $z(x_1, \xi_1) = f(\xi_1) and \; z(x_2, \xi_2) = f(\xi_2)$. As a result,

$$\lambda f(\xi_1) + (1-\lambda) f(\xi_2) = \lambda z(x_1, \xi_1) + (1-\lambda) z(x_2, \xi_2)$$

$$\geq z(\lambda(x_1, \xi_1) + (1-\lambda)(x_2, \xi_2))$$

$$\geq \min_x z(x, \lambda\xi_1 + (1-\lambda)\xi_2)$$

$$= f(\lambda\xi_1 + (1-\lambda)\xi_2)$$

which indicates the convexity of $f(\xi)$. .According to Jensen's inequality, $Ef(\xi) \geq f(E\xi)$ for every convex function $f(\xi)$ of $\xi$.

### 5.8.3 EVPI and VSS of the Developed Two-Stage Stochastic Model (TSLP)

When comparing here-and-now and wait-and-see strategies, the difference $EVPI = WS - RP$ represents the expected value of perfect information. A modest additional benefit specifies when one reaches a small perfect information.

From this study, the $EVPI = WS - RP = 6848290 - 6594310$

$= 253980$ (Results found by using Tables 5.4, 5.5 and 5.6)

$VSS = RP - EEV$ Compares the here-and-now and expected values approaches and indicates the value of the stochastic solution. A minimal VSS indicates that the program using expected values rather than random variables is a good approximation of the stochastic program.

$VSS = RP - EEV = 6594310 - 6582454$

$= 11856$

The positive value of EVPI and VSS indicates that the benefits of applying stochastic models while uncertainties arise.

## 5.9 Conclusion

This chapter includes some general stochastic models. Five different models are developed here under five different economic conditions. These models are showing the obvious results. In the case of reduced costs and increased selling prices, optimal profits are maximized, while minimal profit is given with the increased costs and reduced prices. The expected profit is assumed to remain steady in the case of stable economic conditions. The model, a combination of all scenarios, is also formulated in this chapter and shows a smart result that is more optimal than the deterministic situations. The RMG industry can be benefitted by applying these models in different economic circumstances. A two-stage stochastic (TSLP) is also formulated with EVPI and VSS

calculations. By applying this TSLP model they can manage the few product's uncertainties through involving stages. The (Wait and See) WS solution gives the maximum result in terms of profit, the deterministic equivalent model (RP) shows a value less than the WS and more than the expected value solution (EEV). The positive EVPI and VSS indicate the benefits of applying of the stochastic model in case of any kind of uncertainty. Finally, a stochastic model is offered to the garment industry in which any number of upper bounds and lower bounds can be applied as per the company's demand, which implies a real-life alignment.

# Chapter 6

# Two-Stage and Multi-Stage Stochastic Models for Managing Uncertainties of RMG Industry

# Chapter 6

# Two-Stage and Multi-Stage Stochastic Models for Managing Uncertainties of RMG Industry

## 6.1 Introduction

In the previous chapter, the developed stochastic models indicate that all the manufactured products would be sold. That means profit or loss would be calculated by considering all the produced products as they are all sold out. But it is not the actual reality. It is going to develop now some other models, both in two-stage and multistage stochastic ground, in which the actual sold amount or actual demand is considered, and this amount is the uncertain component of these models which is to be revealed. The further production planning will be taken as a $2^{nd}$ stage or recourse choice after the realization of customer demand. Again, by observing the customer demand of the $2^{nd}$ stage, the production planning of the immediate next stage will be finalized and so on. That directs, after revealing the demand of each stage, further production planning would be fixed. And this process will be continued from one stage to another. From the beginning of this study, the 3 products have fluctuation in demand, and for the proper distribution or selecting the appropriate manufacturing quantity, the monthly demand should be measured properly. For moving forward to this aim and to formulate a two-stage stochastic model, it is considered that the possibilities of monthly demand could be three (3) with equal probabilities. Firstly, it is presented that the sell can be any quantity and less than the production and secondly, it is shown that the company should meet the customers' demand completely despite their underproduction. If they have less production compared to demand they would have to collect the shortage amount from external source/s to meet the extra demand in this case. And the two-stage stochastic models are formulated in both of the cases in the next section. The models then extended to multistage stochastic context and the monthly demand is divided into two parts ($1^{st}$ half and $2^{nd}$ half) to form MSLP to observe the customers' demand rigorously. For the smoothness of operation, only one product's demand is considered as uncertain. Thus, the models are formulated by considering only one product's (leggings ($x_4$)) fluctuating demand and the corresponding parameters of leggings are also considered as uncertain as well. Models for other additional uncertain variables could be ended similarly. Thus, in this chapter, two-stage stochastic models (TSLP) are formulated by considering two

dissimilar cases: firstly, the sell can be of any amount of the production and secondly, the sell should actually be the amount that the customers want to get as well as multiple scenarios would be measured in both of the cases. Since accuracy rises with the increasing of number of stages, the models are then expanded to multistage stochastic models (MSLP) with different scenarios. The outcomes of all of the models with interpretation are also presented in this chapter. Finally, a comparative discussion between TSLP and MSLP is set with graphical presentation.

## 6.2 Two-Stage Stochastic Linear Programming Model (TSLP)

A mathematical framework for making the best choices in the face of uncertainty is a two-stage stochastic linear programming model. In the first stage, a decision is made before the uncertainty is revealed, and in the second stage, a corrected decision is made after the uncertainty is realized. This method is especially helpful in domains where choices must be made in the face of unpredictable future events, such as supply chain management, energy planning, and finance.

From the perspective of mathematical programming, this scenario illustrates a two-stage stochastic program with recourse, expressed as follows:

$$min \;\; C^T x + E_\xi Q(x,\xi)$$
$$subject\ to$$
$$Ax = b$$
$$x \geq 0$$

In this formulation, $Q(x,\xi)$ is defined $as\ max\{q^T y \mid Wy = h - T\,x, y \geq 0\}$, where $\xi$ is the vector comprising the elements of $q^T, h^T, and\ T, and\ E_\xi$ signifies the mathematical expectation concerning $\xi$.

### 6.2.1 Advantages of the TSLP Compare to the MSLP

This model involves a single recourse action after a "here-and-now" choice, which simplifies and reduces computational complexity.

- Since this two-stage model uses fewer processing resources than a multistage model, it is appropriate for problems that need to be solved quickly.

- TSLP is best when choices are taken in advance of unforeseen circumstances, like long-term investment planning.

### 6.2.2 Disadvantages of the TSLP in Comparison to the MSLP

Two-stage stochastic models are unable to modify choices in response to new knowledge over time, which could result in less-than-ideal results in dynamic contexts.

- It is less equipped to reduce the dangers brought on by uncertainties that change over time.

## 6.3 Case Studies for Model Formulation of TSLP

Two cases with different scenarios will be discussed in this section for developing two distinct two-stage stochastic models. Here, three (3) possibilities of monthly demand are considered with equal probabilities. Firstly, it is presented that the sell can be any of the quantity less than the production and secondly, it is shown that the company should meet the customer's demand completely despite their underproduction. If they have less production compared to demand they would collect the shortage amount from external source/s to meet the extra demand in the 2nd case.

### 6.3.1 Case 1: The Sell or Demand Can be of any Number of the Production

According to the history, there is a fluctuation of demand and so there is a restriction on the production of three products, leggings ($x_4$), shirts ($x_5$) and girls' tops ($x_{10}$). But to ease the process, only one variable leggings ($x_4$) is considered as uncertain now in this study to formulate the models (in both of the cases), and a similar calculation could be followed for the other additional products. Generally, the monthly demand for leggings ($x_4$) is more or less than 5000 for most of the months. Now it is considered that the demand of leggings could be either 4000, 5000 or 6000 for a specific month on the basis of seasonal demand, with a probability of 50%, 25% and 25% respectively. Suppose the expected demand is about 4000 for a particular month and the selling quantity or actual demand is $y_{21}$ in this case, with price is \$2.6 per piece of legging, while the production cost of per piece of legging is \$2.16. Initially, the industry produces $x_4$(r) pieces of leggings (here, the monthly production of leggings is denoted by $\boldsymbol{r}$

instead of $x_4$) without realizing the demand. This $\boldsymbol{r}$ is the **1st stage decision** without revealing the uncertainty.

#### 6.3.1.1 TSLP Model Formulation with Scenario Generation (case I)

Now, three scenarios could arise.

**Scenario 1:** Due to the fluctuation of seasonal demand, the expected demand of leggings ($x_4$) is 4000 pieces for a particular month with probability 0.5 and in this case, the actual demand is considered $y_{21}$ (uncertain state which is to be revealed) and the selling price is $2.6.

**Scenario 2:** The expected demand for leggings ($x_4$) could be 5000 pieces for a particular month with probability 0.25, due to the seasonal demand and in this case, the actual demand is $y_{22}$.

**Scenario 3:** The expected demand for leggings ($x_4$) is 6000 pieces for a particular month with a probability of 0.25 and in this case, the actual demand is considered as $y_{23}$.

These three scenarios with probability distribution are shown in Figure 6.1 as pictorial presentation.

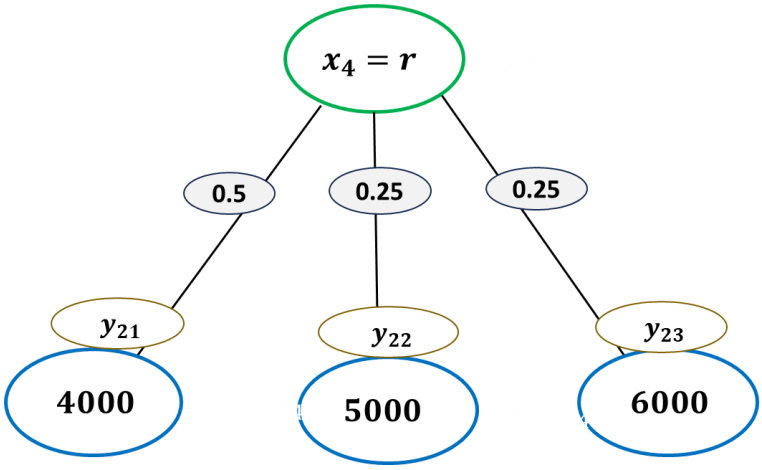



Figure 6.1: Scenario generation with probability distribution of Two-stage model (Case-I)

Here $\boldsymbol{y_{21}, y_{22}\ and\ y_{23}}$ are the demand **and are the uncertain quantities for the scenario $\boldsymbol{1, 2\ and\ 3}$ respectively which are to be revealed**

The extra restrictions in this case are

$$y_{2s} \leq r \qquad (s = 1, 2, 3)$$
$$y_{2s} \leq 4000; \quad (s = 1)$$
$$y_{2s} \leq 5000; \quad (s = 2)$$
$$y_{2s} \leq 6000; \quad (s = 3)$$

The objective function is considered as a cost minimization type.

$$\textbf{\textit{Minimize } z} = 2.16r - (x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + 1.5 * x_5 + 0.95 * x_{10}) + \sum_{s=1}^{3}(Ps * (-2.6 * y_{2s}))$$

*Subject to*

$$\sum_{j=1}^{10} A_{ij}x_j \leq b_i \ , \ \ where\ i = 1,\ 2,\ 3, \dots, 57$$

Where

$P_s = 0.5, 0.25\ and\ 0.25\ respectively\ for\ s = 1, 2, 3$
$r$ = Production of leggings in the first stage without revealing uncertainty
$y_{21}$= Actual demand for scenario 1
$y_{22}$= Actual demand for scenario 2
$y_{23}$= Actual demand for scenario 3

Thus the algebraic form of this two-stage stochastic model (TSLP) for case I is

$$\textbf{\textit{Minimize } z}: 2.16 * r + 0.5 * (-2.6 * y_{21}) + 0.25 * (-2.6 * y_{22}) + 0.25 * (-2.6 * y_{23}) - (x_1 + 0.8 * x_2 + 1.24 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + 1.5 * x_5 + 0.95 * x_{10})$$

***Subject to***

$$1.05667 * x_1 + 0.6317 * x_2 + 1.334 * x_3 + 1.5021 * r + 1.903 * x_5 + 2.465 * x_6 + 2.88 * x_7 + 1.4317 * x_8 + 1.96 * x_9 + 1.528 * x_{10} \leq 20000000;$$

$$0.01405 * x_1 + .00985 * x_2 + .0331 * x_3 + .05781 * r + 0.0168 * x_5 + .1726 * x_6 + .12 * x_7 + .448 * x_8 + 0.01853 * x_9 + 0.0452 * x_{10} \leq 300000;$$

$$0.013470 * x_1 + .03616 * x_2 + 0.02338 * x_3 + .02577 * r + .02577 * 20.00829 * x_5 + 0.041 * x_6 + .02098 * x_7 + 0.0111 * x_8 + 0.00833 * x_9 + 0.00648 * x_{10} \leq 220000;$$

$$.001717 * x_3 + .00182 * x_6 \leq 29000;$$

$$.00044 * x_1 + .00000580 * x_2 + .01204 * x_3 + 0.0053 * x_5 + 0.00209 * x_8 + .00569 * x_9 + .00118 * x_{10} \leq 35000;$$

$$.00527 * x_1 + .01386 * x_2 + .00928 * x_3 + .011338 * r + 0.0004 * x_5 + .01677 * x_6 + .0047 * x_7 + .00662 * x_8 + .00516 * x_9 + .006 * x_{10} \leq 100000;$$

$$.01089 * x_1 + .0063 * x_2 + .01804 * x_3 + .02221 * r + 0.0101 * x_5 + .25224 * x_6 + .133824 * x_7 + 0.00924 * x_8 + .01015 * x_9 + .009077 * x_{10} \leq 90000;$$

$$.001807 * x_1 + .00135 * x_2 + .003008 * x_3 + .003497 * r + 0.0002 * x_5 + .002568 * x_6 + .004176 * x_7 + .00084 * x_8 + .00197 * x_9 + .000117 * x_{10} \leq 23000;$$

$$.142257 * x_2 + .1143 * x_6 + 0.1254 * x_7 \leq 50000;$$

$$.0145 * x_1 + .0393 * x_2 + .05088 * x_3 + .0155 * r + 0.00148 * x_5 + .01245 * x_8 + .013467 * x_9 + .00137 * x_{10} \leq 237000;$$

$$.0001322 * x_1 + .000175 * x_2 + .0018048 * x_3 + .000144 * x_8 + .0001472 * x_{10} \leq 35000;$$

$$.0009135 * x_1 + .01117 * x_2 + .02305 * x_3 + .080802 * r + 0.0007 * x_5 + .07 * x_6 + 0.05 * x_7 + .0007 * x_8 + .008771 * x_{10} \leq 100000;$$

$$1.5 * x_1 + 2 * x_2 + 3 * x_3 + 1.5 * r + 3.5 * x_5 + 3 * x_6 + 3 * x_7 + 1.5 * x_8 + 2.5 * x_9 + 1.6 * x_{10} \leq 25000000;$$

$$6 * x_1 + 8 * x_2 + 12 * x_3 + 8 * r + 10 * x_5 + 12 * x_6 + 9 * x_7 + 6 * x_8 + 6 * x_9 + 5.5 * x_{10} \leq 60000500;$$

$$.1538 * x_1 + .1538 * x_2 + .1538 * x_3 + .1538 * r + .1538 * x_5 + .1538 * x_6 + .1538 * x_7 + .1538 * x_8 + .1538 * x_9 + .1538 * x_{10} \leq 950000;$$

$$.28976 * x_1 + .28976 * x_2 + .28976 * x_3 + .28976 * r + 0.5 * x_5 + 0.3 * x_6 + 0.2 * x_7 + 0.3 * x_8 + 0.27 * x_9 + .28 * x_{10} \leq 1700000;$$

$$.014 * x_1 + 0.0108 * x_2 + .0188 * x_3 + .0218 * r + 0.2 * x_5 + .1 * x_6 + .02 * x_7 + .03 * x_8 + .015 * x_9 + .011 * x_{10} \leq 5500000;$$

Another 2 sets of constraints are added here with the different costs of the raw materials associated to that specific product legging ( $x_4$) with according to demand where $i = 18, 19, \ldots, 51$

$$r \geq 4000;$$
$$y_{2s} \leq r; \ (s = 1, 2, 3)$$
$$y_{2s} \leq 4000; \ (s = 1)$$
$$y_{2s} \leq 5000; \ (s = 2)$$
$$y_{2s} \leq 6000; \ (s = 3)$$

#### 6.3.1.2 Outcome of TSLP (Case-I) after Solving by AMPL

| Product name | Amount to produce |
|---|---|
| $x_1\ (T-shirt)$ | 0 |
| $x_2(Vest)$ | 0 |
| $x_3(Sweat\ shirt)$ | 17 |
| $x_4 = r\ (Leggings\ as\ 1st\ stage\ production)$ | 4000 |
| $y_{21}(Demand\ of\ 2nd\ stage\ in\ 1st\ scenario)$ | 4000 |
| $y_{22}(Demand\ of\ 2nd\ stage\ in\ 2nd\ scenario)$ | 4000 |
| $y_{23}(Demand\ of\ 2nd\ stage\ in\ 3rd\ scenario)$ | 4000 |
| $x_5(Shirt)$ | 207501 |
| $x_6(Jacket)$ | 0 |
| $x_7(Jogger)$ | 220474 |
| $x_8(Polo\ shirt)$ | 0 |
| $x_9(Casual\ shirt)$ | 5745740 |
| $x_{10}(Girl's\ top)$ | 0 |
| $\boldsymbol{Minimum\ cost\ i.e.\ Maximum\ profit}$ | $\mathbf{6354270} |

Table 6.1: Outcome by AMPL of the TSLP (case I)

Table 6.1 shows that, this two-stage stochastic model (TSLP) is suggested to produce 4000 pieces of leggings ($x_4\ and\ in\ this\ case\ \boldsymbol{r = 4000}$) as a first stage production, and as the demand must not exceed the production, the proposed actual demand is 4000 pieces for all three 3 scenarios $\left(y_{21} = y_{22} = y_{23} = 4000\right)$ no other option is open here. After revealing this demand, the next stage production would be decided. The production of the other products is also offered as per presented in Table 6.1. In this case the optimal profit is $6354270.

### 6.3.2 Case 2: The Company has to Meet the Customers' Demand Completely

In general, the monthly demand of leggings ($x_4$) is more or less than 5000 for most of the months. To formulate the model for this case 2, it is considered that the demand of leggings could be either 4000, 5000 or 6000 for a specific month with a probability of 30%, 40% and 30% respectively. Suppose the expected demand is 4000 for a particular month and the production is less than 4000, then they have to collect an extra amount of $y_{21}$ from an external source, with the cost of $2.5 per piece to meet the clients' demand, while the production cost of per piece of leggings is $2.16. Initially, the industry produces $x_4$(r) pieces of leggings. This $\boldsymbol{r}$ is the **1st stage production** without revealing the uncertainty.

### 6.3.2.1 TSLP Model Formulation with Scenario Generation (case II)

Three scenarios may arise in this case

**Scenario 1:** Due to the fluctuation of seasonal demand, the expected demand of leggings ($x_4$) is 4000 pieces for a particular month with a probability of 0.30 and if the production is less than 4000, then they have to collect $y_{21}$ from an external source at the cost of $2.5 per piece to meet the extra demand.

**Scenario 2:** The expected demand of leggings ($x_4$) is 5000 pieces for a particular month with a probability of 0.40 and in case of shortage, they will go for buying $y_{22}$ from an exterior source with the cost of $2.5 per piece to meet the customer demand.

**Scenario 3:** The expected demand of leggings ($x_4$) is 6000 pieces in this scenario for a particular month with a probability 0.30 and they will go for a collection of $y_{23}$ from the other source/s to meet the customer demand in case of shortage of production due to the excess of seasonal demand.

Here, the collected amount $\boldsymbol{y_{21}, y_{22}}$ ***and*** $\boldsymbol{y_{23}}$ **are the uncertain quantties for the scenario 1, 2** ***and*** **3 respectively which are to be revealed**

And the pictorial appearance of scenarios with probabilities is presented in the Figure 6.2

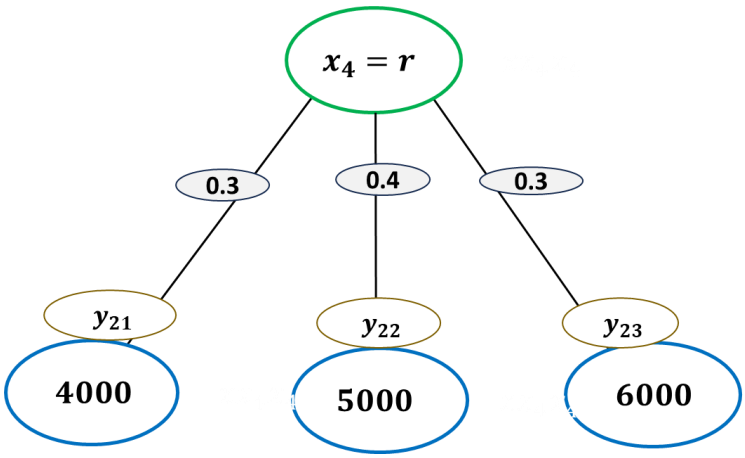



Figure 6.2: Scenario generation with probability distribution of TSLP (Case-II)

The restrictions for scenario 1 is

$$r\ (regular\ production\ of\ leggings\ (x_4)) \ + \ y_{21}\ (collection\ from\ external\ source) \geq 4000$$

Consider the first stage, the constraints to ensure customer demand is satisfied are:

$$r + y_{2s} \geq 4000; (s = 1)$$
$$r + y_{2s} \geq 5000; (s = 2)$$
$$r + y_{2s} \geq 6000; (s = 3)$$

At the second stage they will have units left over (i.e. inventory) to help meet future demand. This inventory level will be:

$$r + y_{2s} - 4000; (s = 1)$$
$$r + y_{2s} - 5000; (s = 2)$$
$$r + y_{2s} - 6000; (s = 3)$$

Here the objective function is considered as cost minimization type, and the model is

$$\textbf{\textit{Minimize } z} = 2.16r - (x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + 1.5 * x_5 + 0.95 * x_{10}) + \sum_{s=1}^{3} P_s * (2.5 * y_{2s})$$

*subject to*

$$\sum_{j=1}^{10} A_{ij} x_j \leq b_i \ , \ where\ i = 1, 2, 3, \dots, 46 \text{ and}$$

Where

$P_s = 0.3, 0.4\ and\ 0.3\ respectively\ for\ s = 1, 2, 3$

$r$ = Production of leggings (for the 1st half of a month) in the first stage without revealing uncertainty
$y_{21}$= purchased amount in the 2nd stage of scenario 1 from the external supplier after observing demand (if demand exceeds the production)

$y_{22}$= purchased amount in the 2nd stage of scenario 2 from the external supplier after observing demand (if demand exceeds the production)

$y_{23}$= purchased amount in the 2nd stage of scenario 3 from an external supplier after observing demand (if demand exceeds the production)

Thus, the algebraic form of the model is

$$\textbf{\textit{Minimize } z}: 2.16 * r + 0.3 * (2.5 * y_{21}) + 0.4 * (2.5 * y_{22}) + 0.3 * (2.5 * y_{23}) - (x_1 + 0.8 * x_2 + 1.24 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + 1.5 * x_5 + 0.95 * x_{10})$$

***Subject to***

$$1.05667 * x_1 + 0.6317 * x_2 + 1.334 * x_3 + 1.5021 * r + 1.6 * y_{2s} + 1.903 * x_5 + 2.465 * x_6 + 2.88 * x_7 + 1.4317 * x_8 + 1.96 * x_9 + 1.528 * x_{10} \leq 20000000; (s = 1, 2, 3)$$

$$0.01405 * x_1 + .00985 * x_2 + .0331 * x_3 + .05781 * r + .07 * y_{2s} + 0.0168 * x_5 + .1726 * x_6 + .12 * x_7 + .448 * x_8 + 0.01853 * x_9 + 0.0452 * x_{10} \leq 300000; (s = 1, 2, 3)$$

$$0.013470 * x_1 + .03616 * x_2 + 0.02338 * x_3 + .02577 * r + .04 * y_{2s} + .02577 * 20.00829 * x_5 + 0.041 * x_6 + .02098 * x_7 + 0.0111 * x_8 + 0.00833 * x_9 + 0.00648 * x_{10} \leq 220000; (s = 1, 2, 3)$$

$$.001717 * x_3 + .00182 * x_6 \leq 29000;$$

$$.00044 * x_1 + .00000580 * x_2 + .01204 * x_3 + 0.0053 * x_5 + 0.00209 * x_8 + .00569 * x_2 + .00118 * x_{10} \leq 35000;$$

$$.00527 * x_1 + .01386 * x_2 + .00928 * x_3 + .011338 * r + .02 * y_{2s} + 0.0004 * x_5 + .01677 * x_6 + .0047 * x_7 + .00662 * x_8 + .00516 * x_9 + .006 * x_{10} \leq 100000; (s = 1, 2, 3)$$

$$.01089 * x_1 + .0063 * x_2 + .01804 * x_3 + .02221 * r + .03 * y_{2s} + 0.0101 * x_5 + .25224 * x_6 + .133824 * x_7 + 0.00924 * x_8 + .01015 * x_9 + .009077 * x_{10} \leq 90000; (s = 1, 2, 3)$$

$$.001807 * x_1 + .00135 * x_2 + .003008 * x_3 + .003497 * r + .005 * y_{2s} + 0.0002 * x_5 + .002568 * x_6 + .004176 * x_7 + .00084 * x_8 + .00197 * x_9 + .000117 * x_{10} \leq 23000; (s = 1, 2, 3)$$

$$.142257 * x_2 + .1143 * x_6 + 0.1254 * x_7 \leq 50000;$$

$$.0145 * x_1 + .0393 * x_2 + .05088 * x_3 + .0155 * r + .02 * y_{2s} + 0.00148 * x_5 + .01245 * x_8 + .013467 * x_9 + .00137 * x_{10} \leq 237000; (s = 1, 2, 3)$$

$$.0001322 * x_1 + .000175 * x_2 + .0018048 * x_3 + .000144 * x_8 + .0001472 * x_{10} \leq 35000;$$

$$.0009135 * x_1 + .01117 * x_2 + .02305 * x_3 + .080802 * r + .16 * y_{2s} + 0.0007 * x_5 + .07 * x_6 + 0.05 * x_7 + .0007 * x_8 + .008771 * x_{10} \leq 100000; (s = 1, 2, 3)$$

$$1.5 * x_1 + 2 * x_2 + 3 * x_3 + 1.5 * r + 1.6 * y_{2s} + 3.5 * x_5 + 3 * x_6 + 3 * x_7 + 1.5 * x_8 + 2.5 * x_9 + 1.6 * x_{10} \leq 25000000; (s = 1, 2, 3)$$

$$6 * x_1 + 8 * x_2 + 12 * x_3 + 8 * r + 8.5 * y_{2s} + 10 * x_5 + 12 * x_6 + 9 * x_7 + 6 * x_8 + 6 * x_9 + 5.5 * x_{10} \leq 60000500; (s = 1, 2, 3)$$

$$.1538 * x_1 + .1538 * x_2 + .1538 * x_3 + .1538 * r + .2 * y_{2s} + .1538 * x_5 + .1538 * x_6 + .1538 * x_7 + .1538 * x_8 + .1538 * x_9 + .1538 * x_{10} \leq 950000; (s = 1, 2, 3)$$

$$.28976 * x_1 + .28976 * x_2 + .28976 * x_3 + .28976 * r + .3 * y_{2s} + 0.5 * x_5 + 0.3 * x_6 + 0.2 * x_7 + 0.3 * x_8 + 0.27 * x_9 + .28 * x_{10} \leq 1700000; (s = 1, 2, 3)$$

$$.014 * x_1 + 0.0108 * x_2 + .0188 * x_3 + .0218 * r + .03 * y_{2s} + 0.2 * x_5 + .1 * x_6 + .02 * x_7 + .03 * x_8 + .015 * x_9 + .011 * x_{10} \leq 5500000; (s = 1, 2, 3)$$

$$r + y_{2s} \geq 4000; \ (s = 1)$$
$$r + y_{2s} \geq 5000; (s = 2)$$
$$r + y_{3s} \geq 6000; \ (s = 3)$$

### 6.3.2.2 Outcome of TSLP (Case-II) after Solving by AMPL

| Product name | Amount to produce |
|---|---|
| $x_1\ (T-shirt)$ | 3031690 |
| $x_2(Vest)$ | 0 |
| $x_3(Sweat\ shirt)$ | 2796180 |
| $x_4 = r\ (Leggings\ as\ 1st\ stage\ variable)$ | 4000 |
| $y_{21}(purchased\ amount\ from\ external\ source\ in\ 1st\ scenario)$ | 0 |
| $y_{22}(purchased\ amount\ from\ external\ source\ in\ 2nd\ scenario)$ | 1000 |
| $y_{23}(purchased\ amount\ from\ external\ source\ in\ 3rd\ scenario)$ | 2000 |
| $x_5(Shirt)$ | 0 |
| $x_6(Jacket)$ | 0 |
| $x_7(Jogger)$ | 47770 |
| $x_8(Polo\ shirt)$ | 0 |
| $x_9(Casual\ shirt)$ | 0 |
| $x_{10}(Girl's\ top)$ | 0 |
| $\boldsymbol{Minimum\ cost\ i.e.Maximum\ profit}$ | $\mathbf{6545630}$ |

Table 6.2: Outcome by AMPL of Two stage model of case II

So this two-stage stochastic model is suggested to produce 4000 pieces of leggings ($x_4\ and\ in\ this\ case\ \boldsymbol{r = 4000}$) as a first stage production and after revealing the demand, there is no need to collect or buy any amount from external supplier/s for scenario 1. And in the case of scenario 2 & 3 the collection from the external source would be 1000 and 2000 pieces, respectively $(y_{22} = 1000\ \&\ y_{23} = 2000)$ to comply with the client's demand of 5000 and 6000. The production of other products is also proposed as per shown in Table 6.2. The optimal profit in this case is $6545630.

In the next section, these two-stage stochastic (TSLP) models are extended to a multistate stochastic model (MSLP) by adding additional stages, as the accuracy would be better in adding more stages by allocating more time to reveal uncertainty.

## 6.4 Multistage Stochastic Linear Programming Model (MSLP)

In a multiple-stage model, decisions are made over several time periods (or stages), and uncertainty is gradually exposed over time. As the process progresses, new information

becomes available, and each step builds upon earlier decisions (e.g., demand in future periods is revealed gradually). Typically, the problem is presented as a multi-stage decision-making process with decision variables that are contingent on the state at each stage. An extension of two-stage stochastic programming (TSLP), a Multistage Stochastic Linear Programming (MSLP) model is intended to manage decision-making processes under uncertainty over a number of time periods. In MSLP, choices are made in a step-by-step manner over a number of phases, with each step gradually revealing more uncertainty. This method is especially helpful in situations where choices made at one point in time affect possibilities and results later on, such as supply chain management, energy systems, and financial planning etc.

$$\min_{x_1,\ x_2(\xi_1),\ldots,\ x_T(\xi_{1:T-1})} E\left[\sum_{t=1}^{T} c_t(x_t,\ \xi_t)\right]$$

subject to

$$A_t x_t + B_t x_{t-1} = b_t(\xi_t), \quad \forall t = 1, \ldots, T$$

where

- $x_t$: *Decision variables at stage* $t$
- $\xi_t$ : *Random variables at stage* $t$ *representing uncertainty*.
- $A_t, B_t$ : *Matrices defining constraints at stage* $t$.
- $b_t(\xi_t)$: *Constraints that depend on uncertain parameters*.

$$x_t \geq 0, \quad \forall t = 1, \ldots, T$$

### 6.4.1 Advantages of MSLP Compare to TSLP

- MSLP captures better real-world issues where choices change over time (e.g., financial planning, supply chain management).
- Captures how decisions are made step-by-step, with uncertainty being revealed in phases, which can result in better-informed, flexible choices.
- Can offer superior solutions for issues where decisions are based on past performance and uncertainty changes over time.

### 6.4.2 Disadvantages of MSLP in Comparison to TSLP

- Compared to a single-stage SLP, it is far more difficult to formulate and solve.
- Increased computational cost, particularly for big problems with lots of steps and possible outcomes.
- Calls for more complex strategies (such as progressive hedging, scenario tree representation, or decomposition techniques).

## 6.5 Extension of the Two-Stage Stochastic (TSLP) Model to Multistage (MSLP) Model

The term "multistage" refers to the fact that choices are taken in stages. Multistage Stochastic Linear Programming (MSLP) Model is a mathematical framework used to optimize decisions over multiple time stages under uncertainty. So, for revealing the uncertainty noticeably and after realizing the stochastic amount, the two-stage one is extended to the 3rd stage i.e. multi-stage stochastic model, and the similar two cases of (TSLP) are measured for presenting the multistage stochastic models.

### 6.5.1 Case 1: The Sell or Demand can be of any Number of the Production

Generally, the monthly demand of leggings ($x_4$) is more or less than 5000 for most of the months. Now in this case to formulate a MSLP model a full month's expected demand is divided into two portions for smooth or appropriate production quantity. Here, 2 situations can arise for the 2nd stage (1st half of a month) and 4 scenarios can occur for the 3rd stage (2nd half of a month). Primarily, it is considered that the industry produces $x_4(r)$ pieces of leggings for the 1st (first) half of a month or for the first 15 days (here the monthly production of leggings is denoted by $\boldsymbol{r}$ instead of $x_4$) of a month without realizing the demand and the production cost is $2.16 per piece. This $\boldsymbol{r}$ is the **1st stage production** without revealing the uncertainty. Next, after revealing the demand of 1st half of a particular month, the production schedule for the last half (the 2nd stage decision) of that month will be fixed, and again, after observing the demand of the immediate next stage, the production schedule of the further stage will be fixed and this procedure will go on similarly.

#### 6.5.1.1 MSLP Model Formulation with Scenario Generation (case I)

The two (2) raised scenarios for the 2nd stage stochastic model are:

**Scenario 1:** Due to the fluctuation of seasonal demand, the expected demand of leggings ($x_4$) could be either 2500 with probability $2/3$ for the 1st (first) 15 days of a specific month, and the actual sell or demand would be $y_{21}$ or $y_{22}$ for this scenario with a selling price of \$2.6 per piece.

**Scenario 2:** The expected demand of leggings ($x_4$) could be 3500 pieces with probability $1/3$ for the 1st (first) 15 days of a month, and the actual sell or demand would be $y_{23}$ or $y_{24}$ for this scenario.

After realizing the actual demand in this stage $\boldsymbol{y_{21}/y_{22}}$ ***and*** $\boldsymbol{y_{23}/y_{24}}$, now they will produce $r_{21}$ or $r_{22}$ for the rest 15 days of the month. And this production $\boldsymbol{r_{21}}$**or** $r_{22}$ and $r_{23}$ or $r_{24}$ are the 2nd stage decision for 1st scenario and 2nd scenario respectively which are to be decided.

And four (4) other scenarios will arise for the last half of a month in the third (3rd) stage.

**Scenario 1:** If scenario 1 occurs in the 2nd stage, then again other two situations may arise in the 3rd stage. The expected demand can be either 3000 with probability $1/4$ for the last 15 days of that specific month. The actual demand is measured as $y_{31}$.

**Scenario 2:** Or the demand of leggings ($x_4$) would be 2000 with probability $3/4$ for the last 15 days of that specific month. The actual demand is measured as $y_{32}$.

**Scenario 3:** If the expected demand is 3500 (associated with scenario 2 of the 2nd stage) in 1st (first)15 days of that month then the demand can be either 2500 with probability $1/2$ in next 15 days of a month and the actual demand is measured as $y_{33}$.

**Scenario 4:** Or the expected demand is 4500 with probability $1/2$ for the last 15 days of that particular month. The actual demand is measured as $y_{34}$.

Thus the sell will be $\boldsymbol{y_{31}, y_{32}, y_{33}}$ ***and*** $\boldsymbol{y_{34}}$ **for the scenario** $\boldsymbol{1, 2, 3}$ ***and*** $\boldsymbol{4}$ **respectively which are the 3rd stage's uncertain decision and are to be revealed** according to the demand with a selling price of \$2.6 per piece. After realizing the actual demand of this stage next production schedule would be finalized and this procedure will go on similarly. In this study it is shown up to this stage but by applying the proposed general form of multistage stochastic model, RMG organization can be benefited to follow a better scheduling of production so that cost could be minimized along with profit maximization and of course the uncertainty would be mitigated.

The scenarios with all the probabilities are shown in Figure 6.3 as a diagram.

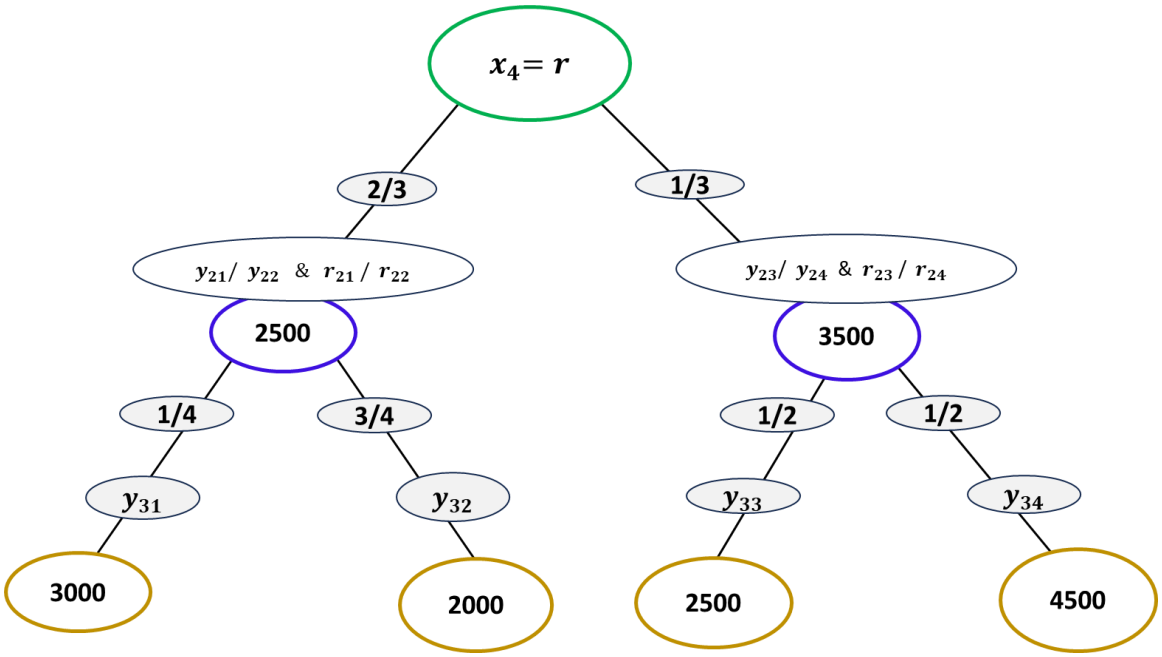



Figure 6.3: Scenario generation with probability distribution of MSLP (Case-I)

Here the objective function is considered as cost minimization type and the model is

$$\textbf{\textit{Minimize Z}} = 2.16r - (x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + 1.5 * x_5 + 0.95 * x_{10}) + \sum_{s=1}^{4}(P_s * (2.16 * r_{2s} - 2.6 * y_{2s} - 2.6 * y_{3s}))$$

where $P_s = \left(\frac{2}{3} * \frac{1}{4}\right), \left(\frac{2}{3} * \frac{3}{4}\right), \left(\frac{1}{3} * \frac{1}{2}\right)$ *and* $\left(\frac{1}{3} * \frac{1}{2}\right)$ *respectively for* $s = 1, 2, \ldots, 4$

***subject to***

$$1.05667 * x_1 + 0.6317 * x_2 + 1.334 * x_3 + 1.5021 * r + 1.5021 * r_{2s} + 1.903 * x_5 + 2.465 * x_6 + 2.88 * x_7 + 1.4317 * x_8 + 1.96 * x_9 + 1.528 * x_{10} \leq 20000000; \ (s = 1, 2, 3, 4)$$

$$0.01405 * x_1 + .00985 * x_2 + .0331 * x_3 + .05781 * r + .05781 * r_{2s} + .0168 * x_5 + .1726 * x_6 + .12 * x_7 + .448 * x_8 + 0.01853 * x_9 + 0.0452 * x_{10} \leq 300000; \ (s = 1, 2, 3, 4)$$

$$0.013470 * x_1 + .03616 * x_2 + 0.02338 * x_3 + .02577 * r + .02577 * r_{2s} + .02577 * x_5 + 0.041 * x_6 + .02098 * x_7 + 0.0111 * x_8 + 0.00833 * x_9 + 0.00648 * x_{10} \leq 220000; \ (s = 1, 2, 3, 4)$$

$$.001717 * x_3 + .00182 * x_6 \leq 29000;$$

$$.00044 * x_1 + .00000580 * x_2 + .01204 * x_3 + 0.0053 * x_5 + 0.00209 * x_8 + .00569 * x_9 + .00118 * x_{10} \leq 35000;$$

$$.00527 * x_1 + .01386 * x_2 + .00928 * x_3 + .011338 * r + .011338 * r_{2s} + 0.0004 * x_5 + .01677 * x_6 + .0047 * x_7 + .00662 * x_8 + .00516 * x_9 + .006 * x_{10} \leq 100000; (s = 1,2,3,4)$$

$$.01089 * x_1 + .0063 * x_2 + .01804 * x_3 + .02221 * r + .02221 * r_{2s} + .0.0101 * x_5 + .25224 * x_6 + .133824 * x_7 + 0.00924 * x_8 + .01015 * x_9 + .009077 * x_{10} \leq 90000; (s = 1,2,3,4)$$

$$.001807 * x_1 + .00135 * x_2 + .003008 * x_3 + .003497 * r + .003497 * r_{2s} + 0.0002 * x_5 + .002568 * x_6 + .004176 * x_7 + .00084 * x_8 + .00197 * x_9 + .000117 * x_{10} \leq 23000; (s = 1,2,3,4)$$

$$.142257 * x_2 + .1143 * x_6 + 0.1254 * x_7 \leq 50000;$$

$$.0145 * x_1 + .0393 * x_2 + .05088 * x_3 + .05055 * r + .05055 * r_{2s} + 0.00148 * x_5 + .01245 * x_8 + .013467 * x_9 + .00137 * x_{10} \leq 237000; (s = 1,2,3,4)$$

$$.0001322 * x_1 + .000175 * x_2 + .0018048 * x_3 + .000144 * x_8 + .0001472 * x_{10} \leq 35000;$$

$$.0009135 * x_1 + .01117 * x_2 + .02305 * x_3 + .080802 * r + .080802 * r_{2s} + 0.0007 * x_5 + .07 * x_6 + 0.05 * x_7 + .0007 * x_8 + .008771 * x_{10} \leq 100000; (s = 1,2,3,4)$$

$$1.5 * x_1 + 2 * x_2 + 3 * x_3 + 1.5 * r + 1.5 * r_{2s} + 3.5 * x_5 + 3 * x_6 + 3 * x_7 + 1.5 * x_8 + 2.5 * x_9 + 1.6 * x_{10} \leq 25000000; (s = 1,2,3,4)$$

$$6 * x_1 + 8 * x_2 + 12 * x_3 + 8 * r + 8 * r_{2s} + 10 * x_5 + 12 * x_6 + 9 * x_7 + 6 * x_8 + 6 * x_9 + 5.5 * x_{10} \leq 60000500; (s = 1,2,3,4)$$

$$.1538 * x_1 + .1538 * x_2 + .1538 * x_3 + .1538 * r + .1538 * r_{2s} + .1538 * x_5 + .1538 * x_6 + .1538 * x_7 + .1538 * x_8 + .1538 * x_9 + .1538 * x_{10} \leq 950000; (s = 1,2,3,4)$$

$$.28976 * x_1 + .28976 * x_2 + .28976 * x_3 + .28976 * r + .28976 * r_{2s} + 0.5 * x_5 + 0.3 * x_6 + 0.2 * x_7 + 0.3 * x_8 + 0.27 * x_9 + .28 * x_{10} \leq 1700000; (s = 1,2,3,4)$$

$$.014 * x_1 + 0.0108 * x_2 + .0188 * x_3 + .0218 * r + .0218 * r_{2s} + 0.2 * x_5 + .1 * x_6 + .02 * x_7 + .03 * x_8 + .015 * x_9 + .011 * x_{10} \leq 5500000; (s = 1,2,3,4)$$

$$y_{2s} \leq r; \quad (s = 1,2)$$
$$y_{2s} \leq 2500; \quad (s = 1,2)$$
$$y_{21} = y_{22};$$
$$y_{2s} \leq r; \quad (s = 3,4)$$
$$y_{2s} \leq 3500; \quad (s = 3,4)$$
$$y_{23} = y_{24};;$$
$$y_{3s} \leq r_{2s} + (r - y_{2s}); \quad (s = 1,2,3,4)$$
$$y_{3s} \leq 3000 \; (s = 1)$$
$$y_{3s} \leq 2000; (s = 2)$$
$$y_{3s} \leq 2500; \quad (s = 3)$$
$$y_{3s} \leq 4500; (s = 4)$$
$$r_{21} = r_{22};$$
$$r_{23} = r_{24};$$

where

$r$ = Production of leggings (for the 1st half of a month) in first stage without revealing uncertainty

$y_{21}$= $y_{22}$ = Demand of 2nd stage for situation 1

$y_{23}$= $y_{24}$= Demand of 2nd stage for situation 2

$r_{21}$/ $r_{22}$ = production in the 2nd stage for scenario 1 (after knowing the demand of 1st 15 days)

$r_{23}$ / $r_{24}$ = production in the 2nd stage for scenario 2 (after knowing the demand of 1st 15 days)

$y_{31}$= Demand of 3rd stage for scenario 1

$y_{32}$ = Demand of 3rd stage for scenario 2

$y_{33}$ = Demand of 3rd stage for scenario 3

$y_{34}$ = Demand of 3rd stage for scenario 4

### 6.5.1.2 Outcome of MSLP after Solving by AMPL (Case-I)

| Product name | Amount to produce |
|---|---|
| $x_1\ (T-shirt)$ | 3034120 |
| $x_2(Vest)$ | 0 |
| $x_3(Sweat\ shirt)$ | 2796100 |
| $x_4 = r\ (Leggings\ as\ 1st\ stage\ production)$ | 3500 |
| $r_{21} = r_{22} = (2nd\ stage\ production\ in\ 1st\ scenario)$ | 0 |
| $r_{23} = r_{24} = (2nd\ stage\ production\ in\ 2nd\ scenario)$ | 2500 |
| $y_{21} = y_{22} = (Demand\ of\ 2nd\ stage\ in\ 1st\ scenario)$ | 2500 |
| $y_{23} = y_{24} = (Demand\ of\ 2nd\ stage\ in\ 2nd\ scenario)$ | 3500 |
| $y_{31}(Demand\ of\ 3rd\ stage\ in\ 1st\ scenario)$ | 1000 |
| $y_{32}(Demand\ of\ 3rd\ stage\ in\ 2nd\ scenario)$ | 1000 |
| $y_{33}(Demand\ of\ 3rd\ stage\ in\ 3rd\ scenario)$ | 2500 |
| $y_{34}(Demand\ of\ 3rd\ stage\ in\ 4th\ scenario)$ | 2500 |
| $x_5(Shirt)$ | 0 |
| $x_6(Jacket)$ | 0 |
| $x_7(Jogger)$ | 48116 |
| $x_8(Polo\ shirt)$ | 0 |
| $x_9(Casual\ shirt)$ | 0 |
| $x_{10}(Girl's\ top)$ | 0 |
| $\boldsymbol{Minimum\ cost\ i.e.\ Maximum\ profit}$ | $\mathbf{6580820} |

Table 6.3: Outcome by AMPL of the MSLP (Case I)

Table 6.3 presents that this multistage stochastic model is suggested to produce 3500 pieces of leggings ($x_4$ *and in this case* $\boldsymbol{r = 3500}$) as first stage production for the first half of a month, and the demand is either 2500 (scenario 1), then there is no need for extra production for the last half of that month ( $r_{21}$ *and* $r_{22}$= 0). If the demand is 2500 for the first half, then the demand for the last half could be any amount in scenario 1 and 2 in the 3rd stage, respectively and in this case, the model offered 1000 pieces of demand for the 1st and 2nd scenarios of the 3rd stage ($y_{31}$ *and* $y_{32} = 1000$) for the optimum profit. If scenario 2 occurs in the 2nd stage, then the recommended production of the 2nd stage is ($r_{23} = r_{24} = 2500$) and the demand obtained by the solution of 3rd and 4th scenarios is 2500 pieces ($y_{33}$ *and* $y_{34} = 2500$). The production of other products is also shown in the Table 6.3. The optimal profit in this case is $6580820.

### 6.5.2 Case 2: The Company has to Meet Customers' Demand Completely

According to the demand, there is a restriction of the production of leggings. Here it is considered that the monthly demand of leggings is divided in two parts for developing a MSLP model. Firstly, the demand of leggings ($x_4$) is either 2500 with probability 1/2 or 3500 with probability 1/2 in the 1st half of a month, and if the demand is 2500, then the expected demand can be either 3000 with probability 2/3 or 2000 with probability 1/3 for the last 15 days of that month. Similarly, if the demand is 3500 then in 1st half (scenario 2 of 2nd stage) of that month then the demand can be either 2500 with probability 1/2 or 4500 with probability 1/2 for the last 15 days of that month. Here, it is considered that they must meet the demand. If their production is less than the demand or in the case of a shortage of products of any amount, they have to collect it from an external supplier to meet the extra demand. Let them first produce $x_4$ $(r)$ pieces. If it is less than 2500, then they buy $y_{21}/y_{22}$ from an external supplier so that they can meet the demand of 2500 pieces and the purchasing price of $2.5 per piece. In the next stage, after realizing the demand of 1st 15 days, they are going to produce $r_{21/}\, r_{22}$ and $r_{23}/r_{24}$ for scenario 1 and scenario 2 respectively, to meet the demand of 3000, 2000 and 2500, 4500, respectively. And if they can't fulfill their demand by their production in the next stage, they have to again collect $y_{31}, y_{32}, y_{33}$ *and* $y_{34}$ for scenario $1, 2, 3,$ *and* 4 respectively from external suppliers in the third stage to meet the current demand. And the algebraic model for mandatorily meeting the demand is presented in the next section.

#### 6.5.2.1 MSLP Model Formulation with Scenario Generation (case II)

The two (2) raised situations for the 2nd stage decision are:

**Scenario 1:** Due to the fluctuation of seasonal demand, the expected demand of leggings ($x_4$) could be either 2500 with probability 1/2 for the 1st(first) 15 days of a specific month and suppose the production is less than 2500, then they are going to collect $y_{21}/y_{22}$ from external source with the cost of \$2.5 per piece.

**Scenario 2:** The demand of leggings ($x_4$) could be 3500 pieces with probability 1/2 for the 1st(first) 15 days of a month and suppose the production is less than 3500, then they are going to accumulate $y_{23}/y_{24}$ for this scenario, from the external source with the cost of \$2.5 per piece.

After realizing the demand in this stage now the organization will produce $r_{21}$ /$r_{22}$ and $r_{23}/r_{24}$ respectively for scenario 1 and scenario 2 for the rest 15 days of the month. And this production $r_{21}/r_{22}/r_{24}/\ r_{24}$ **is the 2nd stage decision**.

And four (4) other scenarios will be arisen for the last half of that particular month in third (3rd stage)

**Scenario 1:** If the expected demand is 2500 (associated with scenario 1 of 2nd stage) occurs for the 1st (first) 15 days, then the demand can be either 3000 with probability 2/3 for the last 15 days of that specific month. and again if the production is less than 3000 then they are going to buy $y_{31}$ for this scenario from the external source to fulfill the customer demand with the cost of \$2.5 per piece.

**Scenario 2:** Or the expected demand of leggings ($x_4$) would be 2000 with probability 1/3 for the last 15 days of that specific month and if the production is less than 2000 then they would buy $y_{32}$ for meeting the extra demand from the external source with the cost of \$2.5 per piece.

**Scenario 3:** If the expected demand is 3500 (associated with scenario 2 of 2nd stage) occurs in 1st (first)15 days of that month then the actual demand can be either 2500 with probability $1/2$ in next 15 days of a month. They will go for a collection of $y_{33}$ amount from other sources to meet the customer demand in case of shortage of production.

**Scenario 4:** Or demand is 4500 with probability 1/2 for the last 15 days of that particular month. They will again go for a collection from other source/s to meet the customer demand in

case of a shortage of production. And for this scenario the collected amount from external source is considered as $y_{34}$.

And these bought amount $\boldsymbol{y_{31}, y_{32}, y_{33}}$ ***and*** $\boldsymbol{y_{34}}$ **for the scenario 1, 2, 3** ***and*** **4 respectively are the 3rd stage's uncertain decisions which are to be revealed** according to the demand.

The scenarios with all the probabilities are shown in Figure 6.4 as a diagram.

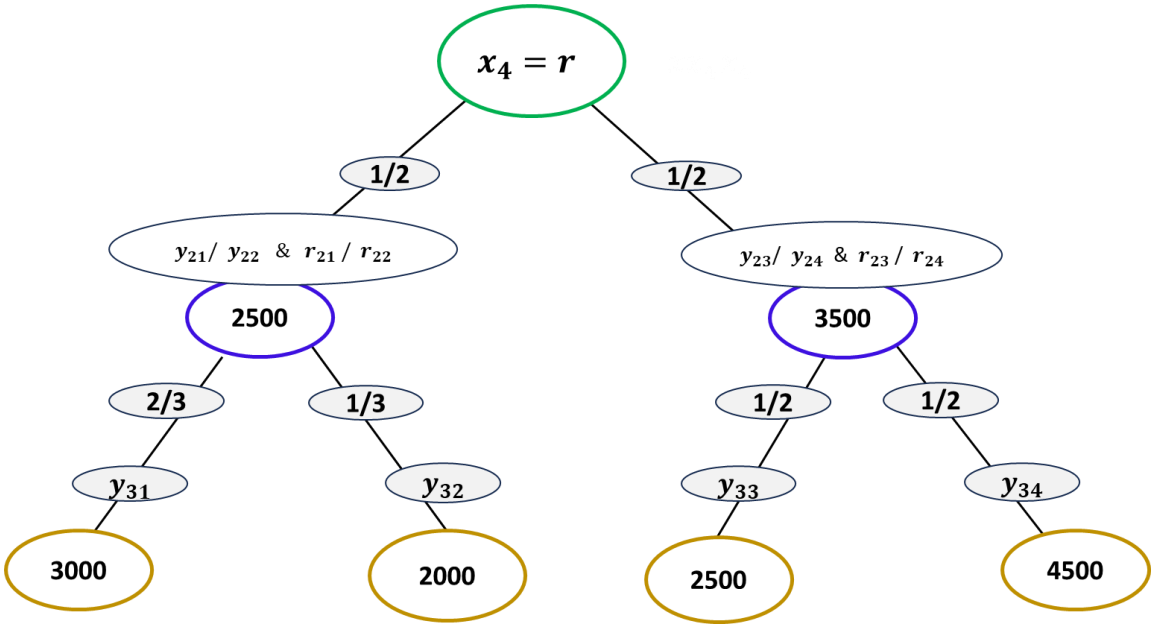



Figure 6.4: Scenario generation with probability distribution of Multistage model (Case-II)

Consider the first stage, the constraints to ensure customer demand is satisfied are:

$$r + y_{2s} \geq 2500; (s = 1, 2)$$
$$r + y_{2s} \geq 3500; (s = 3,4)$$

At the second stage they will have units left over (i.e. inventory) to help meet future demand. This inventory level will be:

$$r + y_{2s} - 2500;$$
$$r + y_{2s} - 3500;$$

To ensure that demand is met in third stage we have: inventory + amount produced + amount bought externally ≥ demand

$$r + y_{2s} - 2500 + r_{2s} + y_{3s} \geq 3000; (s = 1)$$
$$r + y_{2s} - 2500 + r_{2s} + y_{3s} \geq 2000; (s = 2)$$
$$r + y_{2s} - 3500 + r_{2s} + y_{3s} \geq 2500; (s = 3)$$
$$r + y_{2s} - 3500 + r_{2s} + y_{3s} \geq 4500; (s = 4)$$

Thus, the algebraic form of the model is

$$\textbf{\textit{Minimize z}} = 2.16r - (x_1 + 0.8 * x_2 + 1.2469 * x_3 + 0.99 * x_6 + 1.21 * x_7 + 0.79 * x_8 + 1.005 * x_9 + 1.5 * x_5 + 0.95 * x_{10}) + \sum_{s=1}^{4} P_s * (2.16 * r_{2s} + 2.5 * y_{2s} + 2.5 * y_{3s})$$

where $P_s = \left(\frac{1}{2} * \frac{2}{3}\right), \left(\frac{1}{2} * \frac{1}{3}\right), \left(\frac{1}{2} * \frac{1}{2}\right)$ *and* $\left(\frac{1}{2} * \frac{1}{2}\right)$ *respectively for* $s = 1, 2, \ldots, 4$

***Subject to***

$$1.05667 * x_1 + 0.6317 * x_2 + 1.334 * x_3 + 1.5021 * r + 1.5021 * r_{2s} + 1.6 * y_{2s} + 1.6 * y_{3s} + 1.903 * x_5 + 2.465 * x_6 + 2.88 * x_7 + 1.4317 * x_8 + 1.96 * x_9 + 1.528 * x_{10} \leq 20000000; (s = 1, 2, 3, 4)$$

$$0.01405 * x_1 + .00985 * x_2 + .0331 * x_3 + .05781 * r + .05781 * r_{2s} + .07 * y_{2s} + .07 * y_{3s} + 0.0168 * x_5 + .1726 * x_6 + .12 * x_7 + .448 * x_8 + 0.01853 * x_9 + 0.0452 * x_{10} \leq 300000; (s = 1, 2, 3, 4)$$

$$0.013470 * x_1 + .03616 * x_2 + 0.02338 * x_3 + .02577 * r + .02577 * r_{2s} + .04 * y_{2s} + .04 * y_{3s} + .02577 * x_5 + 0.041 * x_6 + .02098 * x_7 + 0.0111 * x_8 + 0.00833 * x_9 + 0.00648 * x_{10} \leq 220000; (s = 1, 2, 3, 4)$$

$$.001717 * x_3 + .00182 * x_6 \leq 29000;$$

$$.00044 * x_1 + .00000580 * x_2 + .01204 * x_3 + 0.0053 * x_5 + 0.00209 * x_8 + .00569 * x_9 + .00118 * x_{10} \leq 35000;$$

$$.00527 * x_1 + .01386 * x_2 + .00928 * x_3 + .011338 * r + .011338 * r_{2s} + .02 * y_{2s} + .02 * y_{3s} + 0.0004 * x_5 + .01677 * x_6 + .0047 * x_7 + .00662 * x_8 + .00516 * x_9 + .006 * x_{10} \leq 100000; (s = 1, 2, 3, 4)$$

$$.01089 * x_1 + .0063 * x_2 + .01804 * x_3 + .02221 * r + .02221 * r_{2s} + .03 * y_{2s} + .03 * y_{3s} + 0.0101 * x_5 + .25224 * x_6 + .133824 * x_7 + 0.00924 * x_8 + .01015 * x_9 + .009077 * x_{10} \leq 90000; (s = 1, 2, 3, 4)$$

$$.001807 * x_1 + .00135 * x_2 + .003008 * x_3 + .003497 * r + .003497 * r_{2s} + .005 * y_{2s} + .005 * y_{3s} + 0.0002 * x_5 + .002568 * x_6 + .004176 * x_7 + .00084 * x_8 + .00197 * x_9 + .000117 * x_{10} \leq 23000; (s = 1, 2, 3, 4)$$

$$.142257 * x_2 + .1143 * x_6 + 0.1254 * x_7 \leq 50000;$$

$$.0145 * x_1 + .0393 * x_2 + .05088 * x_3 + .0155 * r + .0155 * r_{2s} + .02 * y_{2s} + .02 * y_{3s} + .0.00148 * x_5 + .01245 * x_8 + .013467 * x_9 + .00137 * x_{10} \leq 237000; (s = 1, 2, 3, 4)$$

$$.0001322 * x_1 + .000175 * x_2 + .0018048 * x_3 + .000144 * x_8 + .0001472 * x_{10} \leq 35000;$$

$$.0009135 * x_1 + .01117 * x_2 + .02305 * x_3 + .080802 * r + .080802 * r_{2s} + .16 * y_{2s} + .16 * y_{3s} + 0.0007 * x_5 + .07 * x_6 + 0.05 * x_7 + .0007 * x_8 + .008771 * x_{10} \leq 100000; (s = 1, 2, 3, 4)$$

$$1.5 * x_1 + 2 * x_2 + 3 * x_3 + 1.5 * r + 1.5 * r_{2s} + 1.6 * y_{2s} + 1.6 * y_{3s} + 3.5 * x_5 + 3 * x_6 + 3 * x_7 + 1.5 * x_8 + 2.5 * x_9 + 1.6 * x_{10} \leq 25000000; (s = 1, 2, 3, 4)$$

$$6 * x_1 + 8 * x_2 + 12 * x_3 + 8 * r + 8 * r_{2s} + 8.5 * y_{2s} + 8.5 * y_{3s} + 10 * x_5 + 12 * x_6 + 9 * x_7 + 6 * x_8 + 6 * x_9 + 5.5 * x_{10} \leq 60000500; (s = 1, 2, 3, 4)$$

$$.1538 * x_1 + .1538 * x_2 + .1538 * x_3 + .1538 * r + .1538 * r_{2s} + .2 * y_{2s} + .2 * y_{3s} + .1538 * x_5 + .1538 * x_6 + .1538 * x_7 + .1538 * x_8 + .1538 * x_9 + .1538 * x_{10} \leq 950000; (s = 1, 2, 3, 4)$$

$$.28976 * x_1 + .28976 * x_2 + .28976 * x_3 + .28976 * r + .28976 * r_{2s} + .3 * y_{2s} + .3 * y_{3s} + 0.5 * x_5 + 0.3 * x_6 + 0.2 * x_7 + 0.3 * x_8 + 0.27 * x_9 + .28 * x_{10} \leq 1700000; (s = 1, 2, 3, 4)$$

$$.014 * x_1 + 0.0108 * x_2 + .0188 * x_3 + .0218 * r + .0218 * r_{2s} + .03 * y_{2s} + .03 * y_{3s} + 0.2 * x_5 + .1 * x_6 + .02 * x_7 + .03 * x_8 + .015 * x_9 + .011 * x_{10} \leq 5500000; (s = 1, 2, 3, 4)$$

$$r + y_{2s} \geq 2500 \quad (s = 1, 2)$$
$$r + y_{2s} \geq 3500 \quad (s = 3, 4)$$
$$r + y_{2s} - 2500 + r_{2s} + y_{3s} \geq 3000; (s = 1)$$
$$r + y_{2s} - 2500 + r_{2s} + y_{3s} \geq 2000; (s = 2)$$
$$r + y_{2s} - 3500 + r_{2s} + y_{3s} \geq 2500; (s = 3)$$
$$r + y_{2s} - 3500 + r_{2s} + y_{3s} \geq 4500; (s = 4)$$
$$y_{21} = y_{22}$$
$$y_{23} = y_{24}$$
$$r_{21} = r_{22}$$
$$r_{23} = r_{24}$$

where

$r_{21} = r_{22}$ = Production of the 2nd stage for scenario 1(after knowing the demand of 1st 15 days)

$y_{21} = y_{22}$ = Purchased amount in 2nd stage from the external supplier for situation 1 (if demand exceeds the production)

$r_{23} = r_{24}$ =Production of the 2nd stage for scenario 2 (after knowing the demand of 1st 15 days)

$y_{23} = y_{24}$ =Purchased amount in 2nd stage from the external supplier for scenario 2 (if demand exceeds the production)

$y_{31}$ = Purchased amount in 3rd stage from the external supplier for scenario 1 (if demand exceeds the production)

$y_{32}$ = Purchased amount in 3rd stage from the external supplier for scenario 2 (if demand exceeds the production)

$y_{33}$ = Purchased amount in 3rd stage from the external supplier for scenario 3 (if demand exceeds the production)

$y_{34}$ = Purchased amount in 3rd stage from the external supplier for scenario 4 (if demand exceeds the production)

### 6.5.2.2 Outcome of MSLP (Case-II) after Solving by AMPL

| Product name | Amount to produce |
|---|---|
| $x_1\ (T-shirt)$ | 3029750 |
| $x_2(Vest)$ | 0 |
| $x_3(Sweat\ shirt)$ | 2796250 |
| $x_4 = r\ (Leggings\ as\ 1st\ stage\ production)$ | 3500 |
| $r_{21} = r_{22}\ (2nd\ stage\ production\ in\ 1st\ scenario\ after\ revealing\ the\ demand)$ | 1000 |
| $r_{23} = r_{24}\ (2nd\ stage\ production\ in\ 2nd\ scenario\ after\ revealing\ the\ demand)$ | 2500 |
| $y_{21} = y_{22}(purchased\ amount\ of\ 2nd\ stage\ in\ 1st\ scenario)$ | 0 |
| $y_{23} = y_{24}(purchased\ amount\ of\ 2nd\ stage\ in\ 2nd\ scenario)$ | 0 |
| $y_{31}(purchased\ amount\ of\ 3rd\ stage\ in\ 1st\ scenario)$ | 1000 |
| $y_{32}(purchased\ amount\ of\ 3rd\ stage\ in\ 2nd\ scenario)$ | 0 |
| $y_{33}(purchased\ amount\ of\ 3rd\ stage\ in\ 3rd\ scenario)$ | 0 |
| $y_{34}(purchased\ amount\ of\ 3rd\ stage\ in\ 4th\ scenario)$ | 2000 |
| $x_5(Shirt)$ | 0 |
| $x_6(Jacket)$ | 0 |
| $x_7(Jogger)$ | 47587 |
| $x_8(Polo\ shirt)$ | 0 |
| $x_9(Casual\ shirt)$ | 0 |
| $x_{10}(Girl's\ top)$ | 0 |
| $\boldsymbol{Minimum\ cost\ i.e.Maximum\ profit}$ | **$6560560** |

Table 6.4: Output of MSLP by AMPL of case II

Table 6.4 thus shows that, this multistage stochastic model suggested to produce 3500 pieces of leggings ($x_4\ and\ in\ this\ case\ r = 3500$) as first stage production for the first half of a month without the realization of demand. and when the demand is either 2500 (scenario 1) then the recommended production for last half of that month ( $r_{21} = 1000$). Since the demand is 2500 for the first half then the demand for the last half would be either 3000 *or* 2000 in scenario 1 and 2 in 3rd stage respectively so the rest of the 1000 units in 1st stage and the new production of 2nd stage $r_{21} = 1000$ is needed in this stage to meet the demand 2000 units of scenario $s_{32}$ (3rd stage 2nd scenario) and also extra amount is needed to buy from external source/s $(y_{31} = 1000)$ to meet the demand of 3000 units in scenario 1. But if scenario 2 occurs in 2nd stage then the model proposed 2500 pieces of production in this stage ( $r_{22} = 2500$) and also have to buy 2000 units of products from exterior supplier to meet the extra customers' demand 4500 in $s_{34}$. Thus,

it recommends $(y_{33} = 0 \; and \; y_{34} = 2000)$. The production of other products found by AMPL is presented in Table 6.4. The optimal profit in this case is $6560560.

## 6.6 Comparison between MSLP and TSLP

The particular situation will determine whether to use two-stage or multistage stochastic models. Two-stage models (TSLP) work well in situations when computational simplicity is required and decisions must be taken up front with flexibility.

Despite their greater computing complexity, multistage models (MSLP) are better suited for situations where uncertainties change over time and adaptive decision-making is required.

A Multistage Stochastic LP model is probably preferable if one's decision-making process is dynamic by nature (i.e., decisions made today impact future results) and anyone anticipate that uncertainty will change or evolve over time (as in the case of inventory management, energy production, or financial investment, for example). Over time, it enables ones to make more flexible and knowledgeable decisions (Hu and Hu, 2018).

Because of its easiness to solve and less expensive to compute, a two-stage LP may be adequate if one's decisions are more immediate or if the uncertainty can be regarded as static. But, multistage SLP is better at simulating real-world issues with sequential decisions and changing uncertainty, but it has a higher computational cost and complexity.

For simpler, one-shot decision problems under uncertainty where a simpler solution is adequate, stochastic LP works well in that case.

In conclusion, by enabling adaptive decision-making, improved risk management, and the optimization of long-term objectives, the multistage stochastic model provides notable benefits in production planning under uncertainty. It helps businesses to make better decisions that adapt to the shifting supply and demand dynamics over time, which leads to increased customer satisfaction, lower costs, and increased efficiency.

### 6.6.1 Findings from Both of the cases of TSLP and MSLP Models in terms of Profit

| Name of the two cases | Profit obtained by Two-Stage Model (TSLP) | Profit obtained by Multistage Model (MSLP) |
|---|---|---|
| **The sell or demand can be of any number of the production** | **$6354270** | **$6580820** |
| **The Company has to Meet Customers' Demand Completely** | **$6545630** | **$6560560** |

Table 6.5: Comparison between Two Stage (TSLP) and Multistage Model (MSLP)

From the Table 6.5 above, it is found that the multistage stochastic model (MSLP) gives a better result in both of the cases in terms of profit, though it is very much minimal. And in the 1st case it gives about 3.5% much better result than the two-stage model's (TSLP) solution.

A graphical presentation of the comparison between TSLP and MSLP are presented below in Figure 6.5

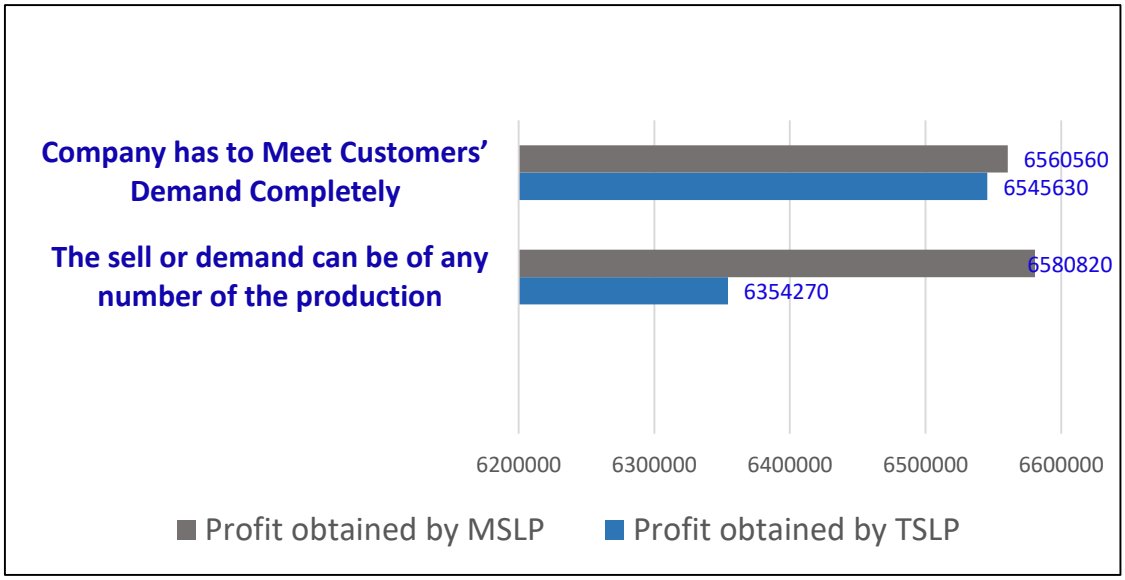



Figure 6.5: Graphical presentation on comparison between TSLP and MSLP

## 6.7 Conclusion

In this chapter, some models on two-stage and multistage stochastic ground are formulated where several cases with various scenarios of RMG industry are discussed. The outcomes of those models presented and offered the production planning for gaining optimal profit along with managing uncertainties in different cases for the garment industry of Bangladesh. Same cases are analyzed in both of the TSLP and MSLP models and it is observed that MSLP gives a bit better result compare to TSLP. This chapter also includes a comparative analysis between two-stage and multistage models with the advantages and disadvantages of both of the models in comparison to other. As different cases through various scenarios are discussed, the models are convenient in several manners. Hope by applying these models industry will be benefitted with attaining the optimal production schedule as MSLP models offers more time and steps to disclose uncertainty. The next chapter will present the conclusion and recommendations of this thesis.

# Chapter 07

# Conclusions and Recommendations

# Chapter 07
# Conclusions and Recommendations

## 7.1 Conclusions and Recommendations

This study methodically creates and assesses a set of optimization models for production planning in Bangladesh's RMG sector, from deterministic to stochastic frameworks, which is a highly volatile sector.

To sum up, this study offers several noteworthy contributions, and they are as follows:

To offer a better production schedule under predetermined parameters, a deterministic linear programming model, for profit maximization along with the optimal utilization of available resources has been developed initially in this study. The model suggested a production schedule of producing 6 products instead of 10, i.e., by rescheduling the production planning of RMG industry, the output is optimal in terms of profit. This deterministic approach is especially helpful for short-term planning in steady market conditions and enables scenario-independent optimization for RMG industry.

In a deterministic model, decision $x$ is the best for the model rather than the situations. So, to mitigate the gap, some general stochastic linear programming (SLP) models are formulated here to manage each situation with uncertainty. As uncertainty is everywhere. Uncertainty in business is like navigating through fog. Any system's operation or design will inevitably involve some level of uncertainty. Five different stochastic models were developed then to address all the relevant parameters as uncertain, which include costs, prices, demand, and process time. In this study, five distinct scenarios are assessed in favorable, stable, and unfavorable economic situations for formulating these five different models. And obviously, the outcomes of these models were identical according to the scenarios. Since it is impossible to foresee which scenario would occur, a general stochastic model was also developed in this study by merging all of the possible scenarios. This method captures holistic uncertainty and offers robust planning in a range of situations in the ready-made garment industry of Bangladesh. This combined model offered an acceptable solution for the RMG industry to make decisions in different volatile situations, which is better than the consequences of the deterministic as well as other general SLP models.

A two-stage stochastic (TSLP) approach was developed then with wait and see "WS", stochastic solution "RP", and Expected value "EEV" calculations, and only preserving uncertainty in a few critical parameters while keeping other parameters as deterministic to strike a balance between flexibility and tractability. This $WS \geq RP \geq EEV$ inequality is seen when profit maximization is involved. Structure provides flexibility with manageable case counts, allowing for "here-and-now" decisions to be made and "recourse" actions to be taken whenever uncertainty is revealed. Additionally, the study carried out thorough studies of EVPI (Expected Value of Perfect Information) and VSS (Value of the Stochastic Solution) in order to quantify the advantages of using stochastic models over deterministic ones. The positive results obtained in this study for both parameters show that using the stochastic model has advantages, even if uncertainty may occur in many situations. Thus, this TSLP model offered a high opportunity for the ready-made garment industry to make decisions with managing uncertainty

The research further handles seasonal demand fluctuation or demand variability. Considering this fluctuated seasonal demand, two distinct two-stage stochastic models were developed again through two different cases such as (1) actual sales of RMG is taken into consideration, and (2) the organization has to meet the customers' demand. After that, a multistage stochastic programming model (MSLP) is developed as an extension of TSLP, considering these same two cases. The best production schedule for handling the cases is found after examining both the cases for both models. MSLP is applied for getting better results as uncertainty in real-world issues often develops across a number of time periods rather than in a single step. Additionally, it is shown that the multistage stochastic model outperforms the two-stage stochastic model in terms of results. Therefore, an MSLP model was also proposed for the RMG industry, which will help to make appropriate decisions in cases where the production process allows abundant time.

Thus, this thesis offers a clear and effective process for moving RMG processes from deterministic planning to multistage stochastic programming. Showing how the EVPI and VSS can quantitatively direct the choice of decision-making models based on uncertainty profiles is the theoretical implication of this research.

The practical significance of this study is that the two-stage stochastic model offers significant profit resilience without incurring prohibitive computing cost, making it a feasible proposal for

RMG enterprises in Bangladesh. Multistage models, on the other hand, work better for businesses with sophisticated computing systems and dynamic demand environments.

This thesis thus improves production planning in Bangladesh's garment industry in the face of economic uncertainty by bridging the gap between fundamental deterministic optimization and sophisticated stochastic programming. The research provides both theoretical clarity and practical insights by combining rigorous modeling with evaluation through the mathematical programming languages LINDO and AMPL. The suggested models give decision-makers of any particular organization the tools they need to strike a balance between computational viability, robustness, and profitability in a dynamic global market, whether they are used as an adaptive multistage system or as a scaled baseline.

Crucially, the study shows how stochastic modeling techniques are better in situations where uncertainty is common and emphasizes how RMG companies must use these quantitative decision-support tools. The results have significant applications, pushing industry participants to adopt more sophisticated, scenario-driven optimization methods for tactical and strategic choices rather than sticking with conventional deterministic planning. Thus, by means of this study both Deterministic and Stochastic models that means a complete production planning is offered for the Garments industry of Bangladesh. Hope the proposed models will minimize the manufacturing cost, optimize the profit and most importantly mitigate the uncertainty as well as the risk of the garment sector of Bangladesh.

In conclusion, this study closes a significant gap in the RMG sector of Bangladesh between theoretical optimization methodologies and real-world business implementation. It lays the groundwork for further study in fields including supply chain network design integration, real-time decision systems with predictive analytics, and multi-objective optimization (e.g., balancing cost, quality, and sustainability). Models developed in this study will be crucial for preserving operational resilience, profitability, and competitiveness as the global garment industry grows more unstable.

## 7.2 Limitations of the Study

- The quality of the analysis is directly proportionate to the quality of data. The quality of data may vary factory to factory.
- Manufacturers fear to disclose actual data in order to maintain their company's confidentiality.
- Those who are familiar with an existing institution-based production planning method may be hesitant to adopt a scientific method for profit optimization.
- Labor unrest sometimes creates an unpredictable environment in our garments sector. It may hamper the research time.

## 7.3 Future Research

In order to strengthen or expand understanding, more investigation is needed in a field. Hence, there is definitely a plan to extend this research.

The extensions of this research can be included:

- More scenarios on real market or price data will be incorporated to enhance the concerned field.
- Supply chain disruption models might be integrated
- Chance constraint model can be merged with the existing models also using integer programming is an option to record subcontracting or lot-sizing choices
- Conformity to sustainability goals such as waste minimization, energy restrictions, and social compliance.

# APPENDIX

# APPENDICES

## Appendix A- Survey Questionnaire

### COVER LETTER FOR SURVEY INSTRUMENT

**CONFIDENTIAL**

**QUESTIONNAIRE FOR COLLECTION INFORMATION OF READY-MADE GARMENTS PRODUCTION AND COSTING SYSTEM**

Thank you for continuing to be part of our research work. This survey asks about your Ready Made Garments (RMG) experiences while producing RMG products, as well as a variety of processes, technologies, and inputs.

This survey is being conducted by PhD researchers at Department of Mathematics, University of Dhaka at Dhaka, Bangladesh. The study will help researchers to maximize RMG production efficiently.

As with all PhD research surveys, your response to this survey, or any individual question in this survey, is completely voluntary. You will not be individually identified and your responses will be used for statistical purposes only.

If you have questions about your rights as a participant in this survey, or are dissatisfied at any time with any aspect of the survey, you may contact researcher at 01816510786.

## GENERAL INFORMATION

Name of the organization:

ECHOTEX LIMITED

Location:

Pallibiddut, Chandra, Kaliakoir,Gazipur

Contact Number (if possible)

## Input the information in monthly basis:

| Outputs / Input | Unit | Product-1 (T Shirt) | Product-2 (Vest) | Product-3 (Sweat Shirt) | Product-4 (Legging) | Product-5 (Shirt) | Product- 6 (Hoodie Jacket) | Product-7 (Jogger) | Product-8 (Polo- shirt) | Product- 9 (Casual shirt) | Product-10 (Girls' Tops) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Monthly Production | Pc s | 2845902 | 684752 | 538560 | 388708 | 2950900 | 280200 | 450049 | 950854 | 2548725 | 845265 |
| Fabrics | Kg | 493791 | 57071 | 101433 | 87407 | 801200 | 920323 | 180020 | 223564 | 714253 | 213578 |
| Thread | Co ne | 71448 | 13493 | 34332 | 16770 | 100200 | 32262 | 36004 | 23675 | 84520 | 75846 |
| Labels | pc s | 3834461 | 1238152 | 1259324 | 1002062 | 2434461 | 575103 | 472551 | 1057000 | 2124523 | 548216 |
| Zipper | PC S | | | 4404 | | | 294565 | | | | |
| Button | PC S | 63628 | 1326 | 324480 | | 75888 | | | 50365 | 69254 | 49875 |
| Polybag | PC S | 750186 | 237435 | 125021 | 220371 | 70853 | 94000 | 42551 | 29547 | 65824 | 254731 |
| Carton | PC S | 26961 | 5427 | 9167 | 13284 | 28411 | 58913 | 59069 | 8452 | 23541 | 6584 |
| Hand tags | PC S | 514310 | 30015 | 162052 | 135949 | 617000 | 24000 | 471000 | 17563 | 502485 | 14523 |
| Rivets lace | YDS | | 98395 | | | | 53385 | 94103 | | | 169053 |
| Hanger | PCS | 516257 | 207300 | 391517 | 131009 | 626245 | | | 158568 | 514658 | 14520 |
| Tapes/Velcro | YDS | 12550 | 4000 | 32400 | | | | | 4587 | | 4200 |
| Elastics | YDS | 52000 | 153000 | 137950 | 392606 | | 210150 | 22522 | 17520 | | 16542 |
| Labor cost (Include OT) | B D T | 70,758,075 | 17,025,088 | 13,390,296 | 9,664,503 | 70821600 | 6724800 | 10801176 | 22820496 | 61169400 | 2028000 |
| Operating cost (Include Machinery Overhead) | B D T | 35,029,143 | 8,428,356 | 6,628,934 | 4,784,461 | 35410800 | 17866148 | 5513100.25 | 11885675 | 31859062.5 | 1056250 |
| Machinary cost | | | | | | | | | | | |
| Inventory Cost | B D T | 3394550.217 | 631278.429 | 860823.5459 | 721600.8116 | 3565547 | 2977691 | 2467485 | 2675210 | 3065547 | 2368470 |
| Total Demand Monthly | pc s | 30,00,000 | 750,000 | 650,000 | 400,000 | 31,25,000 | 3,00,000 | 4,60,000 | 9,00,000 | 25,00,000 | 10,00,000 |
| Total Supply yearly | | | | | | | | | | | |
| Selling Price | $ | 5294974 | 1112414 | 1495089 | 1119900 | 10151096 | 1134810 | 2019369.86 | 2576789.95 | 7773611.25 | 2121615.15 |

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## Numbers of units/cost required for producing one-unit product:

| Outputs / Input | Unit | Product-1 (T-Shirt) | Product-2 (Vest) | Product-3 (Sweat Shirt) | Product-4 (Legging) | Product-5 (Shirt) | Product-6 (Hoodie Jacket) | Product-7 (Jogger) | Product-8 (Polo- shirt) | Product- 9 (Casual shirt) | Product- 10 (Girls' Tops) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Fabrics | Cost$/kg | 6.09 | 7.58 | 7.08 | 6.68 | 7.01 | 7.50 | 7.20 | 6.09 | 7.00 | 6.05 |
| Thread | Cost$/pcs | 0.56 | 0.50 | 0.52 | 1.34 | 0.56 | 1.50 | 1.50 | .56 | .56 | .565 |
| Labels | Cost $/pc | 0.01 | 0.02 | 0.01 | 0.01 | .01 | 0.02 | 0.02 | .01 | .01 | .01 |
| Zipper | Cost $/pc | | | 0.21 | | | 0.70 | | | | |
| Button | Cost $/pc | 0.02 | 0.003 | 0.02 | | .021 | | | .02 | .021 | .02 |
| Polybag | Cost$/pcs | 0.02 | 0.04 | 0.04 | 0.02 | .02 | 0.05 | 0.05 | .02 | .02 | .02 |
| Carton | Cost$/pcs | 1.15 | 0.80 | 1.06 | 0.65 | 1.05 | 1.20 | 1.02 | 1.05 | 1.10 | 1.15 |
| Hand tags | Cost$/pcs | 0.01 | 0.03 | 0.01 | 0.01 | .01 | 0.03 | 0.04 | .01 | .01 | .01 |
| Rivetslace | Cost$/yds | | 0.99 | | | | 0.60 | 0.60 | | | .80 |
| Hanger | Cost$/pcs | 0.08 | 0.13 | 0.07 | 0.15 | .07 | 0.15 | | .075 | .067 | .08 |
| Tapes/Velcro | Cost $/yds | 0.03 | 0.03 | 0.03 | | | | | .03 | | .032 |
| Elastics | Cost$/yds | 0.05 | 0.05 | 0.09 | 0.08 | | 0.05 | 0.08 | .048 | | .045 |
| Labor hour | Minutes/pcs | 1.5 | 2 | 3 | 1.5 | 2.00 | 3.00 | 2.50 | 1.5 | 2.00 | 1.5 |
| Machinery hour | Minutes/pcs | 6 | 8 | 12 | 8 | 6.5 | 12.00 | 9.00 | 6.0 | 6.25 | 6 |

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## Available resource/budget (in dollar/tk)

| Sl No | Items | | U O M | 2020 | | | | | | 2021 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | July | August | September | October | November | December | January | February | March | April | May | June | July | August | September | October | November | December |
| 1 | **T-Shirt, Polo-Shirt,Sweat Shirt, Girls Top, Leggings, Hoodie Jacket** | **A** | **$** | 9,705,892 | 6,111,625 | 10,328,124 | 8,018,869 | 8,951,513 | 8,914,528 | 9,853,307 | 10,063,702 | 14,009,622 | 10,419,525 | 12,928,863 | 11,115,790 | 10,191,186.69 | 6,417,206.58 | 10,844,529.69 | 8,419,812.42 | 9,399,088.29 | 9,360,254.77 |
| 2 | **Fabrics** | **B** | **$** | 5,231,321 | 3,262,335 | 5,913,604 | 4,499,351 | 4,822,595 | 4,794,223 | 5,166,483 | 5,360,607 | 6,910,206 | 4,817,361 | 6,544,943 | 6,124,182 | 5,492,887.12 | 3,425,451.70 | 6,209,283.76 | 4,724,318.26 | 5,063,724.73 | 5,033,934.42 |
| | **Trims Cost:** | **C=(D:N)** | **$** | **1,133,684** | **756,962** | **1,253,708** | **940,244** | **1,052,377** | **1,097,002** | **1,105,939** | **1,148,523** | **1,797,968** | **1,395,226** | **1,644,719** | **1,370,035** | **1,190,368** | **794,810** | **1,316,394** | **987,256** | **1,104,995** | **1,151,852** |
| 3 | **Thread** | **D** | **$** | 204,063 | 136,253 | 225,667 | 169,244 | 189,428 | 197,460 | 199,069 | 206,734 | 323,634 | 251,140 | 296,049 | 246,606 | 214,265.95 | 143,065.70 | 236,950.58 | 177,705.87 | 198,898.91 | 207,333.12 |
| 4 | **Labels** | **E** | **$** | 136,042 | 90,835 | 150,445 | 112,829 | 126,285 | 131,640 | 132,713 | 137,823 | 215,756 | 167,427 | 197,366 | 164,404 | 142,843.97 | 95,377.13 | 157,967.05 | 118,470.58 | 132,599.27 | 138,222.08 |
| 5 | **Zipper** | **F** | **$** | 34,010 | 22,709 | 37,611 | 28,207 | 31,571 | 32,910 | 33,178 | 34,456 | 53,939 | 41,857 | 49,342 | 41,101 | 35,710.99 | 23,844.28 | 39,491.76 | 29,617.64 | 33,149.82 | 34,555.52 |
| 6 | **Button** | **G** | **$** | 22,674 | 15,139 | 25,074 | 18,805 | 21,048 | 21,940 | 22,119 | 22,970 | 35,959 | 27,904 | 32,894 | 27,401 | 23,807.33 | 15,896.19 | 26,327.84 | 19,745.10 | 22,099.88 | 23,037.01 |
| 7 | **Polybag** | **H** | **$** | 63,486 | 42,390 | 70,208 | 52,654 | 58,933 | 61,432 | 61,933 | 64,317 | 100,686 | 78,133 | 92,104 | 76,722 | 66,660.52 | 44,509.33 | 73,717.96 | 55,286.27 | 61,879.66 | 64,503.64 |
| 8 | **Carton** | **I** | **$** | 141,710 | 94,620 | 156,713 | 117,530 | 131,547 | 137,125 | 138,242 | 143,565 | 224,746 | 174,403 | 205,590 | 171,254 | 148,795.80 | 99,351.18 | 164,549.01 | 123,406.85 | 138,124.24 | 143,981.34 |
| 9 | **Hand tags** | **J** | **$** | 22,674 | 15,139 | 25,074 | 18,805 | 21,048 | 21,940 | 22,119 | 22,970 | 35,959 | 27,904 | 32,894 | 27,401 | 23,807.33 | 15,896.19 | 26,327.84 | 19,745.10 | 22,099.88 | 23,037.01 |
| 10 | **Rivets lace** | **K** | **$** | 45,347 | 30,278 | 50,148 | 37,610 | 42,095 | 43,880 | 44,238 | 45,941 | 71,919 | 55,809 | 65,789 | 54,801 | 47,614.66 | 31,792.38 | 52,655.68 | 39,490.19 | 44,199.76 | 46,074.03 |
| 11 | **Hanger** | **L** | **$** | 270,951 | 180,915 | 299,638 | 224,719 | 251,519 | 262,185 | 264,321 | 274,498 | 429,716 | 333,460 | 393,089 | 327,440 | 284,499.07 | 189,960.46 | 314,619.38 | 235,955.15 | 264,094.95 | 275,293.78 |

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| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | **Tapes/Velcro** | **M** | **$** | 90,695 | 60,557 | 100,297 | 75,219 | 84,190 | 87,760 | 88,475 | 91,882 | 143,837 | 111,618 | 131,577 | 109,603 | 95,229.31 | 63,584.75 | 105,311.37 | 78,980.39 | 88,399.51 | 92,148.06 |
| 13 | **Elastics** | **N** | **$** | 102,031 | 68,127 | 112,834 | 84,622 | 94,714 | 98,730 | 99,534 | 103,367 | 161,817 | 125,570 | 148,025 | 123,303 | 107,132.97 | 71,532.85 | 118,475.29 | 88,852.93 | 99,449.45 | 103,666.56 |
| 14 | **Labor cost (Include OT)** | **O** | **$** | 1,446,210 | 923,875 | 1,554,714 | 963,699 | 1,208,407 | 1,288,549 | 1,682,895 | 1,467,025 | 2,056,958 | 1,585,409 | 1,793,562 | 1,613,062 | 1,518,520.72 | 970,068.36 | 1,632,449.92 | 1,011,883.64 | 1,268,827.86 | 1,352,976.92 |
| 15 | **Operating cost (Include Machinery Overhead)** | **P** | **$** | 964,140 | 615,916 | 1,036,476 | 642,466 | 805,605 | 859,033 | 1,121,930 | 978,016 | 1,371,305 | 1,056,939 | 1,195,708 | 1,075,374 | 1,012,347.15 | 646,712.24 | 1,088,299.94 | 674,589.09 | 845,885.24 | 901,984.61 |
| 16 | **Inventory Cost** | **Q=[B+C]** | **$** | **6,365,005** | **4,019,297** | **7,167,312** | **5,439,595** | **5,874,971** | **5,891,225** | **6,272,423** | **6,509,129** | **8,708,175** | **6,212,588** | **8,189,663** | **7,494,217** | **6,683,255** | **4,220,262** | **7,525,678** | **5,711,574** | **6,168,720** | **6,185,787** |
| 17 | **Total Demand** | **R** | **Pcs** | 4,420,691 | 2,746,589 | 4,341,562 | 3,216,736 | 3,892,644 | 4,087,276 | 4,513,094 | 4,420,691 | 2,746,589 | 4,341,562 | 3,216,736 | 3,892,644 | 4,087,276 | 4,513,094 | 4,420,691 | 2,746,589 | 4,341,562 | 3,216,736 |
| 18 | **Total Supply** | **S** | **Pcs** | 4,210,182 | 2,615,799 | 4,134,821 | 3,063,559 | 3,707,280 | 4,093,510 | 4,235,078 | 4,359,457 | 6,098,322 | 5,011,131 | 5,178,740 | 5,280,500 | 4,420,691.10 | 2,746,588.95 | 4,341,562.05 | 3,216,736.95 | 3,892,644.00 | 4,298,185.50 |
| 19 | **Selling Price/Pcs** | **T** | **$** | **2.31** | **2.34** | **2.50** | **2.62** | **2.41** | **2.18** | **2.33** | **2.31** | **2.30** | **2.08** | **2.50** | **2.11** | **2.31** | **2.34** | **2.50** | **2.62** | **2.41** | **2.18** |

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# Appendix B

## Published paper

1. Suraiya, S., & Hasan, M. B. (2023). Approaching a linear programming model for production planning of a ready-made garments industry. *J. Appl. Math. & Informatics Vol*, *41*(1), 215-228. https://doi.org/10.14317/jami.2023.215

## Submitted paper

1. Suraiya, S., & Hasan, M. B. Optimal Production Planning Models for Garment Industries by Converting Deterministic Linear Programming to Stochastic Programming Model, *Journal of Engineering and Technology for Industrial Applications*

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*J. Appl. Math. & Informatics Vol.* **41***(2023), No. 1, pp. 215 - 228*
https://doi.org/10.14317/jami.2023.215

# APPROACHING A LINEAR PROGRAMMING MODEL FOR PRODUCTION PLANNING OF A READY-MADE GARMENTS INDUSTRY

SAYMA SURAIYA* AND MD. BABUL HASAN

ABSTRACT. The ready-made garments (RMG) have been making a crucial contribution about of 81 % of total export and 12.36 % of total GDP of the country which is now the single biggest export earner for Bangladesh. The cheap production cost is the key important factor to explore this RMG sector. But these RMG sector is running on the basis of intuition based decisions. Though they are making profit it is not optimal. In this study, a deterministic model is developed to help the RMG to minimize the production cost and to maximize their profit along with optimal utilization of available resources. 10 different types of products are taken from one of the garments factories of Gazipur, Dhaka to prepare this research work. This model suggests the manufacturer on which products along with how much should be produced to meet the future demand by maintaining the lowest production cost that ultimately maximize the profit of the organization, and also helps Bangladesh to compete in the international market with 'Made in Bangladesh'. LINDO programming is used here to solve this LP model.

AMS Mathematics Subject Classification : 65H05, 65F10.
*Key words and phrases* : Deterministic linear programming (DLP), ready-made garments industry (RMG), optimization.

## 1. Introduction

Bangladesh is a developing country. Bangladesh has faced significant swings in her economy in trade, industrial, fiscal, agricultural and financial policies over the last two decades. Many segments take part in the growth of our economy in the struggling period. RMG sector is one and the most important of these segments. It is now playing a very vital role in Bangladesh's economy. That's why we have to take proper care of this segment. Bangladesh's ready-made garment

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# Optimal Production Planning Models for Garment Industries by Converting Deterministic Linear Programming to Stochastic Programming Model

Sayma Suraiya, Assistant Professor, Dept. of Business Administration, Daffodil International University

Md. Babul Hasan, Professor, Dept. of Mathematics, University of Dhaka

## Abstract

Businesses want to maximize its profit, and this is achieved at the product level by finding ways to make them more profitable. But uncertainty is everywhere. Uncertainty in business is like navigating through fog. Any system's operation or design will inevitably involve some level of uncertainty. This uncertainty can breed hesitation, making it difficult to invest in new ventures or expand operations. However, it also presents opportunities for innovation and adaptation. Successful businesses learn to embrace uncertainty, using it as a catalyst for creativity and agility. In real-world problem solving, optimization and decision-making under uncertainty is a significant challenge. To avoid this vagueness, stochasticity could be incorporated into a deterministic system. The study of determining the optimal course of action under risk management in many scenarios is known as stochastic programming. In this paper, some general Stochastic Linear Programming (SLP) models are developed for profit optimization of RMG sector of Bangladesh. And a two-stage stochastic linear programming approach is presented here that optimizes the profit with the optimal solution of random variables by using optimization. Mathematical language AMPL is applied here to solve this SLP.

**Keywords:** Deterministic Linear Programming (DLP), Stochastic Linear Programming (SLP), Two-Stage Stochastic Linear Programming (TSLP), Deterministic Equivalent Stochastic Linear Programing, Ready-made garments sector (RMG)

### Introduction

Bangladesh, a tiny nation in Southeast Asia with a dense population, is one of the world's growing economies. The leading representative of Bangladesh in the international market is the country's readymade garments industry, or RMG sector. Bangladesh is ranked second only to China in terms of the overall manufacturing and export of clothing. Unquestionably, RMG export earnings have occupied a prominent position in the nation's overall export earnings since the 1990s (Faruque, 2014) and they are a primary contributor to Bangladesh's GDP. Our economy still depends a lot on the RMG sector for employment in manufacturing, foreign exchange reserves, and women's empowerment. 11.2% of the nation's GDP is contributed by this industry (Jia et al., 2020) The largest industrial sector in the nation is made up of more than 4600 RMG enterprises, which employ 4.1 million people and account for 36% of all manufacturing jobs (Rahman and Al-Hasan, 2019) Women workers who are hardworking, disciplined, and inexpensive are the backbone of this sector (Halim and Islam, 2021). With 61% of employed women, this industrial sector was

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