AN APPLICATION OF A RANDOM COEFFICIENT MODEL TO AGRICULTURAL PRODUCTION IN BANGLADESH

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Dear Sir,

In forwarding Md. Nizamul Islam's Thesis entitled, "AN APPLICATION OF A RANDOM COEFFICIENT MODEL TO AGRICULTURAL PRODUCTION IN BANGLADESH" I, hereby, certify that he has completed the research work as a full time student.

I found him to be arduous, assiduous, diligent & sincere & his thesis contains the results obtained by him through research under my supervision.

(Dr. Matiur Rahman)

Supervisor

DEDICATED

TO

MY BELOVED PARENTS

DECLARATION

Except where otherwise indicated
this thesis is my own work.

(Md. Nizamul Islam)

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ABSTRACT

A land of rivers, Bangladesh is a delta with rich alluvial soil that sustains agriculture throughout the year.

Three major rivers --- the Ganges (Padma), the Brahmaputra and the Meghna – and thousands of their tributaries crisscross about 56,966 (147,570 square kilometers) square miles of mainly low, flat floodplain. The rivers enrich the soil, and also bring the disasters --- annually, without fail. Every year, tens of millions across the country go through the ordeal when floods wash away their homes, their crops and cattle. And for ages, the people have faced floods, with stoic endurance and tenacity. In many ways, they have now learnt to live with it.

On the remote coasts and on the many islands offshore, people make a living braving the surging waves of the Bay of Bengal. The 1970's elemental swoop that wiped off a million had revisited as recently as in 1991 when at least a hundred thousand perished.

With 125 million inhabitants on a small area of land, this is world's ninth most populous country. The density of population, 'at 800 per square kilometer, is the highest in the world, a few city states aside. So the demand for land is

enormous. Roughly 80 percent of them live in the rural areas, although poverty propels them in more and more numbers to migrate to the cities and towns.

Agriculture is still the single largest contributor ---- 32 per cent --- to national economy and single biggest employer-64 per cent of total workforce in Bangladesh. After ready-made garments, agricultural produces and agro-based industrial products fetch the biggest chunk of export earnings. Compared to a growth rate of 3.7 percent in 1995-96, agriculture posted a record rate of six percent in the last fiscal.

Over the past decade, the sector has experienced some fairly significant changes in policy and performance. Though over a long period of time (1980-1994) the agricultural growth rate was 2.6 percent, which was higher than the population growth. In the short run the growth rate was not positive always.

However, privatization, a liberal import policy for irrigation equipment and reduction of tariffs on irrigation and other agricultural equipment have contributed to the expansion of new technology and irrigation and increased food grain production.

Rice still dominates agricultural output, and the country is yet to adopt a broader agricultural policy aiming at crop diversification to sustain agricultural growth and to augment farmers' income and nutritional status. This calls, among others, for technological improvement, better marketing facilities, efficient use of scarce land and change of rice-based water management practices. The viable long-term strategy should also aim at the growth of nonagricultural income, such as from industry and services.

Food grains, mostly wheat, have constituted the major agricultural import item and most have come as food aid. Besides wheat, mustard seeds, edible oil, sugar, cotton, some amount of onion, chilli, lentils and other pulses are also imported. The import of rice has varied widely from year to year and the recent trends in import and domestic production would suggest that the country is nearing self-sufficiency in rice.

Along with favorable weather conditions, steps to bring back discipline in fertilizer marketing were instrumental in raising the food production to 20.25 million metric tons (MT) in 1996-97 from 19.06 million MT in 1995-96.

Deficit in food production has been minimized by 744,000 MT from 2.9 million MT in 1995-96 to 210,000 MT in the last fiscal. In an effort to attain self-sufficiency in food production a Tk. 9.78 billion ambitious plan to raise the production to 25 million MT by the year 2002, or by the end of the Fifth Five Year Plan (FFYP).

Agricultural growth and diversification have been emphasized in the Plan. But recently, a panel of economists has raised questions about the Plan targets. "The implications of a projected 4.5 percent growth annually in rice production do not seem to have been looked at seriously. What would be its effect on the rice price and the profitability of rice production?"

In this thesis an attempt has been made to restructure the well-known Cobb-Douglas production function in the framework of random coefficient model and to apply it to the data on agricultural sector of Bangladesh economy. A massive literature review has been done for the work. The rationale of such choice lies in the fact that the assumption of constancy of parameters across observations in linear regression model is often violated. Bangladesh agricultural sector is very much vulnerable to such violation due to the occurrence of sudden disasters such as cyclones, floods, draughts etc.

Thus, constancy in parameters (elasticity) in production function is very unlikely to hold good. However, two forms of the aforesaid production function, one with capital and labor; and the other with seed use and acreage as inputs have been constructed & estimated using time series data of Bangladesh agricultural sector for a period of 20 years. For estimation purpose, Hildreth-Houck estimation technique has been adopted. The estimation results show satisfactory performance of the model in terms of sign and magnitude. Elasticity coefficients with respect to labor & seed use appear to keep highly significant impact on agricultural output in Bangladesh. Capital appears to be the second best input for boosting up agricultural sector.

For both types of random coefficient model elasticities imply increasing return to scale.

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CHAPTER - I

INTRODUCTION

One of the assumptions we make in general linear model is that the parameters are constant over all the observations. It has often been suggested that this may not be a valid assumption to make. In cross-section studies there can be heterogeneity in the parameters across different cross-section units. In time-series studies there can be variation over time in the parameters. Several models to tackle such problems are suggested in the literature. It is always tempting to argue that the parameters in our models cannot, in general, be expected to be constant across observation and hence we have to consider a varying-parameter model in almost all circumstances. However, this type of argument can be made about every assumption we make, and if we are given voluminous data we can afford the luxury of very general models. If we have limited data, as we often do, there is a limit to the generality we can postulate. Further, even if we have a large data set, the more general the models, the "woollier" the questions we ask, and if we ask "woolly" questions all we can expect to get are "woolly" answers.

Anyway, in some situations it becomes necessary to resort to varying-parameter models. Consider the case where the econometric relationships we estimate are derived from some maximization (or minimization) problem which involves some policy variables. Often these policy variables are assumed to enter the econometric models in an additive fashion and the effects of changes in policies are analyzed on this assumption. However, if economic agents are indeed maximizing, they would be taking these policy variables into account in their decisions and thus the variables would be entering the model not in an additive fashion but as determinants of the parameters in the model. Thus, we necessarily end up with a varying-parameter model, and any econometric evaluation of the effects of economic policies would have to be done within the framework of such models.

For about last two decades the concept of perfect certainty in economic theory has been put to doubt and in some cases has been abandoned and this change has improved our understanding of the complex institutional and behavioral phenomena. It has increasingly become clear that the assumption of stable behavioral and technological relation is completely untenable and this fact has been evidenced by the analysis of Phillips curve and other analytical works. The varying coefficient models have received increased attention because of the evidences that the usual regression assumptions often appear to be invalid and impractical.

The rationale of the varying coefficient model lies in the fact that the "true" coefficients themselves can be viewed directly as the outcome of a stocastic process. One is prompted to adopt flexible parameter approach when one takes into account the possible sources of the variation in the parameters. One such source may be the omission of variables in the econometric relationships. Sometimes this omission is necessary because of the problem of degrees of freedom, the problem of measurement error etc. Another source of variation in the coefficients may be the use of proxi variables and so is the case with incorrect functional forms.

Lucas (1976) has argued that fixed coefficient approach to long run policy analysis and policy evaluation is meaningless. If policy variable changes , it affects the structural equations and hence the parameters. Thus, it is necessary to adopt varying coefficient models. Sims (1971) has argued that if the underlying process of aggregated variables is continuous over time then the coefficients of intertemporarily aggregated relation would be varying. Random coefficient models offer a natural means of combining time series and cross section data. Barr Rosenberg (1973) has exploited this fact.

Thus we understand that varying coefficient models are important and necessary for revealing the real facts. The application of random coefficients models has been advocated by a good number of authers such as Klien (1955), Hildreth &Houck (1968), Swamy (1970), Theil (1968). Theil and Zellner (1969) have utilized random coefficient models to circumvent the aggregation bias of macroeconomic relationships.

The publication of the Arrow, Chenery, Minhas and Solow (ACMS) paper in 1961 precipitated a burst of activity in production analysis . Empirical workers attempted to exploit the simple mathematical form of the marginal productivity conditions implied by the CES function in order to estimate the important elasticity of substitution parameter (σ) .These studies have , however ,not been as informative as had been originally hoped .The substantial differences between the various estimates of σ tends to indicate that it is a highly unstable paramater which is sensitive to both the data base and the particular functional form used ."A variety of hypotheses have been advanced to explain the diversity of results , including cyclical changes in the utilization of factors (Nerlove,1967), random measurment errors (Leontief,1964), systematic variation of input prices and product prices (Nerlove, 1967), embodied and disembodied technical change and problems in the measurment of inputs(Griliches, 1967a; Hildebrand and Liu,1965), simultaneous equation bias (Maddala and Kadane, 1966; Nerlove, 1967), serial correlation (Griliches,1967a),and lagged adjustment (Griliches, 1967a, Lucas;1969, Jorgenson;1972)".

In this thesis work an attempt has been made to estimate a varying parameter regression model for adopting the above mentioned reality for agricultural sector of Bangladesh economy. Total thesis work has been split into several chapters. A lucid discussion on various forms of random coefficient models & their associated estimation techniques is provided in chapter II. Chapter III contains formulation of a random coefficient production model (Cobb-Douglas production function) for Bangladesh while chapter IV bears data description. In chapter V we present estimation results and their interpretations. Chapter VI contains concluding remarks and some policy recommendations emerging from estimation results.

CHAPTER -II

LITERATURE REVIEW

2.1 Introduction

In line with our discussion in introductory chapter, we present various types of works related to random co efficient models and their estimation techniques, testing procedures and empirical evidences.

The econometric methods employed for estimating economic relationships often assume the restrictive assumptions that the accompanying regression coefficients are fixed. But, in real life, this restrictive assumption needs to be relaxed and a more flexibility in econometric modelling is needed so that the regression coefficient can be assumed to vary form one observation to another.

In the present thesis our main purpose is to apply the approach of random coefficient models to the agricultural sector of the Bangladesh economy. At first, we present the concepts of varying coefficient models along with the discussion on different estimation techniques, and their properties. Then, we show the construction of the desired model with the choice of estimation method. The estimated results, analysis along with conclusion and some discussion on policy implications are presented thereafter. We interchangeably use the words random coefficient model & varying coefficient model although they both mean changing coefficient model.

2.2 Underlying concept of varying coefficient models along with the discussion on different estimation techniques

Those who work with classical linear models usually assume that the economic structure generating the sample observations remains constant. That is, there exists not only a single parameter vector relating the dependant and independent variables, but also a constant set of error process parameters and a single functional form. Unfortunately, the microparameter systems along with their aggregate counterparts for economic processes and institutions are not constant, and given that our data are generated by uncontrolled and often unobservable experiments, it is frequently argued that the traditional assumption of fixed coefficients is a poor one.

For example, when using cross-sectional data on micro-units, such as firms or households, it is unlikely that the response to a change in an explanatory variable will be the same for all micro-units. Similarly, when using time series data, it is often difficult to explicitly model a changing economic environment and, in these circumstances, the response coefficients are likely to change over time. These considerations have led to the development of a number of stochastic or variable parameter models.

A multiple regression with constant parameters may be termed as structurally homogenous. In actual life, such homogeneity is almost nonexistent. For example, per unit equal change in total income may produce different effects in the aggregate consumption behavior across different points in time depending on consumers' attitudes, expected future income etc. This idea gives rise to emergence of random coefficient model.

Varying coefficient models can be simple as well as multiple. Under a simple linear varying coefficient model the relation between variables y & x is given by -

$y_i = a_i + b_i x_i + u_i$

where, i = 1,2.....n,

a and b takes different values for different observations,

y is the dependent variable,

x is the independent variable

and u is the error term.

For obtaining estimators we need to use prior information about a and b and these priors can be systematic and stochastic priors. The simplest priors assumed are -

$a_i = a$

$b_i = b$

which remain stable over observations.

Although restrictive, these priors lead to BLU estimators (OLS) under the assumptions of $E(u_i) = 0$,

$E(u^2_i) = \sigma^2,$

$E(u_iu_j) = 0, \qquad i \neq j,$

$E(u_ix_j) = 0$

Another prior may be that $a_i = a + \alpha\rho_i, b_i = b + \beta\rho_i$

where ρ is some other known variable, say for example, some economic policy variable. The justification of such priors lies in the fact that the econometric relationships often need to take into account of policy variables. Incorporation of ρ transforms the original relation into a multiple regression

$$y = a + bx_i + \alpha\rho_i + \beta\rho_i x_i + u_i$$

The OLSE's come out to be efficient in this case provided the policy variables do not influence the independent variables. When systematic variations are discontinuous over time, the specification leads to switching regression models. In this case the specification is -

$$a_i = a + \gamma D_i, \; b_i = b + \delta D_i$$

where D is a dummy variable to identify the segments of switches.

This is a special case of systematically varying coefficient models where policy variable is replaced by D. In this case also the OLSE turn out to be efficient provided the errors variances are equal from one observation to another.

Now, we discuss randomly varying coefficient models. In this case the coefficients are partitioned additively into systematic and random components such as-

$$a_i = a + \in_i,$$
$$b_i = b + \eta_i$$

where $\in_i$ & η_i are stochastic components.

This kind of specifications was assumed by Rubin (1950), Theil and Mennes (1956), Rao (1965), Hildreth & Houck (1968) . Under this specification the model becomes -

$$y_i = a + bx_i + \eta_i x_i + u_i + \in_i = a + bx_i + e_i$$

where $e_i = \eta_i x_i + u_i + \in_i$

In this case the estimators are not efficient because of heteroscedasticity in the errors due to the presence of $\eta_i\, x_i$.

2:2:1 TYPES OF VARYING PARAMETERS MODEL

Varying parameter models can be classified into three types. First, the parameters can vary across subsets of observations within the sample but be nonstochastic. Examples of such models are a general systematically varying parameter model, seasonality models, and a variety of “switching regression” models where the sample observations are generated by two (or more) distinct regimes.

A second class of models is where the parameters are stochastic, and can be thought of as being generated by a stationary stochastic process. Examples of such models are the Swamy and Hsiao (1975) random coefficient models, the Hildreth-Houck (1968) random coefficient model, the return to normality model etc.

Finally, the third class of models consists of those where the stochastic parameters are generated by a process that is not stationary. The Cooley-Prescott (1973) model is of this type. A visual overview is provided in Figure 1.

RANDOM COEFFICIENT MODELS BY TYPE

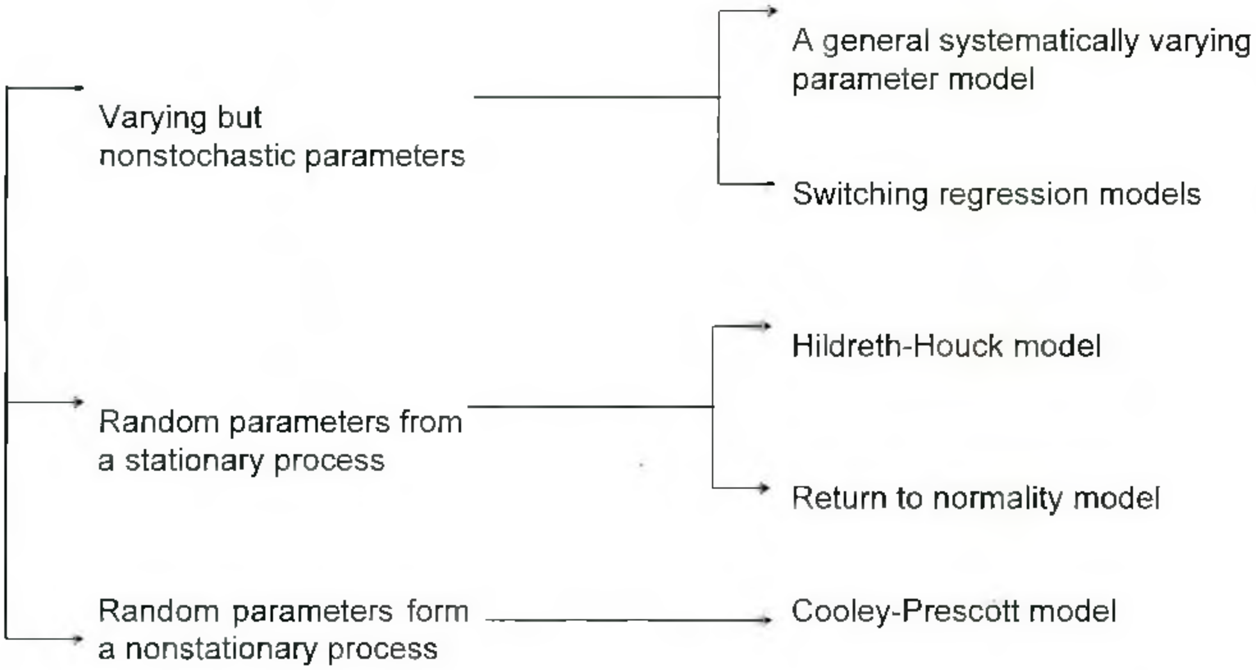

Figure : 1

2:2:2 Estimation techniques as applied to varying coefficient models

GENERAL APPROACH

This is the simplest and most easily interpretable case.

$$\text{Let } y_t = \beta_t x_t + u_t \quad t = 1,2,\ldots\ldots\ldots\ldots,n \qquad (2.1)$$

be the regression equation.

If there is some variable z_t that can explain the movements in β_t, we can write

$$\beta_t = \alpha + \delta z_t \qquad (2.2)$$

Substituting this in Eq. (2.1), we get

$$y_t = \alpha x_t + \delta z_t x_t + u_t \qquad (2.3)$$

The hypothesis that β_t in (2.1) are constant is equivalent to the hypothesis that δ in (2.3) is zero. Thus we can easily test the hypothesis of constancy of β_t . In actual practice if z_t is a policy variable and if a relation like (2.1) is obtained by the maximization behavior of firms or individuals, then z_t should not enter Eq. (2.1) additively. It should be entering as a determining variable for the parameters as in Eq. (2.2).

If Eq. (2.2) is made stochastic, i.e.,

$$\beta_t = \alpha + \delta z_t + v_t \tag{2.4}$$

then substituting (2.4) in (2.1), we get

$$y_t = \alpha x_t + \delta x_t z_t + w_t \tag{2.5}$$

where $w_t = u_t + v_t x_t$

If we assume

$cov(u_t, v_s) = 0$ for all t and s

$var(u_t) = \sigma_u^2$ for all t

$var(v_t) = \sigma_v^2$ for all t

and $cov(u_t u_s) = cov(v_t v_s) = 0$ for all $t \neq s$

then $var((w_t) = \sigma_u^2 + x_t^2 \sigma_v^2$

and $cov(w_t w_s) = 0$ for $t \neq s$

Thus Eq. (2.5) is a usual regression model with heteroscedastic residuals, the variances being proportional to $(1 + \lambda x_t^2)$,

where $\lambda = \sigma_v^2 / \sigma_u^2$.

The maximum-likelihood estimates for this model can be easily obtained as follows : The log likelihood is given by (under the assumption that u_t and v_t are normally distributed)

$$\log L = \text{const.} - \frac{n}{2} \log \sigma_u^2 - \frac{1}{2} \sum_{t=1}^{n} \log(1 + \lambda x_t^2) - \frac{1}{2\sigma_u^2} \sum_{t=1}^{n} \frac{(y_t - \alpha x_t - \delta x_t z_t)^2}{1 + \lambda x_t^2}$$

The estimation would proceed as follows :

For given λ we estimate the regression equation

$$\frac{y_t}{r_t} = \alpha \frac{x_t}{r_t} + \delta \frac{x_t z_t}{r_t}$$

where $r_t = \sqrt{1+\lambda x_t^2}$ and compute the residual sum of squares. $\hat{\sigma}^2{}_u(\lambda)$.

Let $$F(\lambda) = -\frac{n}{2}\log\hat{\sigma}^2{}_u(\lambda) - \frac{1}{2}\sum_{t=1}^{n}\log(1+\lambda x_t{}^2)$$

Then the value of λ for which F(λ) is maximum is the ML estimate of λ, and the corresponding estimates of α and δ are the ML estimates of α and δ.

Generalization of the above models to the case of several explanatory variables is straightforward. In the case where equations like (2.2) are non-stochastic, we have no problems. All we end up with is models like (2.3) with just more variables. In the case where these equations are stochastic as in (2.4), the residual w_t has a complicated structure. Var(w_t) is now $\sigma_u^2 + x_t' \sum_v x_t$, where $\sum_v$ is the covariance matrix of the residuals in the different equations like (2.4) and x_t is the vector of explanatory variables. One can simplify this by assuming $\sum_v$ to be diagonal so that

$$\sum_v = \begin{bmatrix} \sigma_1^2 & & & 0 \\ & \sigma_2^2 & & \\ & & \ddots & \\ 0 & & & \sigma_k^2 \end{bmatrix}$$

where k is the number of explanatory variables. In this case

$$\text{var}(w_t) = \sigma_u^2(1+\lambda_1 x_{1t}^2 + \lambda_2 x_{2t}^2 + \ldots\ldots\ldots + \lambda_k x_{kt}^2)$$

where $\lambda_i = \frac{\sigma_i^2}{\sigma_{\varepsilon}^2}$ i = 1,2, ...k

We can no longer use the search procedure outlined earlier, but we can still compute the maximum-likelihood estimates by an iterative procedure (at least in principle). Hildreth and Houck (1968) considered this model with no error term in Eq.(2.1). Thus var(w_i) = $\sigma_1^2\, x_{1i}^2 + \sigma_2^2\, x_{2i}^2 + \ldots\ldots\ldots + \sigma_k^2\, x_{ki}^2$

2:2:3 HILDRETH AND HOUCK MODEL

Hildreth and Houck (1968) considered a linear model of the type

$$y_t = \sum_{k=1}^{k} Z_{tk} (\beta_k + v_{tk}) \qquad (3.1)$$

$$y_t = \sum_{k=1}^{k} Z_{tk} \beta_k + u_t \qquad (3.2)$$

where $u_t = \sum_{k=1}^{k} Z_{tk} v_{tk}$ (3.3)

In matrix notation the model can be written as

$$y = z\beta + u \qquad (3.4)$$

where z is a TXK matrix

y, β, u are vectors of order T,K,T respectively. Let φ be the variance matrix of u. For fixed K, v_{tk} are assumed to be independently and identically distributed with zero means and

$$E\, V^2_{tk} = \alpha_{kk}, \qquad (3.5)$$

t = 1,2,......, T

For any s,t and j ≠ k, V_{sj} is assumed to be independent of V_{tk} and thus, from (3.3), (3.5) and this independence of V_{tk}

$$\varphi_{st} = 0 \text{ for } s \neq t \qquad (3.6)$$

$$\varphi_{tt} = \sum_{k=1}^{k} Z^2_{tk} \alpha_{kk}, \qquad (3.7)$$

t = 1, ...T

$= \dot{z}_t \bar{\alpha}$

Where $\dot{z}$ is a TXK matrix whose elements are squares of the corresponding elements of z, $\dot{z}_t$ is the tth row of $\dot{z}$ and $\bar{\alpha}$ is a column vector with elements $\bar{\alpha} = \alpha_{kk}$, k = 1,2,........k.

The unbiased but in-efficient OLS estimators of β are -

$\hat{\beta} = (z'z)^{-1} z'y.$ (3.8)

If $\bar{\alpha}$ and Φ were known, the BLUE would be

b = $\left(z'\Phi^{-1}z\right)^{-1} z'\Phi^{-1}y$ (3.9)

For unknown Φ, it may be estimated and substituted in (3.9). In order to obtain b, estimate of $\bar{\alpha}$ is necessary.

Let the vector of residuals be

$r = y - z\hat{\beta} = Mu$ (3.10)

where M = $\left\{I - z(z'z)^{-1}z\right\}$ is a symmetric, idempotent matrix of rank T-K. The variance matrix is $Err' = EMUU'M = M\phi M$ (3.11)

and

$E\dot{r} = M\overline{\varphi}$ (3.12)

where

$\dot{r}_t = r_t^2,\ \dot{m}_{st} = m_{st}^2, \overline{\varphi}_t = \varphi_{tt}$

for s,t = 1,2,............T

Let $w = \dot{r} - E\dot{r},\ G = M\dot{Z}$

From equation (3.7), $\overline{\varphi} = \dot{z}\overline{\alpha}$

$\therefore\ \dot{r} = w + E\dot{r}$ (3.13)

$= M\overline{\varphi} + w$

$= M\dot{z}\overline{\alpha} + w = G\overline{\alpha} + w$

G is known because z is known

Thus, the equation (3.13) is a linear model,

The unbiased OLS estimator of α is

$\dot{\alpha} = (G'\ G)^{-1}G'\ \dot{r}$

with variance - covariance matrix

$E(\dot{\alpha} - \alpha)(\dot{\alpha} - \alpha)' = (G'G)^{-1}G'\phi G(G'G)^{-1}$

Hildreth and Houck (1968) proved $\dot{\alpha}$ to be consistent.

Since, $\dot{\alpha}$ may yield negative estimate, a truncated estimator

$\overline{\alpha}_k = \max(0, \dot{\alpha}_k)$

has been proposed and the above two authors proved $\overline{\alpha}_k$ to have smaller MSE than $\dot{\alpha}$ although it is biased.

Another estimator $\hat{\alpha}$ which minimises the sum of squares subject to α≥0 was proposed i.e.

$$\min_{\alpha} (\dot{r} - G\alpha)' (\dot{r} - G\alpha)$$

Corresponding to each estimate of α, there is an estimator of β, namely,

$$\dot{\beta} = \left(z' \, \dot{\phi}^{-1} z \right)^{-1} z' \dot{\phi}^{-1} y$$

$$\bar{\beta} = \left(z'\bar{\phi}^{-1}z\right)^{-1} z'\bar{\phi}^{-1}y$$

$$\hat{\beta} = (z'\hat{\phi}^{-1}z)^{-1}z'\hat{\phi}^{-1}y$$

where diagonal elements of $\dot{\phi}$, $\bar{\phi}$, $\hat{\phi}$ are $z\dot{\alpha}$, $z\bar{\alpha}$, $z\hat{\alpha}$, respectively. All these estimators are consistent Rubin (1950) derived the likelihood equations for this type of model but was not applied due to high non-linearity.

Theil and Mennes (1959) considered the relation between one or two independent variables and derived similar results. These authors assumed normality of v_{tk} & estimated the diagonal elements of Φ which is then used to obtain a revised estimator

$$\bar{\alpha} = (G'\hat{\Phi}^{-1} G)^{-1} G'\hat{\Phi}^{-1} \mathrm{r}$$

where $\hat{\Phi}$ is the estimated diagonal matrix. This estimator is then used to estimate β by $\bar{\beta}$.

Zellner (1965) considered the implications of similar models to deal with aggregation problems. Rao (1965) , Swamy (1967) have studied the closely related models. Froehlich (1973) proposed a variation of Hildreth & Houck (1968) estimators. He has used a two-step procedure utilizing an estimated ϕ matrix which is obtained by restricted LS and subsequently the LS estimator of $\hat{\alpha}$ is

$$\min_{\alpha}(r - G\alpha)' \hat{\phi}^{-1} (r - G\alpha) \text{ subject to } \alpha \geq 0,$$

This estimator is again used to estimate the β vector. Here $\hat{\Phi}$ is the estimated covariance matrix with off-diagonal elements ignored. Rao (1965) proposed an estimation technique for a linear model with heteroscedastic variances. The model is considered usual fixed coefficient model of the type-

$$y = x\beta + e,$$

$$E(e) = 0,$$
$$E(ee') = \Delta$$

where Δ is a diagonal matrix with the element σ^2_t, the variance of the tth observation y_t. The problem is to estimate different variances for which Rao (1965) proposed Minimum Norm Quadratic Unbiased Estimation (MINQUE). MINQUE has the minimum average variance where the average is taken over

$\sigma_1^2, \sigma_2^2, \ldots\ldots\ldots\ldots\ldots\ldots \sigma_T^2.$

The MINQUE in HH model is $\alpha_{MQ} = (\dot{z}' M z)^{-1} \dot{z}' r$. Froehlich (1973) conducted a Monte carlo experiment and tested the properties of the above mentioned estimators. He assumed that $v_{tk} = N(0, \alpha_k)$ for fixed k. He considered -

K = 3,

$\alpha_1 = 1$,

$\alpha_2 = .2$,

$\alpha_3 = .5$

and samples of sizes 25 and 75

$\beta_i' = 1$, the first column of z = 1.

In order to study the sensitivity of the estimators, Froehlich (1973) considered the different patterns of z, namely, z's are randomly independent, harmonic and three combination of the above two. Thus, these five patterns and two samples give ten distinct structures. The results obtained by Froehlich (1973) show that the bias of $\bar{\alpha}_1$ is negligible, for $\bar{\alpha}_2$ it is substantial and for $\bar{\alpha}_3$, it is more pronounced than that of $\bar{\alpha}_1$.

These results support Zellner (1962) preliminary results that the bias of a truncated parameter is small. Increase in α reduces the frequency of negative $\dot{\alpha}$ estimates and $\overline{\alpha}$ becomes closer to $\dot{\alpha}$. Again, for larger sample, frequency of negative $\dot{\alpha}$ decreases. For all structures, the LS estimators $\hat{\alpha}$ is preferable to $\overline{\alpha}$ in terms of MSE. The variance of $\hat{\alpha}$ is smaller in all cases. In this experiment an increase in sample size from 25 to 75 cuts the relative efficiency of $\hat{\alpha}$ over $\overline{\alpha}$ in half. This suggests that for small sample $\hat{\alpha}$ is more worthy to be calculated but for larger samples $\overline{\alpha}$ is more satisfactory. Both $\dot{\alpha}$ and α_{MQ} are consistent but in this Monte Carlo experiment $\dot{\alpha}$ terns out to be superior to α_{MQ} in terms of MSE and variance for all structures. For sample size 25, $\dot{\alpha}$ is particularly inefficient. This Monte Carlo experiment suggests that for true variances close to zero, the iterated ML procedure should be avoided even for large sample and it has also revealed that the restricted LS estimator is the most efficient .

Dent and Hildreth (1977) applied ML method for estimating random coefficient in the similar type of regression model of Hildreth (1968) which was be discussed before. These authors came to the conclusion that MLE may well be worth the extra computational work involved for both large and small samples leading to efficient estimation. Singh et al (1976) put forth a modified fourth step Hildreth-Houck estimator. Hildreth Houck (1968) $\dot{\alpha} = (G' \; G)^{-1} G' \; r$ is consistent but inefficient estimator. In the third step Singh et al (1976) obtained a GLS estimator of α as -

$$\alpha^* = \left(G' \; \dot{\psi}^{-1} \; G\right)^{-1} G' \; \dot{\psi}^{-1} \; r$$

where $\psi = M\phi M,$

$$\dot{\psi}_i = \psi_i^2$$

In the fourth step, they estimated ϕ^* again as $\phi^* = z\alpha^*$ and then the mean co-efficient estimator as -

$$\beta^* = (z' \; \phi^{*-1} \; z)^{-1} z' \; \phi^{*-1} \; y$$

Griffith (1972) proposed that the actual response $b_k = L\beta_k^* + v_{ik}$ necessitates to look for the best estimator for v_{ik} and appealing way to do this is to allocate $\hat{u}_t$ (residual) among $z_{ik}\,\hat{v}_{ik}$ in the same proportion as the variances of $z_{ik}v_{ik}$ contribute to the variance of u_t. Thus,

$$\hat{v}_{ik} = \frac{z_{ik}\alpha_{kk}}{\sum_{k=1}^{k} z_{ik}^2 \alpha_{kk}} \hat{u}_t$$

Introducing this estimator $\hat{v}_{ik}$ in $\hat{b}_k = L\beta_k^* + \hat{v}_{ik}$,

One can obtain BLUE for b.

Now, if we consider the policy variable, say p_t, then our specification becomes

$$a_i = a + \gamma p_t + \in_t,$$
$$b_i = b + \delta p_t + \eta_t$$

Thus, the model which we assumed in the beginning becomes

$$y_t = a + bx_t + \gamma p_t + \delta p_t x_t + \eta_t x_t + u_t + \in_t$$

Here again, the problem of heteroscedasticity prevails and as a result the estimates will be consistent and unbiased but inefficient.

Sequentially varying coefficient models can be of two types, namely adaptive coefficient models developed by Cooley and Prescott (1973) and stochastically convergent models developed by Rosenberg (1973).

Adaptive regression models assume that

$a_i = a_{i-1} + e_i$, $b_i = b_{i-1} + \eta_i$

This specification requires that the coefficients drift from one period to another continuously. Muth (1960) considered a special case of this by putting b_i=0.

The Rosenberg's (1973) stochastically convergent specification requires that the adaptive process converges to the fixed and unknown population value. If a and b are the fixed norms for stochastically convergent coefficients a_i and b_i, then

$$a_i = \lambda a_{i-1} + (1-\lambda)a + \epsilon_i$$
$$b_i = \lambda b_{i-1} + (1-\lambda)b + \eta_i$$

$0<\lambda<1$

In both of these cases, the OLSE are unbiased and consistent but inefficient since the error term is serially correlated and heteroscedastic.

2:2:4 SWITCHING REGRESSION MODEL

This is the model considered by Quandt (1958) and later studied by Goldfeld and Quandt (1973), Hinkley (1981), McGee and Carlton (1970), among others.

Suppose we have n observations on y and x. The switching regression model says that

$$y_t = \alpha_1 + \beta_1 x_t + u_{1t} \quad \text{for } 1 \leq t \leq n_0 \qquad (4.1)$$

$$y_t = \alpha_2 + \beta_2 x_t + u_{2t} \quad \text{for } n_0 < t < n \qquad (4.2)$$

i.e., the relationship between y and x has switched at the point $t = n_o$ from relationship (4.1) to relationship (4.2). If n_o is known, there is really no problem. One estimates a separate regression equation for each of the "regimes." In actual practice n_o needs to be estimated. If we assume the error variances to be equal for both the regimes, all that we have to do is estimate (4.1) and (4.2) for different values of n_o (i.e., different partitions of the entire sample), look at the sum of the two residual sums of squares, and choose that value of n_o for which this sum is minimum.

If the error variances are unequal, one has to maximize the log-likelihood function (assuming the residuals u_{1t} and u_{2t} to be independently and normally distributed).

$$\log L = \text{const.} - n_0 \log \sigma_1 - (n - n_0) \log \sigma_2 - \frac{1}{2\sigma_1^2} \sum_{1}^{n_0} (y_i - \alpha_1 - \beta_1 x_i)^2 - \frac{1}{2\sigma_2^2} \sum_{n_0+1}^{n} (y_i - \alpha_2 - \beta_2 x_i)^2$$

with respect to α_1, β_1, α_2, β_2, , σ_1 and σ_2 for different values of n_o. There are, however, some problems with this procedure. If $n_o \leq 2$ or $\geq n - 2$, log L tends to ∞.

Goldfeld and Quandt (1973) discuss further modifications of this switching regression model to cases where there is continuous switching back and forth. They also consider deterministic and stochastic switching models. In the deterministic models there are some other variables z_1, z_2, ...z_k, such that we are in regime 1 if $\lambda = \pi_1 Z_1 + \pi_2 Z_2 + \ldots\ldots\ldots + \pi_k Z_k < c$ and in regime 2 if $\lambda > c$. The π's have to be estimated, and c is a given constant. In the stochastic switching model there is a probability λ that each observation belongs to regime 1 and a probability $(1-\lambda)$ that it belongs to regime 2. All this can be generalized to cases of more than one regime.

Hinkley (1971), McGee and Carlton (1970), and Gallant and Fuller (1973) consider the cases where there are no discontinuities in the regression function. For the two-regime case considered in Eqs. (4.1) and (4.2), if the two regression lines meet at the point $t = n_o$, this implies a constraint on the parameters. $\alpha_1 + \beta_1 x^* = \alpha_2 + \beta_2 x^*$, where x^* is the value of x at the point $t = n_o$

If the regression parameters do change in response to any changes in the policy variables, it is questionable that they change in an abrupt fashion as hypothesized in the simplest form of the switching regression model. Any switch from one regime to the other is likely to be smooth. In this respect one modification considered by Goldfeld and Quandt (1973) is promising. This is to combine Eqs. (4.1) and (4.2) as

$$y_i = (\alpha_1 + \beta_1 x_i)D_i + (\alpha_2 + \beta_2 x_i)(1-D_i) + D_i u_{1i} + (1-D_i)u_{2i}$$

where D_i will be a function of the policy variables and other exogenous variables. D_i could be assumed to follow a smooth curve as in Fig. 2. It can be taken to be the cumulative probability corresponding to a normal, Cauchy, or any other convenient density.

Goldfeld and Quandt (1973) suggest estimating this model under the assumption that the residuals u_{1i} and u_{2i} have a common variance σ^2 . This is because the model they consider is a mixture of two normal distributions, and if we assume different variances σ_1^2 and σ_2^2 for the residuals in (4.1) and (4.2), the likelihood function for such a mixture distribution tends to ∞ as $\sigma_1 \rightarrow 0$ and $\sigma_2 \rightarrow 0$.

An alternative to the Goldfeld and Quandt (1973) procedure that is different from the switching regression model, and that does not involve problems of mixture distributions, is obtained by considering a continuously varying parameter regression model (2.1) and changing the specification (2.2) from the linear function to a cumulative density function based on the normal or any other convenient distribution. For instance, if we consider this cumulative density to be described by the logistic curve

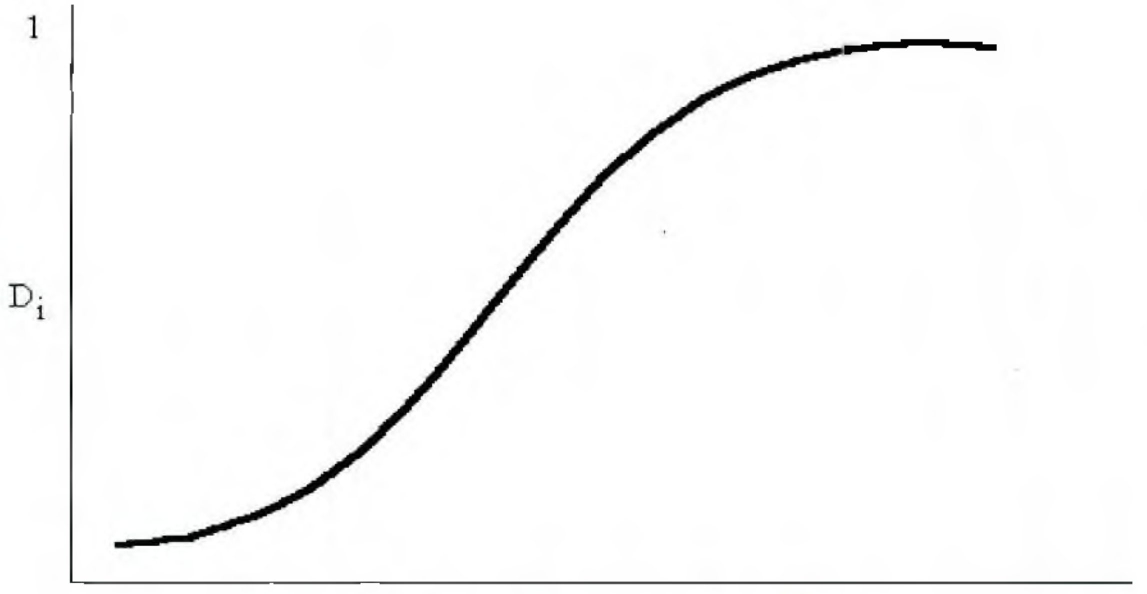

Figure : 2 Cumulative response to a policy.

we can write

$$\beta_t = \bar{\beta} + \frac{c}{1 + e^{\alpha - \delta z_t}}$$

Assuming that $\delta < 0$, we get as $z_t \to -\infty$, $\beta_t \to \bar{\beta} + c$. As $z_t \to \infty$, $\beta_t \to \bar{\beta}$. Substituting this expression for β_t in (2.1), we get, instead of the simple linear equation (2.3), the more complicated equation

$$y_t = \bar{\beta} x_t + \frac{c x_t}{1 + e^{\alpha - \delta z_t}} + u_t$$

The parameters of this equation can be estimated by nonlinear least-squares methods.

2:2:5 ADAPTIVE REGRESSION MODEL

Cooley and Prescott (1973)consider the following model :

$$y_t = \alpha_t + \beta x_t + u_t \qquad (5.1)$$

$$\alpha_t = \alpha_{t-1} + v_{t-1} \qquad (5.2)$$

and call it an *adaptive regression model*. Here $u_t \sim IID(0, \sigma_u^2)$, $v_s \sim IID(0, \sigma_v^2)$, and u_t and v_s are independent for all *t* and *s*. Cooley and Prescott (1973) also allow β in (5.1) to vary in a fashion similar to (5.2) and call it the *varying-parameter regression model*, but we will first consider the model where only the intercept term varies. Cooley and Prescott (1973) argue that many econometric forecasters change their constant terms to take account of structural changes and thereby obtain better forecasts. They, therefore, consider a model given by Eqs. (5.1) and (5.2) as depicting structural change. If economic forecasters change the constant terms in their models, there is one possible alternative explanation of why they do it. Perhaps they have some intuitive feeling of what the forecast should be, and generating this same forecast from an elaborate econometric model lends an aura of respectability (and also salability) to the forecast. The simplest way of doing this is to change the constant term. The term "adaptive regression" originates from the fact that the residual in (5.1) is the sum of a random walk and an independent error. For such structures adaptive forecasting is appropriate. One can also give an alternative justification for the adaptive regression model that is entirely different from that given by Cooley and Prescot (1973). This is in terms of omitted variables. Suppose we have a regression equation

$$y_t = \alpha + \beta x_t + \delta z_t + u_t \qquad (5.3)$$

and z_t is unobservable and hence omitted. Equation (5.3) is the same as (5.1) with

$\alpha_t = \alpha + \delta z_t$.

If z_t is first-order autoregressive with a high autoregression coefficient, (5.2) is a reasonably approximate representation of α_t. With this interpretation, what Eq. (5.2) is supposed to capture is not "structural change" but the effect of omitted variables. Looking at Eq (5.3), it is clear that the omitted variables can as well be combined with the residual u_t In this case we end up with a regression model in which the residuals are both heteroscedastic and autocorrelated. In fact this is also what Eqs (5.1) and (5.2) imply. To write these equations in the form of the usual regression model, we have to choose one of the α's as the basis of reference. Cooley and Prescott (1973) choose α_{n+1} because this facilitates prediction for the first postsample period.

Let $\alpha_{n+1} = \alpha$.

Then

$$\alpha_n = \alpha_{n+1} - v_n = \alpha - v_n$$

$$\alpha_{n-1} = \alpha_n - v_{n-1} = \alpha - v_n - v_{n-1} \quad \text{etc.}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\alpha_2 = \alpha - v_n - v_{n-1} - \ldots\ldots\ldots - v_2$$

$$\alpha_1 = \alpha - v_n - v_{n-1} - \ldots\ldots\ldots - v_1$$

Hence Eq. (5.1) can be written as

$$y_1 = \alpha + \beta x_1 + u_1 - v_1 - v_2 - \ldots\ldots\ldots - v_n$$

$$y_2 = \alpha + \beta x_2 + u_2 - v_2 - v_3 - \ldots\ldots\ldots - v_n$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$y_n = \alpha + \beta x_n + u_n - v_n$$

This implies a regression equation $y_t = \alpha + \beta x_t + w_t$,

where the residuals w_t have a covariance matrix (assuming u_t and v_t are independently and identically distributed with variances σ_u^2 and σ_v^2 respectively)

$$\sigma_u^2 \begin{bmatrix} 1+n\lambda & (n-1)\lambda & \ldots\ldots & 3\lambda & 2\lambda & \lambda \\ (n-1)\lambda & 1+(n-1)\lambda & \ldots\ldots & 3\lambda & 2\lambda & \lambda \\ 3\lambda & 3\lambda & \ldots\ldots & 1+3\lambda & 2\lambda & \lambda \\ 2\lambda & 2\lambda & & 2\lambda & 1+2\lambda & \lambda \\ \lambda & \lambda & & \lambda & \lambda & 1+\lambda \end{bmatrix} \qquad (5.4)$$

where $\lambda = \sigma_v^2 / \sigma_u^2$. Since the covariance matrix is known up to a multiplicative constant if λ is known, the ML estimates can be computed by searching over λ.

Cooley and Prescott (1973) discuss a transformation that reduces the computational burden of inverting the matrix (5.4) at each value of λ that we search on.

The extension of this model to the case where the slope parameters also vary is the model that Cooley and Prescott call the varying-parameter regression model. We can no longer rationalize this type of model in terms of omitted variables. Perhaps one important rationalization is computational tractability and that it picks up structural "drifts" rather than "shifts." The model they consider is

$$\begin{aligned} y_t &= \beta_t' x_t \\ \beta_t &= \beta_t^p + u_t \\ \beta_t^p &= \beta_{t-1}^p + v_t \end{aligned} \qquad (5.5)$$

The superscript p denotes the permanent component of the parameter. They assume the following covariance matrices for u_t and v_t.

$$\begin{aligned} \operatorname{cov}(u_t) &= (1-\theta)\sigma^2 \textstyle\sum_u \\ \operatorname{cov}(v_t) &= \theta\sigma^2 \textstyle\sum_v \qquad 0 < \theta < 1 \end{aligned}$$

where $\sum_u$ and $\sum_v$ are assumed *known*. Since they are assumed known, we can for simplicity, and without any loss of generality, assume them to be identity matrices. Thus our problem is to estimate σ^2, θ, and β_t^p. Again, one cannot estimate the β_t^p for all t. Cooley and Prescott (1973) suggest taking β_{n+1}^p as the reference value, since this is the value needed for prediction for the first postsample period. The method of estimation is similar to the one discussed for the adaptive regression model.

It has been found that for most economic time series adaptive forecasting is better than forecasting from autoregressive models. Cooley and Prescott (1973) also argue that their model has given good results in practice. What their model in effect implies is an ordinary regression model with residuals showing both heteroscedasticity and autocorrelation. Regression models where both these problems are handled simultaneously have not been estimated, and it might be reasonable to assume that both these problems are present if we have omitted variables (which we often do in econometric work). This might also explain why the Cooley and Prescott (1973) model "works." It would be interesting to consider a usual regression model with the residuals exhibiting both heteroscedasticity and autocorrelation and compare its performance with that of the Cooley and Prescott (1973) model. The Monte Carlo study by Cooley and Prescott (1973), though interesting, does not consider all the relevant alternatives. However, other alternative formulations could be computationally more cumbersome. Since the adaptive regression model can also be considered as a model that accounts for the omitted variables, it is important to compare it with models where we use proxies for the omitted variables, With time-series data one commonly used proxy is time - either a linear or polynomial trend term.

2:2:6 STOCHASTICALLY CONVERGENT PARAMETER MODELS

One problem with the adaptive regression model (and also the varying-parameter model considered by Cooley and Perscott 1973) is that the parameters vary over time but do not converge to any fixed values. (Actually this may not be a problem if there are structured "drifts.") Rosenberg (1973) considers a model similar to that of Cooley and Prescott (1973). Instead of making the β_t a random walk, he considers a stochastically convergent parameter structure. The model that Rosenberg (1973) considers is

$$y_t = \beta_t' x_t + u_t \qquad (6.1)$$

where $$\beta_t = (1-\lambda)\bar{\beta} + \lambda\beta_{t-1} + v_t \qquad (6.2)$$

The paper by Rosenberg (1973) is rather lengthy, but the essence of the estimation procedure is as follows : For illustrative purposes let us consider the model where only the constant term is changing, i.e., the adaptive regression model of Cooley and Prescott (1973) :

$$y_t = \alpha_t + \beta x_t + u_t$$

But now we specify

$$\alpha_t = (1-\lambda)\overline{\alpha} + \lambda\alpha_{t-1} + v_t$$

In terms of the lag operator *L* we can write this as

$$(1-\lambda L)\alpha_t = (1-\lambda)\overline{\alpha} + v_t$$

$$\text{or } \alpha_t = \frac{(1-\lambda)\overline{\alpha} + v_t}{1-\lambda L}$$

Hence we have

$$y_t = \frac{(1-\lambda)\overline{\alpha}}{1-\lambda L} + \beta x_t + \frac{v_t}{1-\lambda L} + u_t \qquad (6.3)$$

or $$y_t = \overline{\alpha} + \beta x_t + \left(u_t + \frac{v_t}{1-\lambda L}\right) \qquad (6.4)$$

We can estimate this equation by generalized least squares or maximum-likelihood procedures after deriving the covariance matrix of the residual

$u_t + [\, v_t/(1-\lambda L)]$.

Or else we can write (6.3) as -

$$(1-\lambda L)y_t = (1-\lambda)\overline{\alpha} + (1-\lambda L)\beta x_t + v_t + (1-\lambda L)u_t$$

or $$y_t = (1-\lambda)\overline{\alpha} + \lambda y_{t-1} + \beta x_t - \beta\lambda x_{t-1} + w_t \qquad (6.5)$$

where $w_t = u_t - \lambda u_{t-1} + v_t$.

Again we can estimate (6.5) by generalized least squares or maximum-likelihood procedures after deriving the covariance matrix of w_t. The covariance matrices of the residuals in Eqs. (6.4) and (6.5) are not any more complicated than those in the adaptive regression model.

For the model given by Eqs. (6.1) and (6.2) the procedure is similar. Now the residuals in the equations like (6.4) and (6.5) involve also the variables x_t.

We can write down the expressions formally, but in practice we will have computational difficulties.

2:2:7 KALMAN-FILTER MODELS

In these models we assume $\beta_t = T\beta_{t-1} + v_t$,

where T is a known matrix. Since T is known, as in the case of the adaptive regression model, we can express all the β_t in terms of one of the β's, say β_0.

$$\begin{aligned} \text{Then } \beta_t &= T\beta_{t-1} + v_t \\ &= T(T\beta_{t-2} + v_{t-1}) + v_t \qquad \text{etc.} \\ &= T^t\beta_0 + \sum_{j=1}^{t} T^{t-j} v_j \end{aligned}$$

Thus the regression model reduces to

$$y_t = (T^t\beta_0)' x_t + \sum_{j=1}^{t} \left(T^{t-j} v_j\right)' x_t + u_t$$

We can now formally write the covariance matrix of the residuals and the likelihood function. Since T is a Known matrix, the computational burden in obtaining the ML estimates is not any greater than that in the Cooley-Prescott (1973) varying-parameter regression model. We will not go into the details here.

2:2:8 PURE-RANDOM-COEFFICIENT MODELS

All the above models have some systematic variation in the parameters. The pure-random-coefficient models assume that the β_t are distributed with mean $\bar{\beta}$ and covariance matrix A. Rao (1965) first discussed this model, and Swamy (1970) extended it to the problem of pooling cross-section and time-series data. The model considered by Swamy (1970) is the following :

We have N cross-section units with T observations on each. For illustrative purposes we will consider the case of only one explanatory variables. We have

$$y_{ij} = \beta_i x_{ij} + u_{ij} \qquad i = 1,2,\ldots\ldots\ldots N$$

$$j = 1,2,\ldots\ldots\ldots T \qquad (8.1)$$

The u_{ij} are all independent with mean 0 and var(u_{ij}) = σ_i^2. The β_i are independent with mean β and variance δ^2. We can write $\beta_i = \beta + v_i$,

where $E(v_i) = 0$ and var(v_i) = δ^2. The v_i are independent of the u_{ij}.

Under these assumptions we can write the regression model (8.1) as

$$y_{ij} = \beta x_{ij} + w_{ij} \qquad (8.2)$$

where $$w_{ij} = u_{ij} + v_i x_{ij} \qquad (8.3)$$

Hence

$$\begin{aligned} &\text{var}(w_{ij}) = \sigma_i^2 + \delta^2 x_{ij}^2 \\ &\text{cov}(w_{ij}, w_{ik}) = \text{cov}(u_{ij} + v_i x_{ij})(u_{ik} + v_i x_{ik}) = \delta^2 x_{ij} x_{ik} \\ &\text{cov}(w_{ij}, w_{i'k}) = 0 \end{aligned}$$

for i ≠ i' for all *j* and *k* because v_i are independent and u_{ij} are independent. Thus the covariance matrix of the w_{ij} is

$$V = \begin{bmatrix} \sigma_1^2 I + \delta^2 x_1 x_1' & 0 & \cdots & 0 \\ 0 & \sigma_2^2 I + \delta^2 x_2 x_2' & \cdots & 0 \\ & & \ddots & \\ & & & \ddots \\ 0 & 0 & \cdots & \sigma_N^2 I + \delta^2 x_N x_N' \end{bmatrix} \quad (8.4)$$

where x_i is the vactor of observations $(x_{i1}, x_{i2}, \ldots\ldots x_{iT})$ on the *i*th cross-section unit.

The parameters to estimate are β, δ^2, and σ_i^2(*i = 1,2, ……, N*). If we estimate (8.2) by ordinary least squares, the estimator of β will be consistent but not efficient because $\text{var}(w_{ij})$ is not constant. To get an efficient estimator we have to use generalized least squares. The generalized-least-squares estimator of β is

$$\hat{\beta} = \left[\sum_{i=1}^{N} x_i'(\sigma_i^2 I + \delta^2 x_i x_i')^{-1} x_i\right]^{-1} \left|\sum x_i'(\sigma_i^2 I + \delta^2 x_i x_i')y_i\right| \quad (8.5)$$

Now $$(\sigma_i^2 I + \delta^2 x_i x_i')^{-1} = \frac{1}{\sigma_i^2} I - \frac{\delta^2}{\sigma_i^2(\sigma_i^2 + \delta^2 x_i' x_i)} x_i x_i'$$

Substituting this and simplifying, we get

$$\hat{\beta} = \sum_{i=1}^{N} w_i \hat{\beta}_i \tag{8.6}$$

where $\hat{\beta}_i = (x_i' y_i)/(x_i' x_i)$ is the estimator of β_i from Eqs.(8.1) corresponding to the ith cross-section unit and

$$w_i = \frac{1/\left[\delta^2 + \sigma_i^2/(x_i' x_i)\right]}{\sum_{j=1}^{N}\left\{1/\left[\delta^2 + \sigma_j^2/(x_j' x_j)\right]\right\}} \tag{8.7}$$

Note that $\sigma_i^2/(x_i' x_i)$ is var ($\hat{\beta}_i$). Hence if $\delta^2 = 0$, what Eqs.(8.6) and (8.7) say is that $\hat{\beta}$ is a weighted average of $\hat{\beta}_i$, the weights being inversely proportional to the variances. If δ^2 is very large compared with $\sigma_i^2/(x_i' x_i)$, the weights (8.7) are almost equal, and $\hat{\beta}$ is then close to a simple unweighted average of $\hat{\beta}_i$. The same would be the case if $\sigma_i^2/(x_i' x_i)$ are almost equal.

Suppose $\sigma_i^2/(x_i' x_i)$ are all equal. Then the estimator $\hat{\beta}$ in (8.6) would be the same no matter what δ^2 is. Thus the estimator obtained with the random coefficient model would be the same as the estimator obtained from a model where the coefficients are nonrandom (i.e., $\delta^2 = 0$). However, the variance of $\hat{\beta}$ would be different. It is given by

$$\frac{1}{\sum_{j=1}^{N}\left[1/\delta^2 + \sigma_j^2/(x_j' x_j)\right]} \tag{8.8}$$

which, for different values of δ^2, assumes different values.

In actual practice the GLS estimator (8.6) connot be computed because the parameters δ^2 and σ_i^2 in (8.7) are not known. We can proceed by using some preliminary consistent estimators for these parameters. To obtain these, we estimate Eqs. (8.1) separately for each cross section. We get $\hat{\beta}_i$ and the vector of residuals $\hat{u}_i$. Then we can use

$$\hat{\sigma}_i^2 = \frac{1}{T}\hat{u}_i'\hat{u}_i$$

and
$$\hat{\delta}^2 = \frac{1}{N}\sum\hat{\beta}_i^2 - \left(\frac{1}{N}\sum\hat{\beta}_i\right)^2 \qquad ((8.9)$$

Swamy (1970) suggests using unbiased estimators, which in the case under consideration are

$$\bar{\sigma}_i^2 = \frac{1}{T-1}\hat{u}_i'\hat{u}_i$$

and
$$\bar{\delta}^2 = \frac{1}{N-1}\left[\sum\hat{\beta}_i^2 - \frac{1}{N}\left(\sum\hat{\beta}_i\right)^2\right] - \frac{1}{N}\sum\left(\frac{\bar{\sigma}_i^2}{x_i'x_i}\right)$$

One major problem with this procedure is that the estimator for δ^2 can assume negative values. Further, there is no reason to insist on using unbiased estimators for the parameters in the weight functions to be used in generalized least squares. Thus the estimators in (8.9) should prove adequate in actual practice.

CHAPTER - III

CONSTRUCTION OF THE RANDOM COEFFICIENT PRODUCTION MODEL AND SELECTION OF THE ESTIMATION TECHNIQUE.

Although Theil (1954) performed comprehensive works on the aggregation issues, many of the econometric models used for estimating empirical macro-relationship are generated merely by expediency rather rigorous economic theory. Macro analysts commonly postulate that relationships based on axiomatically founded micro theories can also be meaningfully entertained at the macro level. This practice and procedures have been followed in the production literature. The form of the relationships and the interpretation of the parameters implied by micro concepts are directly projected into the macro sphere and estimated using aggregate data with a hope that the estimated coefficients will yield information about the average relationship which exists in the micro units. But unfortunately there has been rare attempt to establish the nature of the relationship between the analytically derived specification and the micro theory. In view of the above it is clear to us that careful consideration must be taken in moving from micro theory to a macro estimating equation. We do not go far into the micro-macro conflict and do not try to resolve the issue for our present purpose but we construct a random coefficient production model for aggregate data of the agricultural sector of Bangladesh as follows :

Unlike the fixed-coefficient type of production function or the variable elasticity of substitution type of production function used by Arrow, Chenery, Minhas & Solow (1961), we use the simple type of Cobb-Douglas production function which provides easy measures of elasticity. Thus, for the agricultural sector (Crops, livestock, fishery and forestry) of Bangladesh we form the production function using two variants of it.

The first model we use as

$$y_i = A\, k^{\alpha}_i\, L^{\beta}_i$$

where i = 1,2,......n periods,

y = the gross domestic products (per year) in Agriculture sector (Crops. Livestock, Forestry & Fishery) in Bangladesh at constant price (base 1984-85) in million taka.

K = the development expenditure (per year) by the Government of Bangladesh in Agriculture sector (Crops. Livestock, Forestry & Fishery) in Bangladesh at constant price (base 1984-85) in million taka.

L = the number of persons (10 years and above) employed in Agriculture sector (Crops. Livestock, Forestry & Fishery) in Bangladesh.

A is a constant term which is assumed to be 1.

α and β are the elasticity coefficients which are assumed to vary over time so far our present model is concerned.

The second model we use as

$$y_i = KA_i^{\alpha}S_i^{\beta}$$

where y represents the gross domestic products (per year) in Agriculture sector (Crops) in Bangladesh at constant price(base 1984-85) in million taka.

A represents the total cropped area (acres) in Bangladesh and S represents the distribution of improved seeds (in Kg) in agriculture sector (crops) by the government of Bangladesh.

K is a constant term which is assumed to be 1.

α and β are the elasticity coefficients which are assumed to vary over time so far our present model is concerned.

The total output is influenced not only by the variables included in the model but also by the omitted variables as well as by random factors which again vary over time.

In the general frame work the model can be written as

$$y_t = \sum_{i=1}^{k} \beta_{ti} X_{ti}, \quad t = 1,2,\ldots\ldots\ldots\ldots n, \qquad i = 1,2,\ldots\ldots\ldots k$$

where y and X are the log of the output and input respectively. The idea behind omitting the error term lies in the fact that the effects of the omitted variables are supposed to be reflected in the varying elasticity coefficients. Of course, we shall have to apply the appropriate test to ascertain the randomness of the elasticity coefficients. The above model requires nk coefficients to be estimated by n observations. Thus, we need to make some assumptions about the probabilistic behavior of the response coefficients. We assume that the individual response coefficient fluctuates around its mean as follows :

$$\beta_{it} = \bar{\beta}_i + e_{it}$$

where $\bar{\beta}_i$ is the mean of ith response coefficient and e_i is the error term such that -

$E(\beta_{it}) = \bar{\beta}_i$,

$E(e_{it}) = 0$,

$v(e_{it}) = \sigma_{ii}^2 \; \forall$ i&t. $Cov(e_{it}\; e'_{it}) = 0,\; i \neq i' \quad t \neq t'$

Here $x's$ are assumed to be non-stochastic and fixed in repeated samples and independent of e`s. Now, putting $\beta_{it} = \bar{\beta}_i + e_{it}$ in the model we get

$$y_t = \sum_{i=1}^{k} \bar{\beta}_i x_{it} + w_t \quad \text{where} \qquad w_t = \sum x_{it} e_{it}$$

Thus, it is clear that $E(w_t) = 0$, but $V(w_t) = \sum x_{it}^2 \sigma_{ii}$ & $Cov(w_t,\; w'_t) = 0$

Now, we are encountered with the problem of inefficiency of estimators due to heteroscedasticity. To circumvent that problem, we estimate $\bar{\beta}_i$ using Hildreth and Houck (1968) procedure modified by Singh et al (1976).

For estimating the s.e. of our estimates we have used the following procedure :

$$e = y - x\hat{\beta},$$

$$S^2 = \frac{e' \; \hat{\phi}^{-1} \; e}{n-k},$$

$$V(\hat{\beta}) = S^2 (z' \; \hat{\phi}^{-1} \quad z)^{-1}$$

where n = number of observations

k = number of parameters.

CHAPTER - IV

NATURE AND SOURCE OF DATA

In this thesis work we have used two simple type of Cob-Douglas production function model. First model involves y, representing the gross domestic products (per year) in Agriculture sector (Crops. Livestock, Forestry & Fishery) in Bangladesh at constant price (base 1984-85) in million taka. The data cover a period of twenty one years (1974-1994) has been collected from Statistical Year Book of respective year, published by the Bangladesh Bureau of Statistics. K representing the development expenditure (per year) by the Government of Bangladesh in Agriculture sector in Bangladesh at constant price (base 1984-85) in million taka. The data cover a period of twenty one years (1974-1994) has been collected from statistical year book of respective year published by the Bangladesh Bureau of Statistics and L representing the number of persons (10 years and above) employed in Agriculture sector in Bangladesh. The data of the year 1961, 1974, 1981, 1984, 1985, 1986, 1990 and 1992 has been collected from census survey and labor force survey conducted by Bangladesh Bureau of Statistics. But the data which are not available has been estimated by Interpolation using exponential method in the following way-

$$P_1 = P_0 e^{rt}$$

where P_1 = Population of the current year.

P_0 = Population of the base year.

r = Growth rate

t = Number of year between the base year and the current year

In this method at first we have estimated the growth rate of the number of persons (10 years and above) employed in Agriculture sector in Bangladesh considering the year 1961 and 1974 as base year and current year respectively. Then we have estimated the number of persons (10 years and above) employed in Agriculture sector in Bangladesh of each year from 1961 to 1974 with the help of obtained growth rate through Interpolation using exponential method [$P_1 = P_0 e^{rt}$]

Again considering 1974 and 1981 as base year and current year respectively we have estimated the growth rate of the number of persons (10 years and above) employed in Agriculture sector in Bangladesh and then we have estimated the number of person (10 years and above) in Bangladesh of each year from 1974 to 1981.

In this way we have estimated the number of persons (10 years and above) employed in Agriculture sector in Bangladesh of those year which are not available.

Similarly the data of non Agriculture sector in Bangladesh have been estimated by the above formula. We have compared these data with the total population of corresponding year in Bangladesh. Again we compare this data with the population growth rate and literacy rate of Bangladesh and adjust this data.

The second model involves the y, representing the gross domestic products (per year) in Agriculture sector (Crops) in Bangladesh at constant price(base 1984-85) in million taka. The data cover a period of twenty one years (1974-1994) has been collected from Statistical Year Book of respective year, published by the Bangladesh Bureau of Statistics. A representing the total cropped area (acres) in Bangladesh. The data cover a period of twenty one years (1974-1994) has been collected from Statistical Year Book of respective year, published by the Bangladesh Bureau of Statistics and S representing the distribution of improved seeds (in Kg) in agriculture sector (crops) by the government of Bangladesh. The data cover a period of twenty one years (1974-1994) has been collected from Statistical Year Book of respective year, published by the Bangladesh Bureau of Statistics. The data of the year 1974 to 1983 are available in maund but the data of the year 1984 to 1994 are available in Kilo gram. All the data have been used in kilo gram converting the data of the year 1974 to 1983 from maund into kilo gram to avoid this discrepancy.

The data for both types to models are presented in appendix -1

In both of the above two models, the effect of explanatory variables not introduced in the model is supposed to be captured by random disturbance term. In our case, the effect of omitted variables like management, quality of inputs etc is supposed to be reflected in randomly varying coefficients.

CHAPTER - V

ESTIMATION RESULTS AND ANALYSIS

In this chapter, we present the numerical estimation results of the model presented in chapter III on the data discussed in chapter IV.

It is noted that the production functions have been estimated both by OLS (fixed coefficient) and Random Coefficient Method (RC). In order to verify the severity of multicollinearity among explanatory variables, we have adopted the suggestions put forth by Judge et al. In both models, pairwise correlation coefficient between regressors comes out to be much below 0.8. Thus, we rule out the possibility of perfect linear relationship among explanatory variables.

Now, as we are dealing with time series data, there is high likelihood of the existence of autocorrelation. In order to investigate the presence of autocorrelation, we have adopted lagrange Multiplier (LM) test developed by Breush and Pagan (1980) for AR(1) process. The test statistic has revealed the presence of autocorrelation. For correcting autocorrelation we have adopted Hildreth-Lu procedure taking grid values of ρ from 0.10 to .45 and for ρ = .42 we have attained convergence. However, taking into account of the fact that heteroscedasticity is not a serious problem for time series data, we have omitted any test for heteroscedasticity.

Before we, present the estimation results and analysis we represent the movement in variable values graphically as follows :

THE MOVEMENT OF YIELD (CROPS, LIVESTOCK, FORESTRY & FISHERY) VALUES

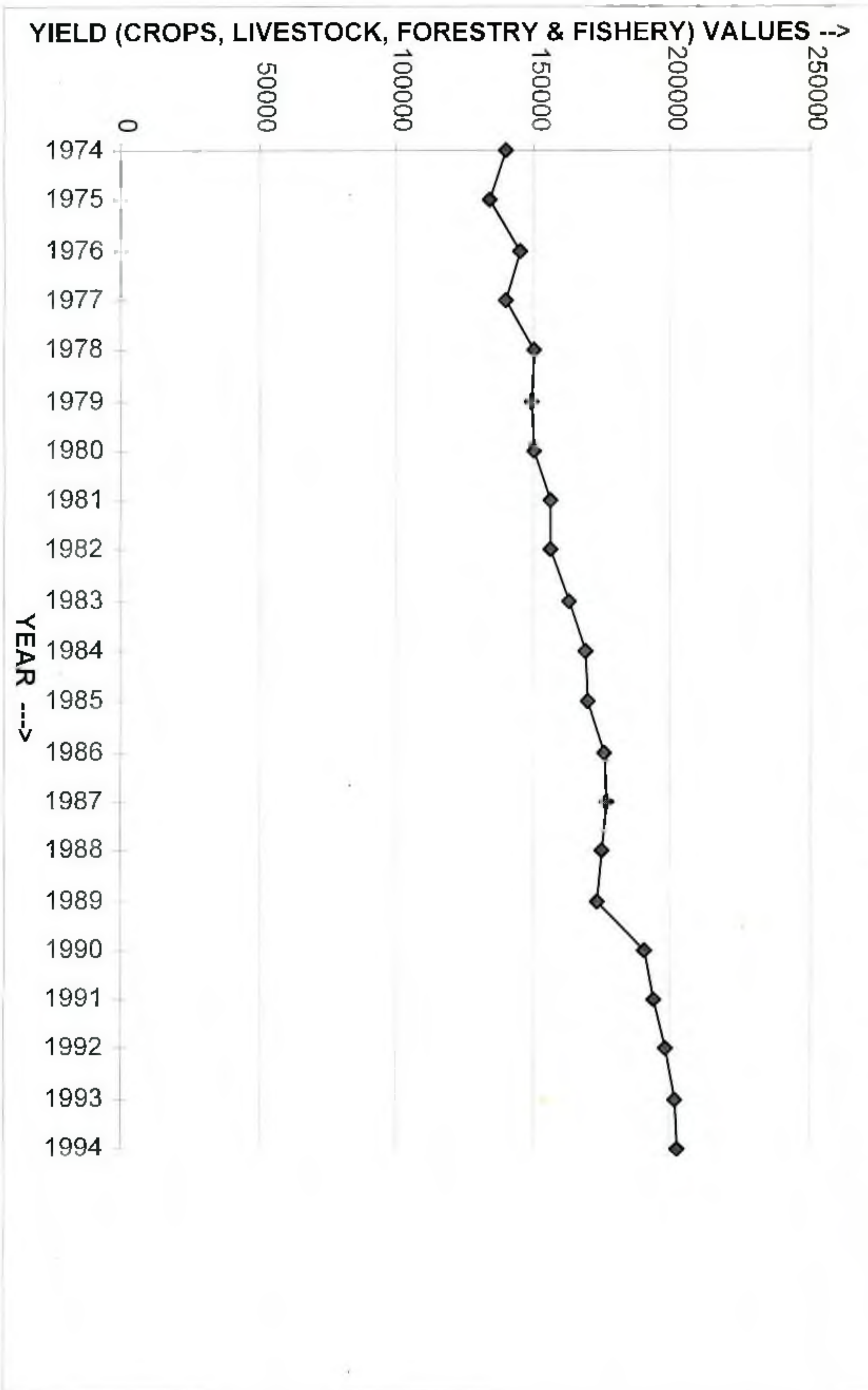

FIGURE - 3

THE MOVEMENT OF LABOR

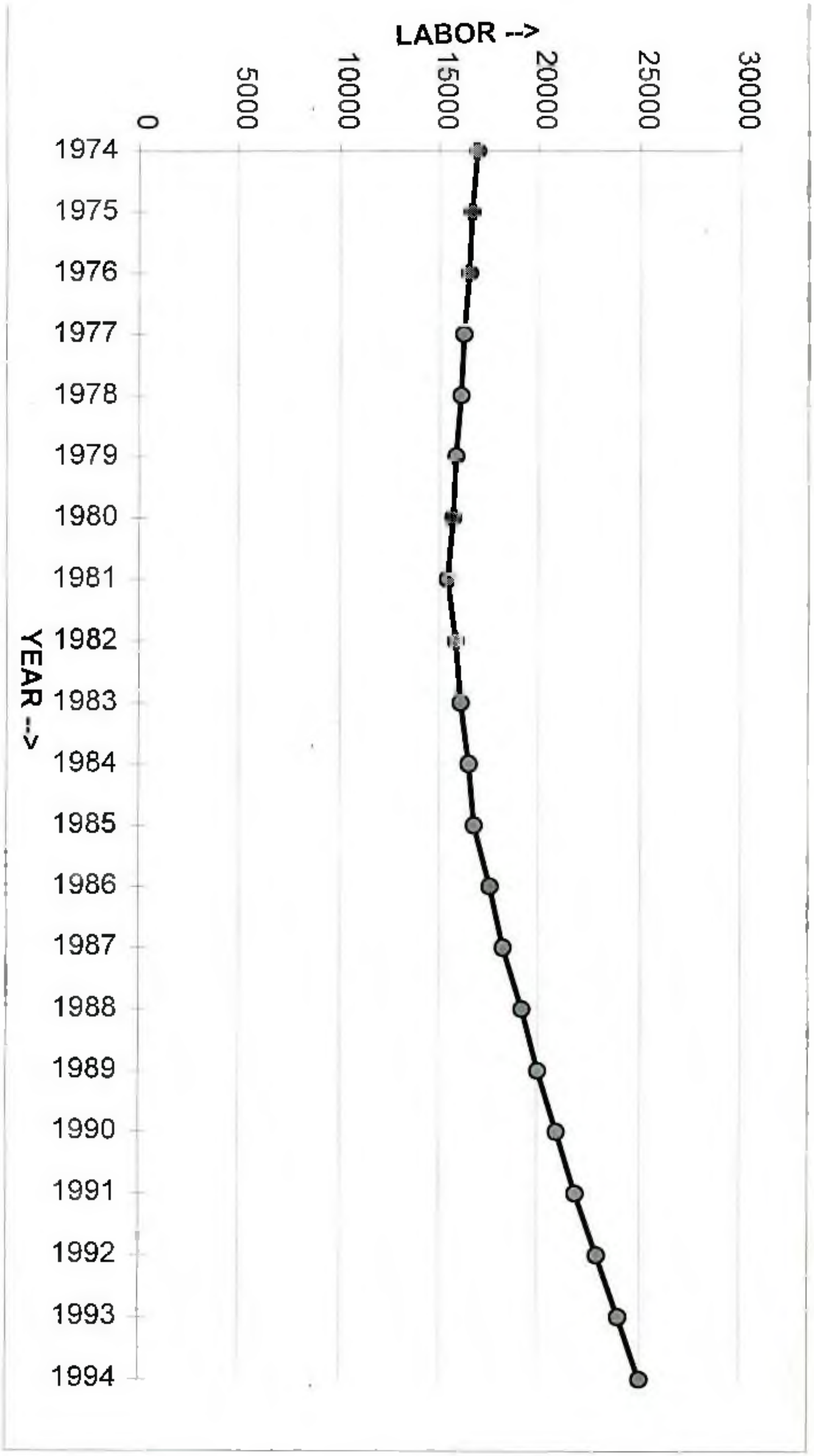

FIGURE - 4

THE MOVEMENT OF CAPITAL

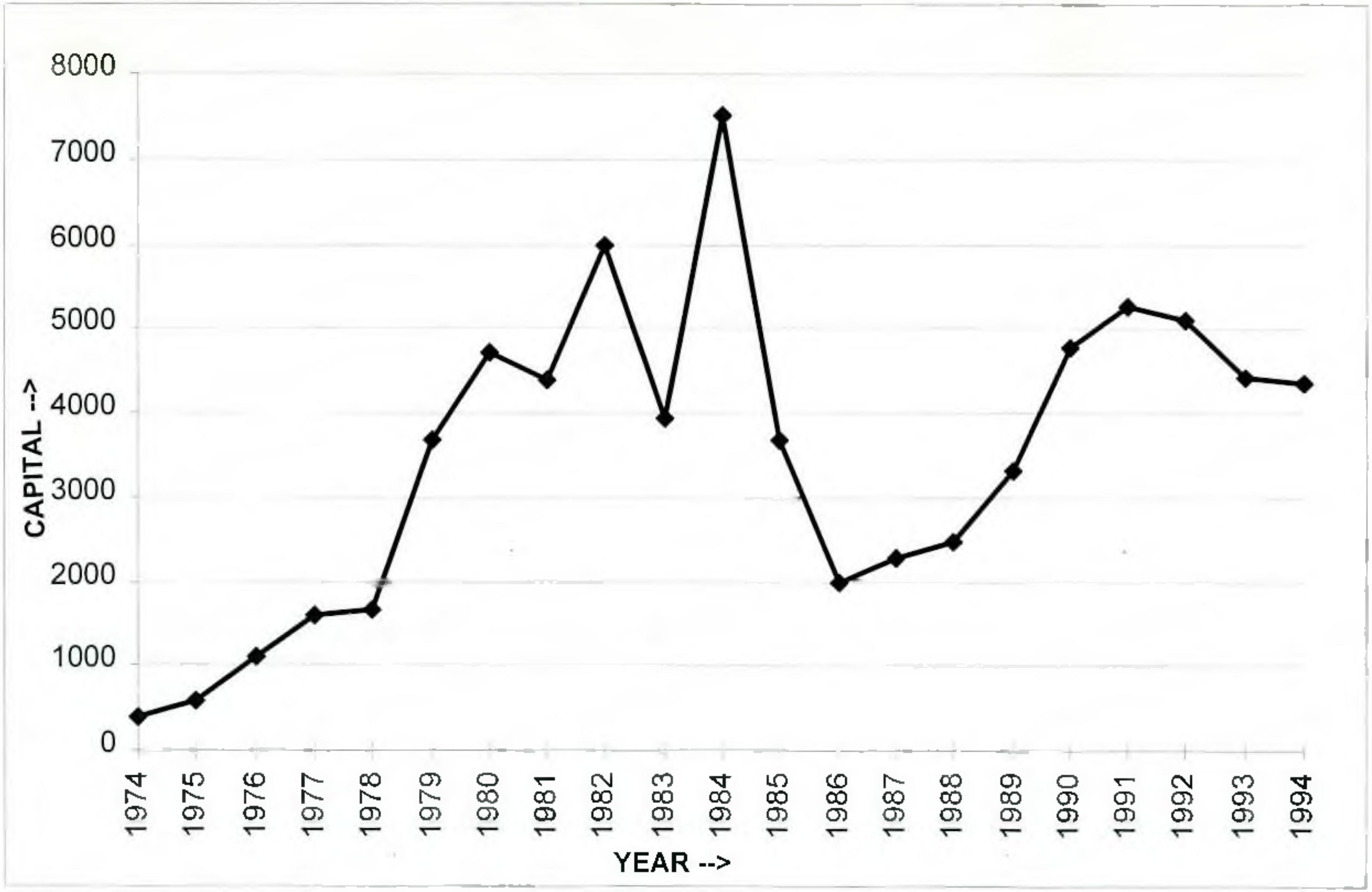

FIGURE - 5

THE MOVEMENT OF YIELD (CROPS) VALUES

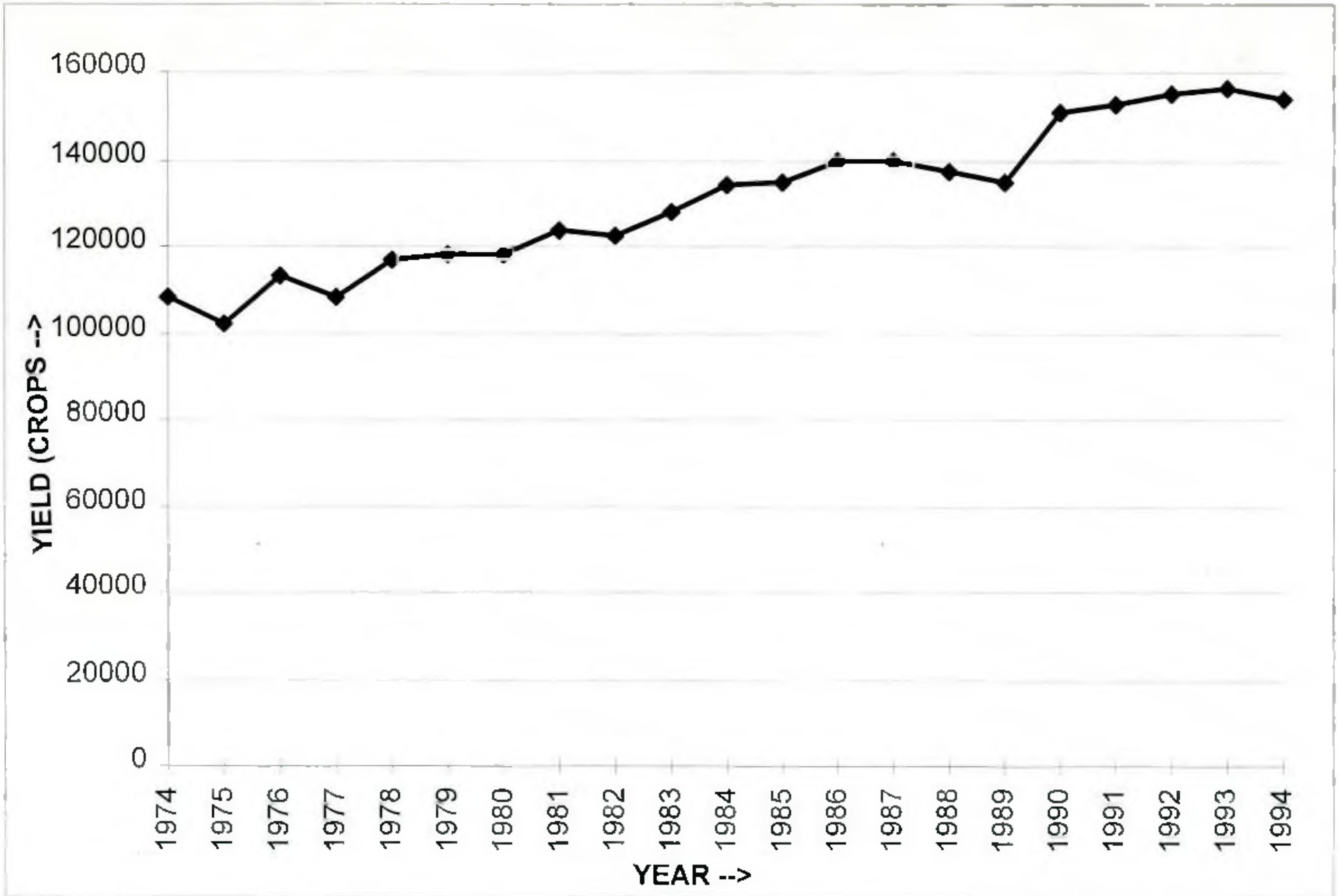

FIGURE - 6

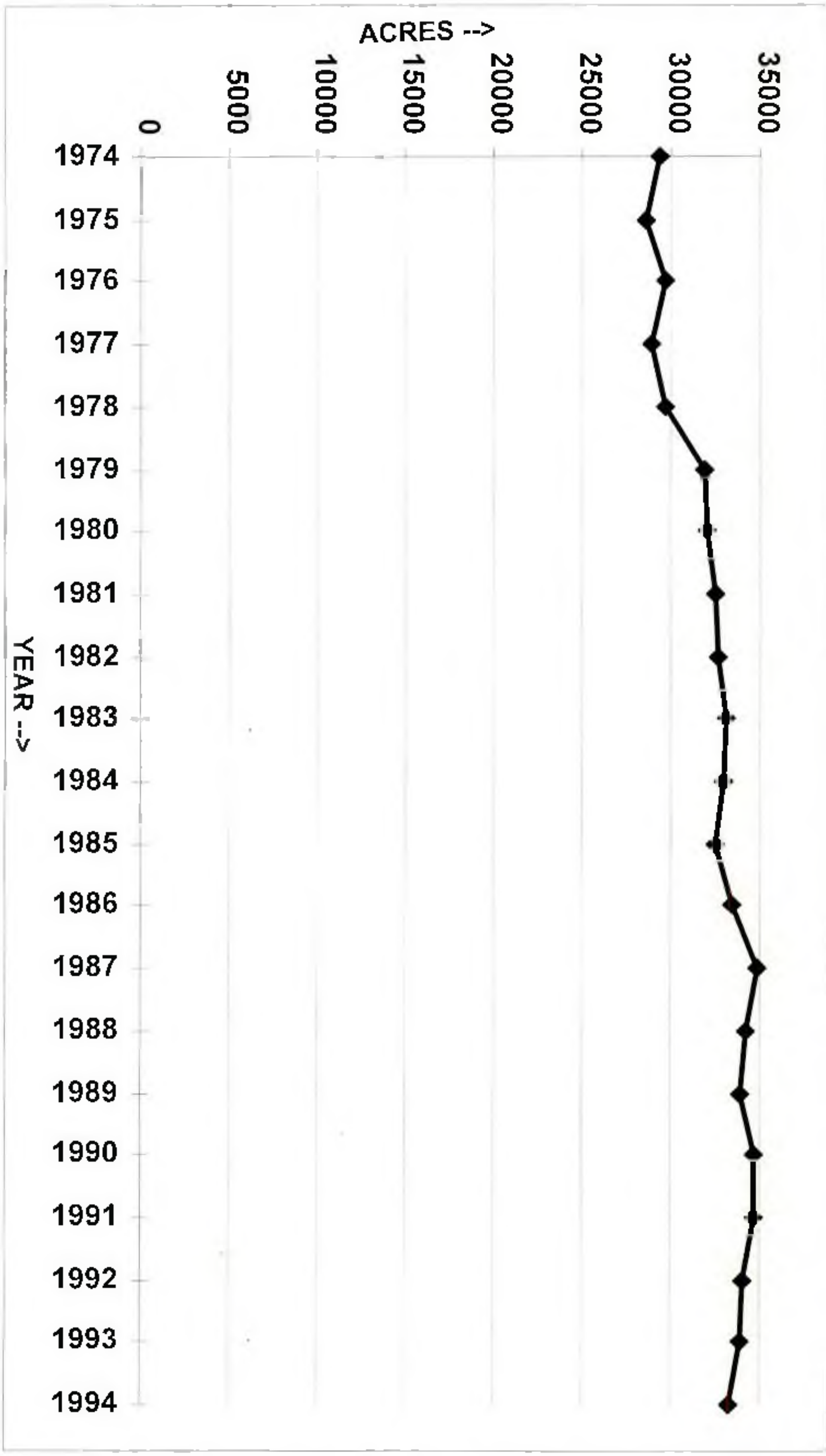
THE MOVEMENT OF ACRES
ACRES -->
0
5000
10000
15000
20000
25000
30000
35000
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
YEAR -->

FIGURE - 7

THE MOVEMENT OF SEEDS

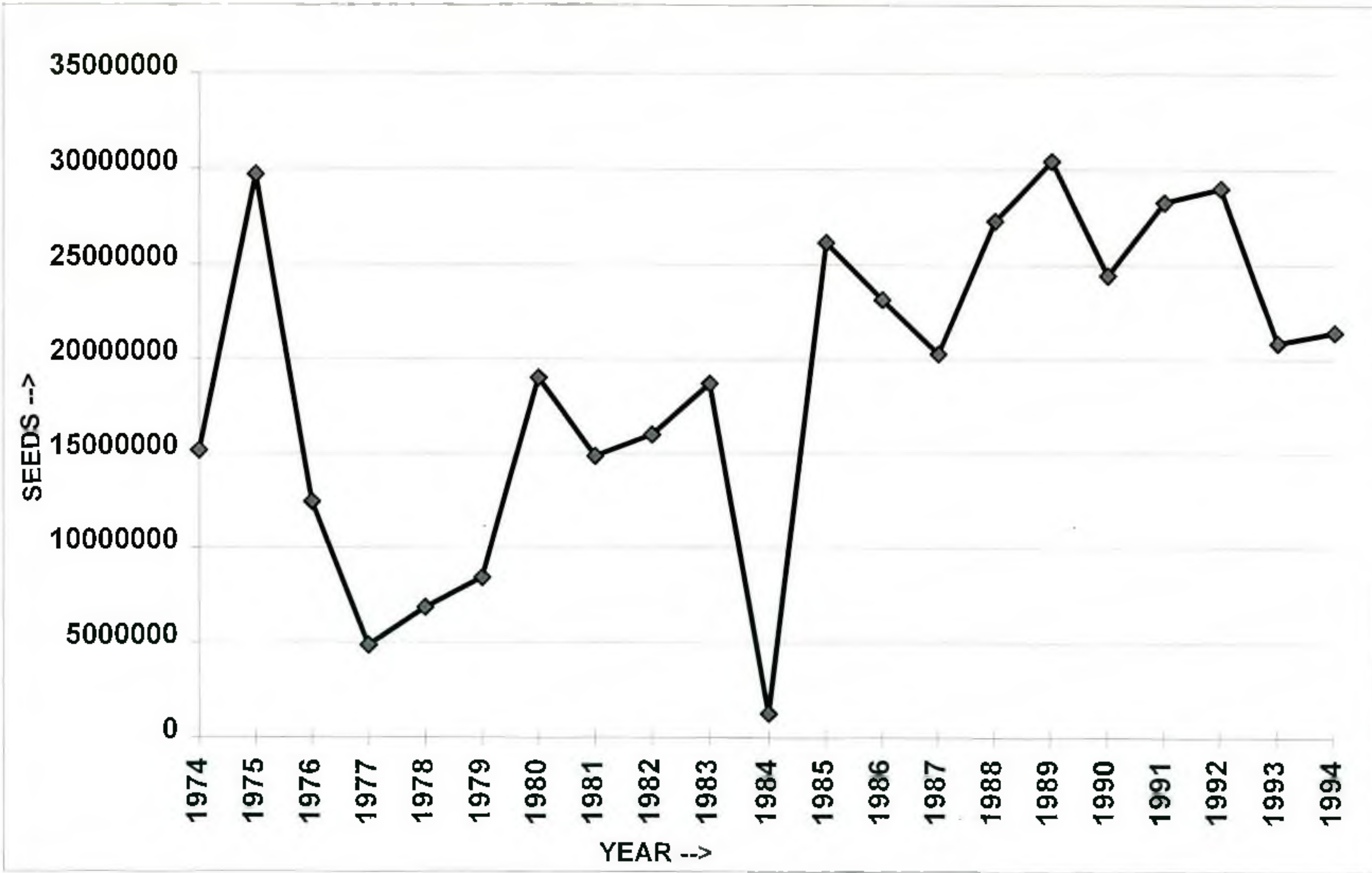

FIGURE - 8

The estimation results of fixed coefficient as well as random coefficients models are presented in Table 1.

ESTIMATES OF ELASTICITY COEFFICIENTS BY FIXED AND RANDOM COEFFICIENT (RC) MODELS.

	MODEL - 1		MODEL - 2	
Regressor	**OLS model**	**RC model**	**OLS model**	**RC model**
Labor	.854 (.0005)	.764 (.0002)	-----	-----
Capital	.634 (.001)	.509 (.0001)	-----	-----
Seed	-----	-----	.896 (.0006)	.790 (.0004)
Acreage	-----	-----	.332 (.071)	.291 (.031)

TABLE 1

(Figures in parentheses indicate standard errors.)

As the results presented in Table 1 show, elasticity coefficients obtained by both fixed & random coefficient models appear to be pretty close. When one considers model I, it is evident from elasticity coefficients that both the inputs, labor and capital keep significant impact on agricultural outputs.

However, labor-coefficient is considerably higher than capital which indicates that Bangladesh agricultural sector is still labor-intensive. This is ascertained both by fixed & random coefficient model. Although random model elasticities appear to be a little lower than those fixed coefficient model, in the former case standard errors are much lower. However in both cases, test of hypothesis at 5% level leads to rejection of the null hypothesis of no effect.

Similar results as discussed above hold good for model 2. In this case, effect of seed is much higher (almost double) than that of acreage. This is true by both fixed and random coefficient model. In this case also, mean elasticities coefficient by random coefficient model is a little lower than that of fixed coefficient model. Results of Model 2 indicate that agricultural output in Bangladesh is very sensitive to seed quality (say, Hyv).

For both Model 1 & 2 and by both fixed and random coefficients, sum of elasticities exceed unity which indicates increasing return to scale and this is more true for capital & labor than that of seed & acreage.

On the whole, Bangladesh agricultural sector, according to current results appears to be highly significantly sensitive with respect to changes in labor, seed use (quality may be). The next important factor appears to be capital investment.

CHAPTER - VI

CONCLUSION

In the present research work an attempt has been made to adopt the approach of random coefficient modelling to time series data of agricultural sector of Bangladesh. The conventional Cobb-Douglas production function has been restructured in the framework of random coefficient model. With variation in inputs two types of such model have been constructed. The estimation technique adopted is Hildreth-Houch procedure. However, for comparative assessment fixed coefficient model has also been estimated for the same data. The estimation results in two cases appear to be almost the same with exception that standard errors of elasticity coefficients for random coefficient models are much lower compared to those of fixed coefficient models. Thus, based on the current results we believe that parameter estimates obtained from random coefficient model are more reliable & stable. However, policy implications of both types of results from both types of models are same. One such policy implication is that Bangladesh agricultural output is sensitive with respect to labor as well as capital. When one considers use of seed & acreage as inputs for production function, seed use appears to be lot more influencial than acreage.

Given the fact that assumption of the fixed coefficient across observations (years) is iikely to be violated in the context of Bangladesh which faces lot of random disturbances such as flood, draught, political instability etc, results obtained from random coefficient model are more acceptable for policy formulation (long-run as well as short-run)

Further Research Horizons.

The present study can be extended in many directions some of which are as follows:

1. Data disaggregation by subsectors of agricultural sector may provide still more dependable results.
2. Assessment of farm size allocative & economic efficiency can be made.
3. Introduction of dynamic elements in the production function may yield more precise results.

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APPENDIX - 1

Year	Yield	Labour	capital
1974	140111	16839	402.62
1975	133717	16637	601.04
1976	144933	16428	1101.15
1977	139631	16221	1603.07
1978	150570	16017	1642.83
1979	149569	15815	3684.36
1980	149806	15616	4719.34
1981	156068	15425	4386.30
1982	156313	15779	5968.57
1983	162935	16110	3945.89
1984	168813	16448	7497.13
1985	169970	16712	3684.36
1986	175549	17478	1982.99
1987	176250	18279	2285.61
1988	174901	19117	2487.06
1989	173037	19993	3327.00
1990	190354	20910	4775.00
1991	193421	21868	5267.00
1992	197662	22870	5115.00
1993	201230	23919	4415.00
1994	201915	25015	4361.00

Yield - Gross domestic Product (In million Tk.) by agriculture sectors (Crops, Livestock, Forestry and Fishery) of Bangladesh at constant price base year 1984-85.

Labor - Employed persons (In million) 10 years and above by agriculture sectors (Crops, Livestock, Forestry and Fishery) of Bangladesh.

Capital - Development expenditure (In million Tk.) of the Government of Bangladesh by agriculture sectors (Crops, Livestock, Forestry and Fishery) of Bangladesh.

Year	Yield	Acres	Seeds
1974	108345	29424	15164676
1975	102332	28637	29730744
1976	113196	29686	12395844
1977	108084	28979	4811400
1978	116706	29702	6791256
1979	117982	31846	8483688
1980	118133	31973	18994896
1981	123936	32521	14803740
1982	122674	32638	15929820
1983	127784	33130	18691668
1984	133921	33013	1328026
1985	135031	32496	26102156
1986	139599	33459	23115344
1987	139596	34883	20228532
1988	137119	34148	27216176
1989	134509	33887	30369361
1990	150828	34750	24358462
1991	152575	34680	28256652
1992	155101	34121	28958469
1993	156392	33856	20885325
1994	153852	33331	21459807

Yield - Gross domestic Product (In million Tk.) by agriculture sectors (Crops) of Bangladesh at constant price, base year 1984-85

Acres - Total cropped area (In thousand acres.) by agriculture sectors (Crops) of Bangladesh.

Seeds - Distribution of improved seeds (In Kg.) by agriculture sectors (Crops) of Bangladesh.

Employment in Agriculture
(Employed Persons 10 Years & above)

Year	Number in 000				
	Agriculture		Non-Agriculture		Population
	Total	Percent	Total	Percent	
1961	14239	84.6	2589	15.4	55222.66
1972	16412	79.8	4163	20.2	
1973	16624	79.3	4345	20.7	
1974	16839	78.7	4569	21.3	76398.00
1975	16637	76.5	5117	23.5	
1976	16428	74.1	5731	25.9	
1977	16221	71.6	6419	28.4	
1978	16017	69.2	7139	30.8	
1979	15815	66.7	7902	33.3	
1980	15616	63.9	8818	36.1	
1981	15425	61	9869	39	89912.00
1982	15779	60.1	10482	39.9	
1983	16110	59.4	11022	40.6	
1984	16448	58.8	11528	41.2	98000.00
1985	16712	57.7	12265	42.3	100500.00
1986	17478	57.2	13084	42.8	102900.00
1987	18279	58.3	13094	41.7	
1988	19117	59.3	13105	40.7	
1989	19993	60.4	13115	39.6	110300.00
1990	20910	61	13355	39	
1991	21868	61.7	13597	38.3	111455.18
1992	22870	62.3	13843	37.7	
1993	23919	62.9	14094	37.1	
1994	25015	63.5	14349	36.5	
1995	26161	64.2	14609	35.8	
1996	27361	64.8	14873	35.2	
1997	28615	65.4	15142	34.6	
1998	29926	66	15416	34	

The data of employed persons 10 years & above in agriculture sector (Crops, Livestock, Forestry and Fishery) which are not available has been estimated by interpolation using exponential method.

THE END

Substituting this and simplifying, we get

$$\hat{\beta} = \sum_{i=1}^{N} w_i \hat{\beta}_i \tag{8.6}$$

where $\hat{\beta}_i = (x_i' y_i)/(x_i' x_i)$ is the estimator of β_i from Eqs.(8.1) corresponding to the ith cross-section unit and

$$w_i = \frac{1/\left[\delta^2 + \sigma_i^2/(x_i' x_i)\right]}{\sum_{j=1}^{N}\left\{1/\left[\delta^2 + \sigma_j^2/(x_j' x_j)\right]\right\}} \tag{8.7}$$

Note that $\sigma_i^2/(x_i' x_i)$ is var ($\hat{\beta}_i$). Hence if $\delta^2 = 0$, what Eqs.(8.6) and (8.7) say is that $\hat{\beta}$ is a weighted average of $\hat{\beta}_i$, the weights being inversely proportional to the variances. If δ^2 is very large compared with $\sigma_i^2/(x_i' x_i)$, the weights (8.7) are almost equal, and $\hat{\beta}$ is then close to a simple unweighted average of $\hat{\beta}_i$. The same would be the case if $\sigma_i^2/(x_i' x_i)$ are almost equal.

Suppose $\sigma_i^2/(x_i' x_i)$ are all equal. Then the estimator $\hat{\beta}$ in (8.6) would be the same no matter what δ^2 is. Thus the estimator obtained with the random coefficient model would be the same as the estimator obtained from a model where the coefficients are nonrandom (i.e., $\delta^2 = 0$). However, the variance of $\hat{\beta}$ would be different. It is given by

$$\frac{1}{\sum_{j=1}^{N}\left[1/\delta^2 + \sigma_j^2/(x_j' x_j)\right]} \tag{8.8}$$