THE WELFARE COST OF PROTECTION:

A PARAMETRIC SIMULATION

by

Mohammad Ali Akbar

A Dissertation Presented to the

FACULTY OF THE GRADUATE SCHOOL

UNIVERSITY OF SOUTHERN CALIFORNIA

In Partial Fulfillment of the

Requirements for the Degree

DOCTOR OF PHILOSOPHY

(Economics)

August 1970

ACKNOWLEDGMENTS

For the basic idea of this dissertation, and for the arduous editing of my English and of my ideas I gratefully acknowledge my great debt to Professor Jeffrey B. Nugent. His keen interest and assistance helped me throughout the study, and I appreciate it fully. In addition I wish to acknowledge my debt to Professors Aurelius Morgner and Park T. Ewart for their helpful supervision, guidance and encouragement.

I am much indebted to USAID for financial assistance and permission to stay in the United States and to the Government of Pakistan for granting me leave for completion of the study. I also gratefully acknowledge the assistance rendered by Dr. Charles Rankin of the Seaver Science Center in programming the model, and to the Computer Science Laboratory of the University of Southern California for granting me computer time. My gratitude for and to the libraries at both USC and UCLA for the invaluable service they have provided me through the years. Finally, I deeply appreciate my wife's constant care and love during the years of study at the University of Southern California. Without her encouragement this work would not have been finished.

TABLE OF CONTENTS

Page

ACKNOWLEDGMENTS ii

LIST OF TABLES v

LIST OF FIGURES vi

Chapter

I. INTRODUCTION 1

1.1 Objectives and Conclusions
1.2 Advantages and Limitations of the Study
1.3 Organization of the Dissertation

II. A Survey of Partial and General Equilibrium APPROACHES TO MEASUREMENT OF THE WELFARE EFFECTS OF TRADE RESTRICTIONS 5

2.1 The Brigden Model
2.2 The Young Model
2.3 Harberger's Method
2.4 Scitovsky's Method
2.5 Johnson's Method
2.6 Hause's Model
2.7 Krueger's Model
2.8 The Balassa-Schydlowsky Effective Tariff Method
2.9 The Dardis Model
2.10 General Equilibrium Approaches
2.11 McKinnon's Simulation Model
2.12 Lage's Linear Programming Model
2.13 Harberger's General Equilibrium Model
2.14 Johnson's Simulation Model
2.15 Other Studies
2.16 An Evaluation

III. JUSTIFICATION FOR THE USE OF A THREE COMMODITY MODEL 52

Chapter Page

IV. AN EXTENSION OF JOHNSON'S GENERAL EQUILIBRIUM MODEL OF WELFARE COST MEASUREMENT . . . 58

4.1 Consumption Cost in the Single Tariff Case
4.2 Consumption Cost in the Differential Tariff Case
4.3 Production Cost in the Single Tariff Case
4.4 Production Cost in the Differential Tariff Case
4.5 The Total Cost of Protection (with the CES Utility Function)

A. The Single Tariff Case
B. The Differential Tariff Case

4.6 Total Cost of Protection in the Unitary Elasticity of Substitution Case

V. SIMULATION OF THE MODEL: EMPIRICAL ESTIMATES 73

5.1 Consumption Cost Calculations
5.2 Production Cost Calculations

VI. POLICY IMPLICATIONS AND CONCLUSIONS 92

BIBLIOGRAPHY 96

APPENDIXES

A. THE DERIVATION OF THE CONSUMPTION COST MEASURE IN THE SINGLE TARIFF CASE 106

B. THE DERIVATION OF THE CONSUMPTION COST MEASURE IN THE DIFFERENTIAL TARIFF CASE . 130

LIST OF TABLES

Table | Page

1. Weighted Averages of Theoretical Incidence of Customs Duties and Charges of Equivalent Effect on the C.I.F. Value of Imports in Selected Latin American Countries 53

2. Weighted Averages of Theoretical Incidence of Customs Duties and Other Duties or Charges of Equivalent Effect on the C.I.F. Value of Imports in Selected Latin American Countries 54

3. Consumption Cost ($1-U_t/U_o$) of 20 Percent, 60 Percent, and 100 Percent Tariff on a Single Commodity for Various Values of r and σ under Two Commodity and r_1, r_3 and σ under Three Commodity Models 78

4. Consumption Cost ($1-U_t/U_o$) of a 60 Percent Single Tariff and Differential Tariff of 20 Percent and 100 Percent on Y and Z Commodity with the Weighted Average of 60 Percent for Various Values of r and σ . . . 83

5. Consumption Cost ($1-U_t/U_o$) of a Weighted Average of 80 Percent, 40 Percent, and 60 Percent Tariff for Various Values of r and σ 86

6. Protection Cost ($1-M_t/M_o$) for Various Values of M, L, N and t (% of M_o) 90

LIST OF FIGURES

Figure		Page
1.	The Brigden Committee's Measure of the Excess Cost of Protection	7
2.	Young's Measure of the Cash Cost of Protection	11
3.	Harberger's Measure of Resource Misallocation Due to Trade Restrictions	13
4.	Scitovsky's Measure of Gain from Specialization of Loss from Trade Diversion	15
5.	Johnson's Measure of Welfare Loss	16
6.	Welfare Loss through Pegging Exchange Rates .	19
7.	Dardis' Measure of Welfare Cost of Final Goods and Intermediate Goods	25
8.	Welfare Effects of Tariffs under General Equilibrium	28

CHAPTER I

INTRODUCTION

1.1 Objectives and Conclusions

Almost all of the several empirical attempts to estimate the welfare cost of tariffs by different authors for different countries and different time periods, using both partial and general equilibrium approaches, have yielded estimates that show the welfare cost of tariffs to be small. It is our hypothesis that these estimates vastly understate the welfare cost of protection. Among the assumptions common to most such attempts has been the simplifying assumption that all taxed commodities are subject to a single, uniform tariff rate.

Since in the real world there is considerable evidence to suggest that tariff rates vary considerably from commodity to commodity, the assumption of a uniform tariff rate would seem to be a particularly unrealistic one. It is the purpose of this dissertation to determine the empirical significance of this assumption by comparing the welfare cost calculations obtained from an extended three-commodity version of Johnson's two-commodity general equilibrium model under uniform tariff rates with the results obtained under comparable conditions when two different tariff rates apply.

As in Johnson's study the consumption and production costs of the tariff are estimated using a range of different values for the elasticity of substitution in consumption, the elasticity of supply, the proportions of income spent on taxed and untaxed commodities, and for the tariff rates. Johnson's definition of welfare cost in terms of the difference in real income that would be obtained at a maximum with and without tariffs is adopted in the present study.

The results of our study support the hypothesis that previous estimates may have considerably understated the welfare cost of tariffs due to their common, though highly unrealistic, assumption that there exists a single uniform tariff.

Our results suggest that the high tariff rates and haphazard structure of the tariff rates existing in most less developed countries may be responsible to a much larger extent than had previously been thought for the low level of national income and low growth rates in many such countries. While there exist some theoretical arguments in favor of differential tariff rates, they are not likely to be sufficiently applicable to such situations or sufficiently valid to diminish the importance of the results obtained in the present study as to the high welfare cost imposed by differential tariffs.

1.2 Advantages and Limitations of the Study

This is a simulation study which stands midway between a full-fledged theoretical study and a comprehensive empirical study--a sort of hothouse of a hypothetical world. However, its parametric values are drawn from the real world. Its limitations are, however, obvious since its policy conclusions cannot be prescribed without considerable further study.

It is believed that the extension of Johnson's two-commodity general equilibrium model to three commodities is an important improvement which permits consideration of differential tariffs as well as tariffs on intermediate commodities etc., and is consistent with similar extensions in other aspects of the theory of international trade in recent years. Nevertheless, this extended model still shared with Johnson's model some of its other limitations, such as the use of a CES function for substitution in consumption, the assumption of a perfectly competitive and static framework, and the absence of multiple equilibria and externalities.

1.3 Organization of the Dissertation

Chapter II surveys the various empirical methods that have been applied in measuring the welfare cost of trade restrictions in both partial and general equilibrium situations. The methods are compared and evaluated and several suggestions are made as to possible sources of bias in the estimating procedures.

Chapter III attempts to demonstrate the empirical relevance of differential tariffs. In Chapter IV we develop a three-commodity model for measuring the welfare cost of tariffs. This model permits the comparison of the welfare cost of tariffs when different tariff rates apply as compared with when a uniform tariff rate applies. In Chapter V the model is subjected to parametric simulation as to yield a range of estimates of the welfare cost of uniform and differential tariffs.

Our conclusions and policy recommendations are presented in Chapter VI, along with some suggestions for further research.

CHAPTER II

A SURVEY OF PARTIAL AND GENERAL EQUILIBRIUM APPROACHES TO MEASUREMENT OF THE WELFARE EFFECTS OF TRADE RESTRICTIONS

The literature on the measurement of welfare loss of protection or trade restrictions can be divided into two broad categories, namely partial equilibrium studies and general equilibrium studies. Both of these types of studies shall be devoted to attempts to measure loss empirically. It may be noted that the discussion is confined to static models.

The two most important studies that initiated the idea of the measurement of welfare cost were the Brigden Committee Report[1] for Australia published in the 1920's, and an individual report by Young[2] on Canada's commercial policy which appeared in 1957. Following the Brigden Report, a number of criticisms appeared, especially those of

---

[1]J. B. Brigden et al., The Australian Tariff: An Economic Enquiry (Melbourne: Melbourne University Press, 1929).

[2]J. H. Young, Canadian Commercial Policy (Ottawa: Royal Commission on Canada's Economic Prospects, 1957).

Viner,[3] Loveday,[4] and Reddaway[5] pertaining to the concept of the welfare cost of tariff, to the particular assumptions made, to the methods of measurement, and to the implications of their results.

In recent years welfare cost calculations have gained a considerable currency reflecting the continuing interest in operationalizing the theory of tariffs on the part of a good number of well-known economists, including Corden,[6] Reitsma,[7] Johnson,[8] and Krueger.[9] Another reason for current interest in the topic is that one after another, the developing countries have resorted to tariff and other protective measures as part of a deliberate effort to pro-

---

[3]J. Viner, "The Australian Tariff--An Economic Enquiry," The Economic Record, V (November, 1929); also, International Economics, Vol. XI, Memoranda on Commercial Policy (Glencoe, Ill.: The Free Press, 1936).

[4]A. Loveday, "The Australian Tariff: A Criticism," The Economic Record, VI (November, 1930).

[5]W. B. Reddaway, "Some Effects of the Australian Tariff," The Economic Record, XIII (June, 1937).

[6]W. M. Corden, The Calculation of the Cost of Protection," The Economic Record, XXXIII (April, 1957), 28-51.

[7]A. J. Reitsma, "The Excess Costs of a Tariff and Their Measurement," The Economic Record, XXXVII (December, 1961), 442-455.

[8]H. Johnson, "The Cost of Protection and Self-Sufficiency," The Quarterly Journal of Economics, LXXIX (August, 1965), 356-372.

[9]A. Krueger, "Some Economic Costs of Exchange Control: The Turkish Case," Journal of Political Economy, LXXIV (October, 1966), 466-480.

mote growth by import substitution with little regard for the comparative advantage or the cost structure of the products that are protected. In what follows we shall compare a few of the more important of these studies noting their (1) approach, (2) estimation methods, and (3) conclusions.

2.1 The Brigden Model

The Brigden model[10] defined the excess cost of protection as those costs of goods protected by tariffs and produced domestically above the costs of similar goods obtained from foreign sources. The model is illustrated with the following figure:

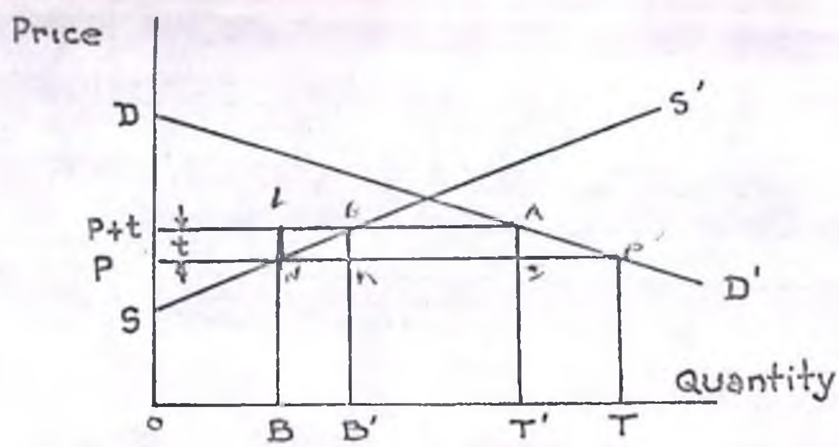

Fig. 1.--The Brigden Committee's measure of the excess cost of protection.

Let DD' and SS' be the home demand and supply curves. At

10Brigden, op. cit., p. 35.

the protected price (P + t), total home consumption is OT', of which OB' is home produced and B'T' is imported. Under free trade total home consumption is OT, of which OB is home produced and BT is imported. Although the Brigden Committee's definition of excess cost of protection referred to the difference between the costs of protected domestically produced goods with a positive tariff and the cost of similar home goods under free trade, in actual measurement the difference between the home market value of the protected output and the cost of equivalent imports, LNKG in Figure 1, was used. The model implied that if P is the price per unit of product without tariff, and t, the absolute tariff per unit of the same, then the domestic price with tariff is given by P + t. Since the duty on imports was known to vary from commodity to commodity, the Brigden Committee employed the "average duty actually paid" on all imports as the measure. Actually protected manufactures were divided into three classes on the basis of the proportion of imports to total consumption. For example, if imports were a substantial proportion of the quantity of any particular good consumed, then domestic price was taken to be equivalent to the price of imports plus duty and the excess cost was the full amount of the duty; for the second category in which imports constituted a relatively small proportion of consumption, the domestic price was taken lower than the import price plus duty, and the excess cost

cost was taken to be half the amount of duty on corresponding imports; in the third category were the sheltered industires in which the excess cost was taken to be one-third of the amount of the duty on corresponding imports. Applying this measure, the Brigden Committee found the cost of Australian tariffs to be approximately 6 percent of national income.

While in percentage terms the welfare cost computed by this method may seem substantial, the definition adopted by the Committee on the one hand overstates the production cost, but on the other hand neglects the consumption cost. By referring to Figure 1, the reader can see that the computed production cost given by the area LNKG is larger than the actual production cost given by the Δ NKG. However, in some cases only one-half or one-third of the proportion was used in the calculations. The consumption cost given by AJP' is ignored. The combination of a possible overestimation of production cost and complete omission of consumption cost in all likelihood yields an overall underestimate of the welfare cost of the tariff.

2.2 The Young Model[11]

Young developed the "cash cost" concept of protection by the illustrative example of broom production. It is assumed that a free trading country may obtain household

---

[11]Young, op. cit., pp. 63-73.

brooms from abroad for $1.00 per broom or for $1.25 per broom from domestic sources. Normally under free trade conditions the country satisfies its entire demand by importing one million brooms. If in an attempt to encourage local broom production, the country imposes a tariff of 25 percent or more on brooms, imports would be completely replaced by domestic production. In this situation consumers must pay a price of $1.25, and as a result they consume fewer brooms, say 900,000 at a total cost of $1,125,000. Resources which normally produced $1 million worth of exports are released, but other resources which produced $1,250,000 worth of domestic goods, are attracted into broom production. Instead of being able to buy 900,000 brooms for $900,000, consumers must pay an extra of $225,000 to subsidize the domestic production of a commodity in which the country evidently does not have a comparative advantage. This amount of $225,000 is what Young calls the "cash cost" of the protective tariff. Specifically it is that amount which consumers must pay for the amount they purchase of a protected domestically produced commodity over and above what they would pay for the same amount of the commodity in world markets without trade restrictions. This calculation is shown in Figure 2.

Young applied the cash cost method from the expenditure side. Once again this method is likely to understate the cash cost of the tariff, since it again excludes consumption cost and in Young's application of this method was applied only to the private sector of the economy.

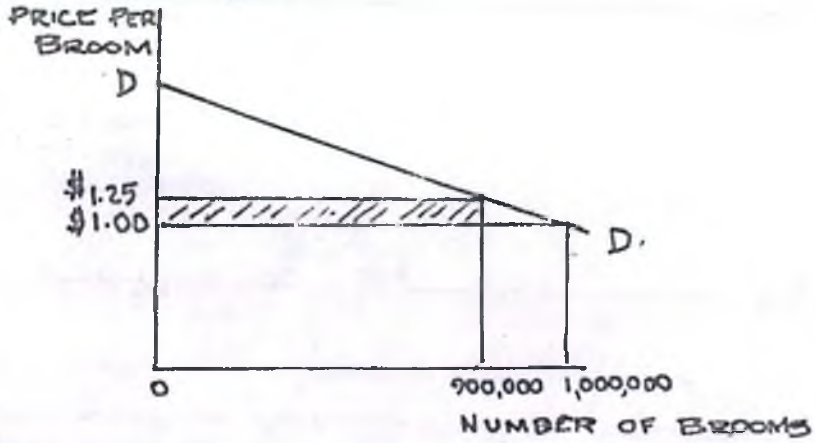

Fig. 2.--Young's measure of the cash cost of protection.

Data limitations on prices, etc., may also have made Young's estimate of the cash cost of the tariff in 1957 (of 3.5 to 4.5 percent of gross expenditure net of indirect taxes) an underestimate. Young himself concluded that "this factor, together with the others (as noted above) has tended to give the final estimate of the cash cost of the tariff a distinct downward bias."[12]

2.3 Harberger's Method[13]

Harberger attempted to measure the welfare cost of tariffs in terms of resource misallocation in economies

---

[12]Young, op. cit., p. 71.

[13]A. C. Harberger, "Using the Resources at Hand More Effectively," American Economic Review, Papers and Proceedings, XLIX (May, 1959), 134-146.

like Chile, Brazil and Argentina. In estimating the welfare cost of trade restrictions, he assumed (1) an average tariff on all imports, (2) a linear domestic demand function, (3) a price given in the world market (the small country assumption), and (4) a small foreign trade sector. Given a transformation function between the quantity of exports (X) and the quantity of imports (M) of the following form T(X,M) = 0, his assumption of an average tariff of 50 percent implies M/X = 1 in the pre-tariff situation and M'/X = 1.5 in the post-tariff situation. His calculations showed that the welfare cost varies with the square of the tariff rate indicating that welfare cost may be underestimated by averaging the tariff rates as is usually done.

Harberger's approach may be demonstrated in Figure 3. Costs are assumed constant at PC and Dd is the demand curve. Under free trade conditions price and quantity are set at the intersection of marginal cost and the demand curve at C. When a tariff t is imposed, OE is consumed and locally produced. Since the imposition of the tariff simply redistributes TPAB from consumers to producers, the triangle ABC represents the "deadweight" loss of the tariff. This is the equivalent to multiplying the price differential AB by one-half the quantity differential BC. In order to convert such an estimate of absolute loss in consumer surplus, this magnitude was multiplied by the proportion of national income which would be subject to such misallocations from tariffs. Using an equivalent tariff of 50 percent, and assuming imports to be 10 percent of national income, Harberger found in the case of Chile that the removal of trade restrictions would raise welfare by not more than 2 1/2 percent of the national income.

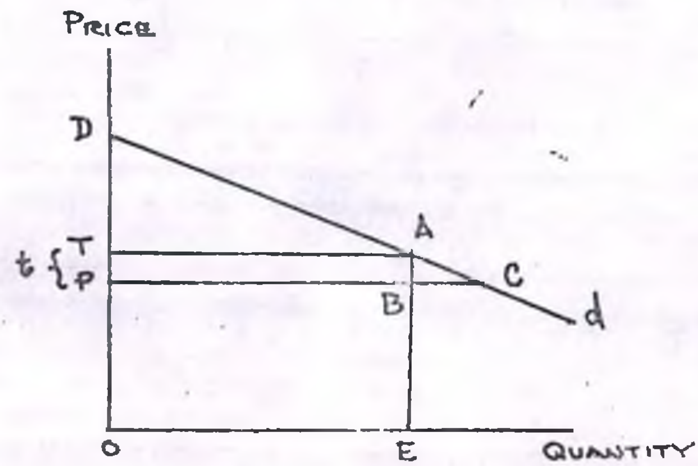

Fig. 3.--Harberger's measure of resource misallocation due to trade restrictions.

2.4 Scitovsky's Method[14]

Although Scitovsky specifically measured the gain of specialization arising out of trade creation and the loss due to trade diversion in a particular customs union, the European Economic Community (EEC), his method is also applicable to calculation of the welfare gain from the elimination of tariffs. If country A exports more of the

---

[14]T. Scitovsky, Economic Theory and Western European Integration (London: Unwin University Press, 1962).

commodity α its production of this commodity will increase and the production of α in country B will decline. This will increase marginal cost of α in country A and decrease the marginal cost of α in country B. There is thus a positive relation between the ratio of the marginal cost of commodity α in country A to the marginal cost of α in country B and the volume of exports of α from country A. The greater the exports of α from country A, the larger will be the ratio of the marginal cost of α in country A to the marginal cost of α in country B. The same argument applies to commodity β and country B. The greater the exports of commodity β from country B, the smaller will be the ratio of the marginal cost of commodity β in country A to the marginal cost of commodity β in country B. Denoting the marginal costs of α in country A as Ac_{α} and of commodity β in country A by Ac_{β}, we can represent Scitovsky's method in Figure 4.

Equilibrium is achieved at E where the value of the exports from country A is equal to the value of exports from country B. The quantity (or volume) units may be chosen in such a way as to make the prices and marginal cost of both products in both countries equal to unity.

Now if country B imposes an ad valorem import duty of t_{α} (indicated by Bt_{α} in Figure 4) on imports of α from country A, the production of the import competing industry

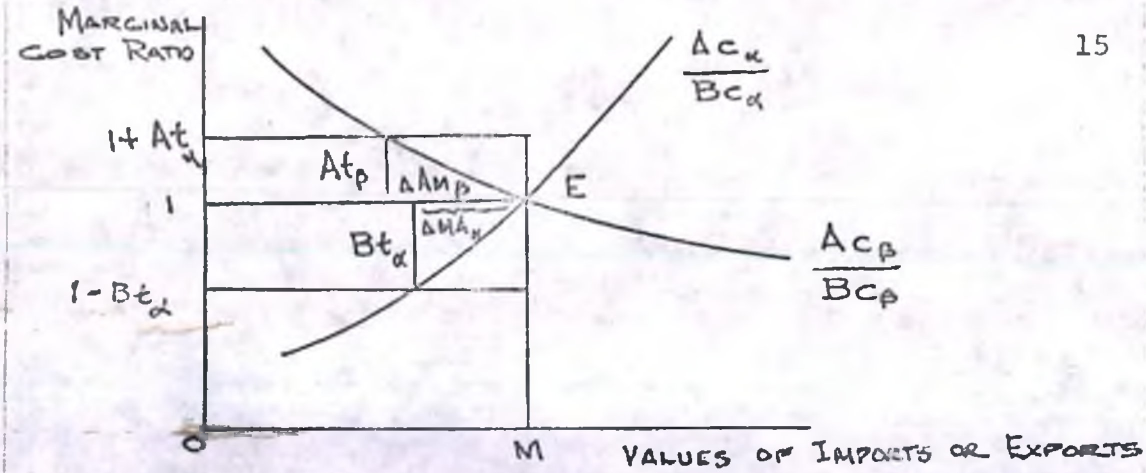

Fig. 4.--Scitovsky's measure of gain from specialization or loss from trade diversion.

ΔBMd = change undergone by the imports of β
At_β = ad valorem tariff on the imports of β commodity into country A
Ac_α = marginal costs of the α commodity in country A

α in country B will increase until the marginal costs of α are t_α percent higher than those of α in country A. Country B is now producing α at costs higher than those it would have to pay if it imported α from country A without a tariff. The figure shows that the loss suffered by country B is not equal to the higher marginal cost of α times the extra production (i.e., reduced imports) of α but is equal to only one-half this magnitude, i.e., $1/2\ Bt_\alpha \times \Delta BM_\alpha$ where ΔBM_α is the change in imports (production) of α by country B. Scitovsky used this method to calculate the benefits of the European Common Market using data provided by Verdoorn.[15] Scitovsky concluded that the gain from the

---

[15] P. J. Verdoorn, "A Customs Union for Western Europe--Advantages and Feasibility," World Politics, VII (July, 1954).

creation of the Common Market which eliminated tariffs between member countries and thereby increased specialization was less than one-twentieth of 1 percent of the gross product of the countries involved. He concluded, "the most striking feature of these estimates is their smallness." Note that although Scitovsky's approach is somewhat different from the approach used in the other studies reviewed in this chapter, the actual calculations depend on measuring the same triangle.

2.5 Johnson Method[16]

Johnson's method can be illustrated by reference to Figure 5.

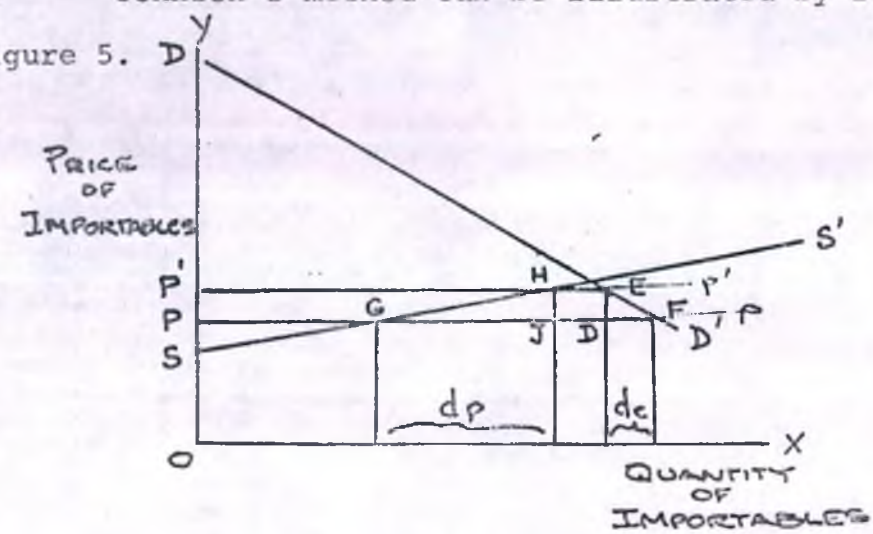

Fig. 5--Johnson's measure of welfare loss.

In this figure quantities of importable goods are measured along the OX axis and the price of importables

---

[16] H. G. Johnson, "The Cost of Protection and the Scientific Tariff," Journal of Political Economy, LXVII (August, 1964), 327-345.

(in terms of exports) along the Y axis. Domestic demand is given by the constant utility demand curve DD', the domestic general equilibrium supply curve by SS' and foreign supply by PP. If the domestic market price of OP' is defined as unity, the PP' is the excess of the domestic price over the foreign price OP'. PP' relative to Op' would then represent the proportion by which free trade would reduce the domestic price. This proportion equals 1/1+t, where t is the tariff rate. With the tariff, consumption is OC, domestic production Op, imports PC. If the tariff were removed and consumers were simultaneously deprived of export goods to the extent required to keep their utility level constant, consumption would expand to OC', and domestic production would fall to OP'. Consumers' surplus would be increased by PP'EF. Of this PP'HG would be offset by a reduction in producers' surplus, and HJDE by the loss of import duties formerly collected. The remainder, consisting of GHJ and DEF, would represent the increase in the value of production and the reduction in the cost of consumption made possible by the change to free trade, which must be extracted if utility is to remain constant; that is GHJ is the production cost and DEF the consumption cost of protection. Thus Johnson uses the conventional partial equilibrium diagram to handle at least some general equilibrium considerations. Once again the magnitudes to be estimated are identical with those designated in other studies.

Indeed, Johnson has gone on to suggest ways in which this traditional model could be generalized. In particular he has discussed the desirability of relaxing the following limiting assumptions:

1. That the domestic price of importables may not be exactly equal to the price of imports plus the tariff rate due to monopolistic positions, etc.

2. The exclusion of relative price effects in terms of home goods and the possibility of substitution effects in production and consumption.

If Johnson had been successful in analyzing the welfare costs of tariffs when the second of these assumptions was relaxed, there would have been little need for the present work. However, Johnson was notably unsuccessful in his attempt to consider the case of relative price and substitution effects (implying the use of a three-goods model) and therefore he ignored such possibilities in his subsequent formulation.[17] He did, however, consider the first of these possibilities, thereby deriving the formula used by Krueger which is presented in section 2.7 below.

---

[17]Johnson, op. cit., p. 333.

Applying this method Johnson found that the entry of Britain to a proposed European Free Trade Area would give her at most a gain of 1 percent of national income.[18]

2.6 Hause's Model[19]

Hause's model attempted to measure welfare loss through Government intervention in the foreign exchange market, such as by pegging exchange rate at points other than those which allow for stable equilibrium under free trade. This may be illustrated with reference to Figure 6.

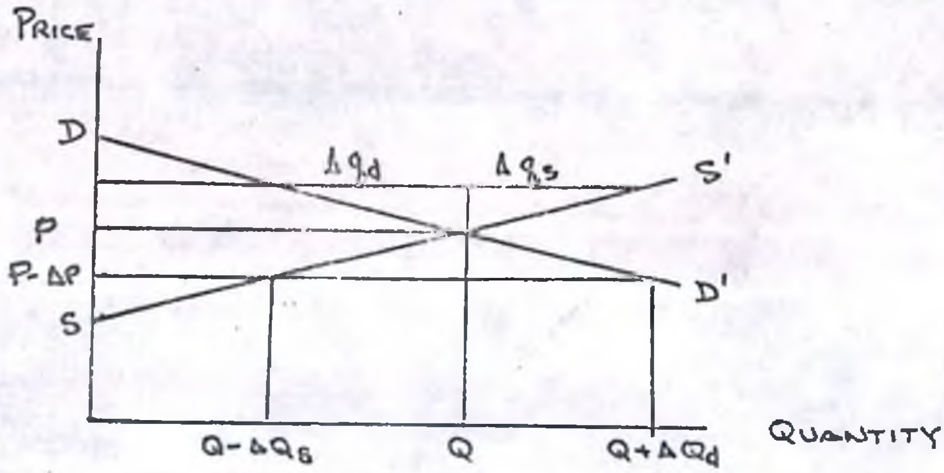

Fig. 6.--Welfare loss through pegging exchange rates.

DD' and SS' are the demand and supply curves of foreign exchange in country 1 (measured in country 1's currency

---

[18]H. G. Johnson, "The Gain from Freer Trade with Europe: An Estimate," Manchester School of Economic and Social Studies, XXVI (September, 1958), 247-255.

[19]J. C. House, "The Welfare Cost of Disequilibrium Exchange Rates," Journal of Political Economy, LXXIV (August, 1966), 333-352.

unit). Let P, Q be the equilibrium exchange rate and the quantity of foreign exchange in country 1, respectively. ΔP is the amount by which country 1's government pegs the exchange rate above and below the equilibrium level in periods 1 and 2. Q_s and Q_d are the additional amounts of foreign exchange that are required respectively to reach the supply curve and the demand curve from the equilibrium point when the exchange rate differs by ΔP.

After intervention the government accumulated ΔQ_d and ΔQ_s units of foreign exchange in the first period at a price $(P + \Delta P)$ and releases it in the second period at $(P - \Delta P)$. The total exchange costs to country 1 are $(Q + \Delta Q_s)(P + \Delta P)$ in period 1, and $(Q - \Delta Q_s)(P - \Delta P)$ in period 2. At the equilibrium exchange rate the total exchange costs to country 1 are given by 2PQ in the two periods. This shows that government policy of intervention raises the exchange costs by $2\Delta P \Delta Q_s$. On the demand side, the policy leads to a decline in imports in the first period which is valued by consumers as $(P + \Delta P/2)\Delta Q_d$, and to an increase in imports in the second period which is valued as $(P - \Delta P/2)\Delta Q_d$. The change in the gross value from imports over the two periods is $-\Delta P \Delta Q_d$. Adding together the change in gross value from imports and the change in exchange costs gives the net change in welfare of $-\Delta P(\Delta Q_d + 2\Delta Q_s)$. This net loss may also be written $(\eta - 2\varepsilon)(\Delta P/P)^2 PQ$ where η is the elasticity of demand for imports (and foreign exchange) with respect to price of foreign exchange in country 1 (and is negative), ε is the elasticity of supply of foreign exchange with respect to the price of foreign exchange for country 1, and PQ is the value of foreign exchange purchases per period at equilibrium measured in terms of country 1's currency units.

Hause's analysis assumed that:

(i) the demand and supply curves of foreign exchange for country 1 are linear

(ii) no speculative demand for foreign exchange exists

(iii) there is full employment

(iv) the distributional effects of governmental policy can be ignored

The welfare measure derived above was applied to Argentina for the period 1959-1962 with the assumed elasticity of demand for imports of 1.5 and that of demand for Argentina's exports of 2.5. Hause found that the welfare loss to Argentina per unit of time as a percentage of foreign trade would be 0.68.

His measure is again of the partial equilibrium triangle variety but differs from the loss measured by ordinary commodity demand and supply curves.

Johnson[20] has extended the model so as to estimate

---

[20] H. G. Johnson, "The Welfare Costs of Exchange

the effect on world welfare as well as on the welfare of the individual country.

2.7 Krueger's Model[21]

Following the general method used by Johnson in measuring production cost of tariffs, but considering the importance of raw material outputs (often imported) in domestic production, Krueger defined a new measure, that of the domestic resource cost of earning foreign exchange. Specifically, the resource cost (R) "of a set of domestic prices differing from world prices" is defined as:

$$R = \sum_i (K_i P_i - C) X_i$$

where X_i is the quantity of output produced in the ith activity (with units so chosen that one unit of output represents one unit of value added in the international market), K_i is the non-rent portion of domestic value added in the ith activity, P_i is the domestic value added in the activity, C is defined as the minimum cost per unit (in domestic currency, with normal profit and no rent) of producing the same international value of output as was actually produced.

---

[21] A. O. Krueger, "Some Economic Costs of Exchange Control: The Turkish Case," Journal of Political Economy, LXXIV (October, 1966), 466-480.

2.8 The Balassa-Schydlowsky Effective Tariff Method[22]

Balassa and Schydlowsky have taken issue with the Krueger-Johnson measure of the welfare cost of protection which calculates the domestic resource cost of earning a unit of foreign exchange. Balassa and Schydlowsky have shown that the domestic cost of a dollar earned or saved for commodity i (B_i) is related to their own effective tariff rate of protection j (Z_i) in the following way:

$$(1)\quad B_i = \sum_j \frac{W_j r_{ji}}{P_i - \sum_j V_j r_{ji}}$$

$$(2)\quad Z_i = \frac{W_j - V_j}{V_j} \quad \text{or} \quad W_j = V_j(1 + Z_j), \text{ substituting}$$

into (1) we get (3)

$$(3)\quad B_i = 1 + \sum_j Z_j \frac{V_j r_{ji}}{\sum_j V_j r_{ji}}$$

where W_j = domestic value added in product j

V_j = value added in product j under free trade

r_{ji} = elements of direct and indirect input requirements

P_i = world price of product i

Thus these authors have demonstrated that the domestic

---

[22] B. Balassa and D. M. Schydlowsky, "Effective Tariffs, Domestic Cost of Foreign Exchange and the Equilibrium Exchange Rate," Journal of Political Economy, LXXVI (May/June, 1968), 348-360.

resource cost per unit of foreign exchange equals one plus a weighted average of the effective rates of protection. By examining a hypothetical example they found that the Krueger-Johnson method may rank an inefficient industry higher in the efficiency scale than an efficient one since the approach ignores the possibility of substitution of foreign for domestic inputs and thereby reducing the apparent or temporarily protected domestic producers. Thus it appears that the welfare cost of protection measured by applying the resource cost approach may overstate the magnitude of loss. These authors do not make the assumptions about foreign demand and domestic supply elasticities necessary computing welfare, but they do suggest that if such assumptions could be made, better estimates of welfare cost could be obtained.

2.9 The Dardis Model[23]

Dardis has extended the conventional method for estimating the welfare cost of protection in such a way as to incorporate tariffs on intermediate goods (in addition to tariffs on final goods). The total welfare cost for both final goods and intermediate goods can be illustrated with reference to Figure 7.

---

[23] R. Dardis, "Intermediate Goods and the Gains from Trade," The Review of Economics and Statistics, XLIX (November, 1967), 502-509.

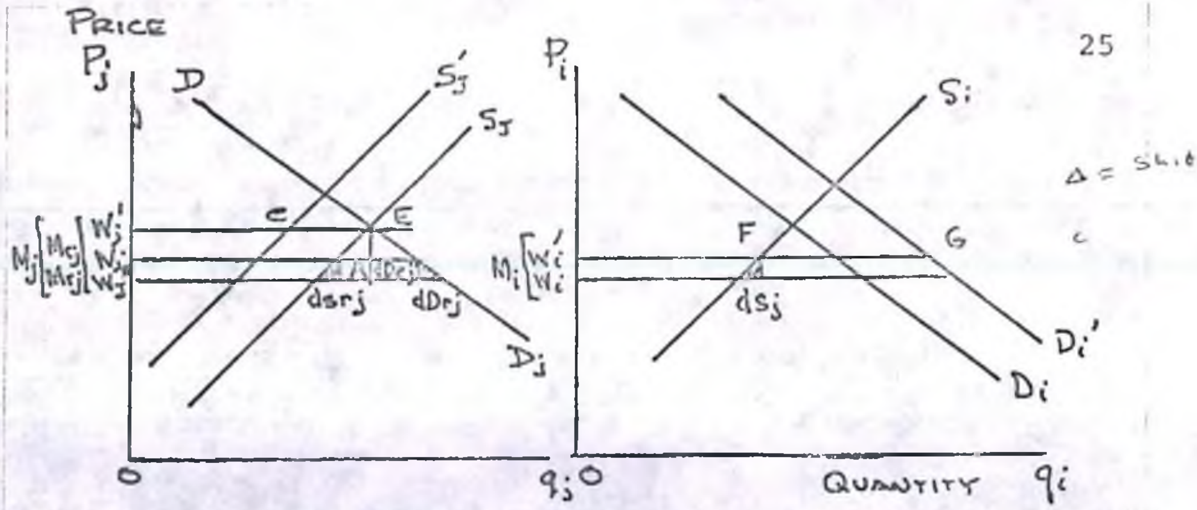

Fig. 7.--Dardis' measure of welfare cost of final goods and intermediate goods.

DD_j, DD_i are respectively the demand curves for final good j and intermediate good i. Similarly S_j and S_i are the respective supply curves. A tariff of Mr_j on good j will raise its price from OW_j to OW_j^o, resulting in a welfare loss equal to 1/2 $Mr_j(dSr_j + dDr_j)$, where dSr_j measures changes in domestic production due to the tariff and dDr_j measures the corresponding change in consumption due to the tariff.

If a gross tariff of M_i is imposed on the intermediate good i it will cause an upward shift in S_j to S_j'. However, an additional tariff of MC_j could be imposed on j which would be sufficient to hold producers' surplus constant and thereby maintain the level of domestic production. The increased compensatory tariff on final good j would increase the value of the marginal product of intermediate good i in production of j, thereby shifting the demand curve for i upward from D_i to D_i'. In the figure $W_j'C$ and

$W_i'F$ indicate domestic production of the two goods j and i and $W_j'E$ and $W_i'G$ show their consumption respectively. The post-tariff price levels are OW_j' and OW_i'. The final good model shows the gain in revenue ($MC_jCE - Mr_jdDC_j$). Deducting this revenue gain from the loss in consumers' surplus one obtains the net welfare loss equal to $MC_jW_j'C + dDC_j(1/2MC_j + Mr_j)$ resulting from a combination of tariffs on final goods and on intermediate goods.

In the ith sector there is a gain in producers' surplus and in revenue amounting to $M_iW_i'G - 1/2M_idS_i$. If fixed coefficients of production are assumed then $MC_jW_j'C = M_iW_i'G$. The cost of the second round tariff is now given by $dDC_j(1/2MC_j + Mr_j) + 1/2M_idS_i$. Thus the total welfare loss is given by the results of the two rounds, i.e., $1/2Mr_jdSr_j + 1/2M_jdD_j + 1/2M_idS_i$.

Dardis applied this measure to estimate the cost of protection of the feed livestock sector in West Germany in 1960. She found the absolute cost of protection in this sector to range from $49 to $50 million while in relative terms the costs varied from 10 to 11 percent of the change in producers' surplus.

2.10 General Equilibrium Approaches

The awareness that partial equilibrium methods of measuring the welfare loss due to protection becomes increasingly unsatisfactory the higher the level of aggregation, and the suspicion on the part of several authors such as Reitsma, Johnson and Corden that partial equilibrium methods may have lent downward bias to such measurements has led researchers to attempt the analysis and measurement of the welfare cost of trade restrictions within a general equilibrium framework. The early attempts to view the theory of measuring the welfare cost of tariffs within general equilibrium can be traced to the writings of Corden,[24] Reitsma[25] and Johnson[26] later followed by McKinnon,[27] Lage,[28] and Johnson[29] himself. Recent attempts have also been made to incorporate the theory of "effective tariffs" into a general equilibrium framework. In what follows we shall first describe a two commodity general equilibrium model giving the restrictive effects of tariffs and then survey the individual studies by McKinnon, Lage and Johnson.

---

[24]Corden, op. cit., pp. 28-51.

[25]Reitsma, op. cit., pp. 442-455.

[26]Johnson, "The Cost of Production and the Scientific Tariff," op. cit., pp. 327-345.

[27]R. I. McKinnon, "Intermediate Products and Differential Tariffs: A Generalization of Lerner's Symmetry Theorem," Quarterly Journal of Economics, LXXX (November, 1966), 584-613.

[28]G. H. Lage, "The Welfare Cost of Trade Restriction: A Linear Programming Analysis" (unpublished Ph.D. dissertation, University of Minnesota, 1967).

[29]Johnson, "The Cost of Production and the Scientific Tariff," op. cit., pp. 327-345.

The welfare effect of a non-prohibitive tariff may be illustrated with reference to Figure 8.

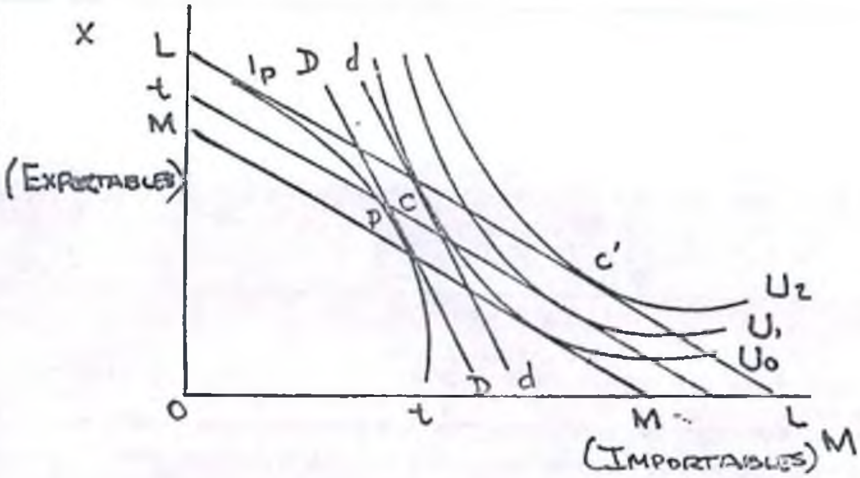

Fig. 8.--Welfare effects of tariffs under general equilibrium.

In the figure tt is the transformation curve. U_0, U_1 and U_2 are the community indifference curves. LL and MM are the internationalized price lines, and DD, dd are the domestic price lines. With restrictions on trade the equilibrium production is at P and equilibrium consumption is at C, the exchange ratio being shown by the slopes of DD and dd. When trade restrictions are removed, the equilibrium positions for production and consumption are at P' and C', respectively. The welfare loss due to tariff may be measured by LM, the amount of the exportable good that could be withdrawn from the community without loss of utility enjoyed at the pretrade situation. The magnitude of the loss can be divided into two components--the exchange loss of welfare from U_1 to U_0 due to the loss in consumption and the loss in welfare from U_2 to U_1 as a result of the reduction in production. Since the exchange loss can be measured by LM, the segment tM measures consumption cost and tL measures production cost. Note that LM is the amount of exportable that can be withdrawn from consumers in order to keep them as well off as before, and that t is the amount of export good which can be withdrawn from the producers with consumption fixed at C so as to keep them as well off. Note also most importantly that this method of measuring welfare loss in terms of one commodity permits one to arrive at a quantitative measure of the welfare cost of protection even if one is not willing to grant the possibility of cardinal utility. This accomplishment is indeed a major breakthrough in the effort to provide statisfactory estimates of the welfare cost of trade restrictions.

McKinnon's Simulation Model[30]

McKinnon derived a simulation formula from production functions relating intermediate inputs and labor to the gross outputs of X_m and X_e of the following Cobb-Douglas form:

$$\text{(i)}\quad X'_m = \phi_m L_m^a X_{em}^{1-a}$$

$$\text{(ii)}\quad X'_e = \phi_e L_e^\beta X_{me}^{1-\beta}$$

[30]McKinnon, op. cit., pp. 584-615.

where X'_m is gross production of commodity m (importables);

X'_e is gross production of commodity e (exportables);

X_{em} is an intermediate input of commodity e in production of m;

X_{me} is an intermediate input of commodity m in production of e; and

L_m and L_e are domestic primary factors in production of m and e.

Setting a = β and choosing units such that $\phi_m = \phi_c = 1$ and $P_m/P_e = 1$ under autarky, McKinnon maximizes $X'_e - X_{me}$ thus obtaining:

$$\frac{\hat{X}_e}{\bar{X}_e} = \left(\frac{P_m}{P_e}\right)^{\frac{1-a}{a}} \qquad \text{where } P_m/P_e < 1$$

where $\frac{P_m}{P_e}$ is the ratio of the price of importables to exportables under free trade relative to the same ratio under autarky;

$\hat{X}_e$ is net consumable output under autarky;

$\bar{X}_e$ is net consumable output under free trade; and

a is the share of the domestic primary factor in the value of gross output of exportables.

Given the parameters P_m/P_e and a, one can determine the ratio of consumable output under autarky to consumable output under free trade. Stipulating a range of values from .70 to 1.0 for a, .25 to 1.0 for P_m/P_c, the loss of national income (in percentage terms when autarky is imposed instead of free trade ranges from 0 to 45 percent when the importable input has a share of 30 percent; it ranges from 0 to 37 percent for an importable input of

25 percent and from 0 to 14 percent when the importable input share is 10 percent. This shows that with the increase in the share of importable input trade in the gross output of exportables the losses from free trade national income due to a policy of economic autarky goes on increasing.

However, McKinnon's use of the Cobb-Douglas production function which restricts the elasticity of substitution to unity despite the evidence summarized by Nerlove[31] which suggests the elasticity of substitution might in fact be less than one, may have biased his estimates downward. If one replaced the Cobb-Douglas production function by another with a lower elasticity of substitution, the inclusion of intermediate inputs along with domestic factors would in all likelihood greatly increase the gains from free trade.

2.12 Lage's Linear Programming Model[32]

Lage's study utilizes a linear programming approach in estimating the welfare cost of protection in Japan for the year 1955. He built two models (i) the basic production model and (ii) the demand inclusive production model.

---

[31] M. Nerlove "Recent Empirical Studies of the CES and Related Production Functions," The Theory and Empirical Analysis of Production, National Bureau of Economic Research, Studies in Income & Wealth (New York: National Bureau of Economic Research, 1967), pp. 55-120.

[32] Lage, op. cit., p. 1.

Since the first model did not give unique solutions for the trade variables, his calculation of the cost of protection is derived from the second model. The second model was based on the assumption of the proportionality of any changes in sectoral final demand to total final demand. In this model he maximized the value of "a," a scalar which, when multiplied by the ratio of the final demand for an individual commodity to the value of total final demand in the base period, gives the sectoral final demand. This was interpreted as maximizing overall consumption in the economy. In this formulation he also abandoned the assumption of fixed and known final demand levels for the domestic sector and instead assumed the expansion of demand for domestic goods in fixed proportions. Two different sets of capacity restrictions and three sets of world prices were used to obtain five different optimal solutions to the programming model

In the first set of three solutions capacity constraints for the agriculture sector were designed to vary between ±2.5 percent, and the non-agriculture sector between ±10 percent. World prices were calculated in the first solution on the basis of a single (unweighted) average tariff, in the second, on the basis of a single (unweighted) tariff twice the size of the tariff in the first case, and in the third, with no tariff. In the second set of solutions the potential change in output of the respective sectors was doubled. In this case only two alternative sectors of world prices were used in the first case with a uniform tariff twice the original size, and in the second case with no tariff. The third solution of the first set and the second solution of the second set (the free trade solutions) have been treated as computed base for alternative calculations of the welfare cost of tariffs.

He has two sets of results for the welfare cost of trade restrictions in Japan. In a comparison between the actual final demand of 1955 and the corresponding computed final demand for the five solutions, the production cost of a relatively moderate tariff is estimated at 4.5 to 6.5 percent of national income. However, by comparing the optimal solutions under free trade with those with the tariff, the cost of protection falls to .3 to .8 percent of national income. Since he does not provide enough explanation as to how he arrived at the second set of these results it is difficult to appraise his figures. Once again the welfare cost of protection is apparently extremely low.

Some possible reasons for such low estimates may be the following:

(1) If trade restrictions have in fact distorted the structure of production one might assume they would have affected the input-output coefficients. However Lage has not accounted for any such distortions;

(ii) He has used two sets of prices--one for the free trade situation and another for the tariff-distorted situation in which the former set is lower than the latter set except for the non-taxed sectors; unless one allows for the demand elasticity in such a situation, the values of the objective function in the free trade situation (with the lower set of prices) will be always lower than that of the tariff distorted case (with the higher set of prices).

(iii) His introduction of the fixed proportion demand assumption does not allow for substitution between goods in the consumption bundle whereas Johnson's results[33] indicate that the consumption cost of protection increases with the elasticity of substitution in consumption.

(iv) He has not explicitly treated imports and exports as activities in his programming model but rather derived their values on the basis of demand functions.

(v) Labor was the only scarce resource in his model other than the other arbitrary limitations on capacity. For most underdeveloped countries the restrictions imposed by shortages of skilled labor, foreign exchange, etc. are likely to impose much more substantial constraints on welfare maximization than the availability of (normal or unskilled) labor. Probably the same is true for Japan.

---

[33]Johnson, "The Welfare Cost of Protection and the Scientific Tariff," op. cit., pp. 356-372.

2.13 Harberger's General Equilibrium Model[34]

Harberger attempts to construct a general model for measuring the welfare loss (or gain) arising from the imposition of any tax on an economic system composed of spenders and government, where the former's money income is a constant percentage of the national income, and the latter balances its fixed budget by neutral taxes or transfers. (He also considers nonneutrality of taxes and distortions other than those caused by taxes.)

Setting up a welfare index W as a function of the tax vector T, that is, $W_{max} = W(0)$ for Pareto optimum position in which T = 0, and given $\overline{Y}$ as the level of national income at this point, the level of welfare in the taxed economy is given by $\overline{Y} + \Delta W$, where

$$\Delta W = \sum_{i=1}^{n} \int_{0}^{T_i} \sum_{j \leq i} T_j \frac{\partial X_j}{\partial T_i} dT_i$$

After making several algebraic manipulations, Harberger obtained the following simpler expression:

$$\Delta W = 1/2 \sum_i \sum_j R_{ij} T_i T_j$$

where $R_{ij} = \frac{\delta X_j}{\delta T_i}$ Reaction coefficients (substitution term)

---

[34] A. C. Harberger, "The Measurement of Waste," American Economic Review, Papers and Proceedings, LIV (May, 1964), 58-76.

T_i = Tax on commodity i

T_j = Tax on commodity j

When the production frontier is linear, X_i are final products and T_i refers to taxes on final products only; this expression may be further reduced to:

$$\Delta W = 1/2 \sum_i T_i \Delta X_i$$

When the X's may be either final or intermediate products, the measure was shown to become

$$\Delta W = 1/2 \sum_i V_i t_i \Delta X_i$$

where V_i represents value added per unit of product in activity i and t_i is percentage of the tax to value added in activity i.

When different tax rates apply to the earnings of capital and labor, the following expression represents the appropriate measure:

$$\Delta W = 1/2 \sum_j E_j \Delta L_j + 1/2 \sum_i B_i \Delta X_i$$

where E_j = possible tax per unit of labor in activity j

L_j = number of labor units employed in activity i

B_i = possible tax per unit of capital in activity i

X_i = number of capital units employed in activity i

Obviously each of these are but simple modifications of the triangle measurement. In his article Harberger makes no attempt to measure the reaction coefficients which would of course be crucial for any empirical application of these formulas. Harberger's work does represent an extension of existing analysis of welfare cost calculations in that the formulation is considerably more general. Especially interesting is Harberger's inclusion of "theory of second best" considerations pertaining to situations in which taxes may compensate for other distortions.

2.14 Johnson's Simulation Model[35]

Johnson's simulation model utilized a CES utility function for determining the consumption cost of protection in a two-good model. In measuring the production cost of tariffs he utilized a transformation function of the form $X^2 + MXY + Y^2 = K^2$, where X and Y are the two bundles of goods.

After necessary algebraic manipulation he derived two formulas for measuring consumption cost and production cost respectively. These are:

$$\text{Consumption cost } U_t/U_o = \frac{[R(1+t)^{\sigma-1} + 1]^{\frac{\sigma}{\sigma-1}}}{1 + R(1+t)^{\sigma}} \quad (R+1)\ \frac{1}{1-\sigma}$$

$$\text{Production cost } \frac{M_t}{M_o} = \frac{(t+2)\sqrt{2-m}}{2\sqrt{2} + 2t + t^2 - (1+t)m}$$

where U_t = utility enjoyed with tariff

U_o = utility enjoyed without tariff

---

[35]Johnson, "The Welfare Cost of Protection and the Scientific Tariff," op. cit., pp. 356-372.

$R = X/Y$, where X, Y are exportable and importable

t = tariff rate

σ = the elasticity of substitution

Also $R = (\frac{A}{a})\beta^{t} + 1$, where A, a are the distribution parameters $\sigma = \frac{1}{\beta+1}$

M_t = national income produced under protection

M_o = national income produced under free trade

m = a parameter (The coefficient of the cross product term in the above transformation function.)

He used a range of values for the following parameter: the proportion of income spent on exports, the elasticity of substitution and tariff rates for simulating estimates of consumption cost. Similarly, alternative estimates of production cost were obtained using alternative values of tariff rates and the coefficients of the cross product term which Johnson showed was related to the elasticity of supply. He also used the model to estimate the total cost of protection and the tariff rates necessary to provide self-sufficiency. Utilizing this model Johnson calculated the consumption cost of a 20 percent tariff, with such parameters as the proportion of income spent on the untaxed good in the absence of a tariff ranging from .05 to .90, and the elasticity of substitution for a CES utility function ranging from .25 to 3.00. The consumption cost calculations turn out to be small. Even with an would seem desirable. Such an extension will be the subject of the remainder of this dissertation.

2.15 Other Studies

A number of other empirical attempts have been made by these and other authors to apply some of the same approaches we have reviewed above to other countries, industries and time periods. For example, utilizing Johnson's partial equilibrium approach Snape[36] has estimated the costs of protection in nine sugar importing countries, and found that this averaged $422 million per annum for the years 1959 to 1961. He estimated that if these protective devices were abandoned the sugar imports of these nine countries would have increased by 73 percent, resulting in additional export receipts of sugar exporters of $480 million to $780 million.

Stern[37] applied the same approach in order to estimate the costs of protection in the United States for the years 1951 and 1960. The costs of protection due to tariffs were estimated to be between $113 million and $183 million and the costs of tariffs and quotas together were estimated at $238 million to $308 million in 1951. The

---

[36] R. H. Snape, "Sugar: Cost of Protection and Taxation," Economica, XXXVI (February, 1969), 29-41.

[37] R. M. Stern, "The U.S. Tariff and the Efficiency of the U.S. Economy," American Economic Review, LIV (May, 1964), 840-852.

corresponding calculations for 1960 were $159 million for tariffs and $349 for tariffs and quotas combined. Stern concluded that "these estimates are striking in view of their relatively small magnitudes, the largest of them in 1951 and 1960 equaled only 0.11 percent of national income in the respective years."[38]

Applying the same approach but including a positive term of trade effect, Basevi[39] calculated the welfare gains derived from tariffs and quotas for the U.S. economy in 1959-1962 to range between $258 million and $558 million. He also concluded "these magnitudes are very small, representing at most 0.11 percent of national income."[40]

Nugent[41] has applied two alternative approaches (the Balassa effective protection method and the Johnson-Krueger excess cost of protection method) to the Central American countries and obtained estimates of the cost of protection ranging between 1.4 percent and 3 percent of gross product of the region.

---

[38] R. M. Stern, op. cit., p. 465.

[39] G. Basevi, "The Restrictive Effect of the U.S. Tariff," American Economic Review, LVII (September, 1968), 840-852.

[40] Ibid., p. 851.

[41] J. B. Nugent, "La estruetrua avancelaria y el costo de proteccion en America Central," El Trimasto Economica, XXXV (October-December, 1968), 756-766.

Thomas[42] has applied a modified version of the Brigden-Young trapezoid approach to estimate the production cost of the trade restrictions prescribed by the Navigation Acts on the thirteen colonies between 1763 and their independence. That cost was estimated to be at most 1/2 of 1 percent of the GNP of the colonies.

Balassa and Kreiner[43] have utilized Johnson's partial equilibrium approach to calculate the extent to which cost of protection for the U.S., Canada, the Common Market, the United Kingdom, the other EFTA countries and Japan would be reduced as a result of a 50 percent reduction in all duties as a result of the "Kennedy Round" negotiations. The net effect in all of these countries combined was estimated to be $326 million or a minute fraction of 1 percent of the gross product of these countries.

Bernard Munk[44] measured the welfare cost of "content protection" for automotive industry in Latin America. Since the policy of "content protection" requires manufacturers to use certain amounts of domestic materials, the

---

[42]R. P. Thomas, "A Quantitative Approach to the Study of the Effects of British Imperial Policy Upon Colonial Welfare: Some preliminary Findings," Journal of Economic History, XXV (December, 1965), 615-638.

[43]B. Balassa and M. E. Kreiner, "Trade Liberalization under the Kennedy Round: The Static Effects," Review of Economics and Statistics, XLIX (May, 1967), 125-137.

[44]B. Munk, "The Welfare Costs of Content Protection The Automotive Industry in Latin America," Journal of Political Economy, LXXVII (January-February, 1969), 85-98.

policy yields an excess cost of production. In the case of the automobile industries in Latin America, Munk has defined excess cost as "the difference between the wholesale price to dealers of a vehicle produced in accordance with the country's content program, in the given year and the C.I.F. cost of supplying the same vehicle to the country via exports from the United States."[45] Thus content protection may be viewed as a tax on the conusmption of the vehicle. And the loss in consumption due to excess cost is measured by the area of the familiar triangle. Applying this measure for the automotive industry in Argentina, Brazil and Mexico, Munk found that the policy of content protection imposes an excess cost of about 30 percent of the actual consumption of automobiles.

Although we have narrowed the scope of the present survey to the consideration of the welfare cost of tariffs or quotas (and thereby not considered discriminatory tariffs or tariff reductions such as those associated with the creation of a customs union or a free trade area), it may be worth pointing out that the several empirical attempts to estimate the static welfare effects of customs unions, such as those of Singh[46] (as reported by Leibenstein),

---

[45]Munk, op. cit., p. 93.

[46]H. A. Singh, "Unpublished Calculations Made by A. Singh Based on Data found in A. A. Faroq, Economic Integration: A Theoretical, Empirical Study" (unpublished Ph.D. dissertation, University of Michigan, 1963).

Janssen,[47] Johnson,[48] Wemelsfelder,[49] Scitovsky,[50] Verdoorn,[51] Balassa,[52] Lawrence,[53] Truman,[54] Virelle,[55] Waelbroeck,[56] and Winford,[57] have all revealed extremely small estimates of welfare gain or loss of these arrangements--both actual and hypothetical.

---

[47]L. H. Janssen, Free Trade, Protection and Customs Union (Leiden: H. E. Stenfertkroese N.V., 1961), p. 132.

[48]Johnson, "The Gains from Freer Trade with Europe," op. cit., pp. 237-249.

[49]J. Wemelsfelder, "The Short-Term Effect of Lowering Import Duties in Germany," Economic Journal, LX (March, 1960), 94-104.

[50]Scitovsky, op. cit., p. 1.

[51]Verdoorn, op. cit., pp. 482-500.

[52]Balassa, "Trade Creation and Trade Diversion in the European Common Market," The Economic Journal, LXXIII (March, 1967), 1-21.

[53]R. Lawrence, "Primary Product Preferences and Economic Welfare: The EEC and Africa," The Open Economy, ed. Peter Kenen and Roger Lawrence (New York: Columbia University Press, 1968).

[54]E. M. Truman, "The European Economic Community: Trade Creation and Trade Diversion" (unpublished Ph.D. dissertation, Yale University, 1967).

[55]L. D. de la Vinelle, "La creation du commerce attributable ou marche common et son incidence sur le volume du produit national de la communoute," Information Statistiques, IV (1965), 61-70; V (1966), 5-31.

[56]J. Waelbroeck, "Le commerce de la Communoute Europeene avec les pays tiers," Integration Europeene et Realitie Economique (Bruges, 1964), pp. 139-164.

[57]W. T. Wilford, "Trade Creation in the Central American Common Market," Western Economic Journal, VIII (March, 1970), 61-69.

2.16 An Evaluation

This review of these partial and general equilibrium approaches reveal that even when the extent of protection is quite high, estimates of the welfare costs of trade restrictions remain quite low. Some of the estimators themselves were not happy with their approach, their method of estimation, or with the particular assumptions they had to make in order to carry out their empirical calculations. The possible reasons for such low estimates may be categorized as follows: (1) matters of definition and approach; (2) data availability; (3) empirical assumptions.

Matters of definition and approach. Most of the empirical estimates attempt to measure either the area under a Marshallian demand curve assuming that money has a constant marginal utility or that under a compensated (or constant utility) demand curve derived from the Hicksian compensating variation in income assumption. Since both these measurements conceptually differ from the observed demand curve which is a function of money income and price, they are likely to bias the estimates. The area under the compensated demand curve is equal to that under the ordinary demand curve only when the latter is inelastic to income changes.[58] But if both income effects and substitution

[58] M. Friedman, "The Marshallian Demand Curve," Journal of Political Economy, LXVII (December, 1948), 47-99; J. R. Hicks, "The Four Consumer's Surplus," Review of

effects are allowed, the compensating variation magnitude may be larger, and is difficult to measure precisely. However, since the income effect will generally cause the ordinary demand curve to be more elastic than the compensated demand curve, the measurement of the change in consumer surplus due to the imposition of a tariff (imposing a negative income effect) from the ordinary demand curve will underestimate the true cost of the tariff although it would overestimate the gain from a price reduction.

Secondly, except for Johnson and Krueger, most studies assume a static competitive model, thereby assuming away dynamic considerations of economies of scale or technological change. Some recent studies[59] have attempted to capture these effects by introducing shifts in both supply and demand curves. It appears that the welfare losses of trade restrictions may be larger if such considerations are included. Hence, the neglect of such considerations has undoubtedly introduced a substantial downward bias to the estimates in the existing studies.

Economic Studies, XI (Winter, 1943-1944), 31-44; F. Machlup, "Professor Hicks' Revision of Demand Theory," Readings in Microeconomics, eds. W. Breit and H. M. Hockman (New York: Holt, Rinehart and Winston, Inc., 1967).

[59] J. B. Nugent, "An Alternative Method for Estimating the Effects of a Customs among Less Developed Countries: The Case of the Central American Common Market," University of Southern California and Economic Growth Center, Yale University, August, 1969. (Mimeographed.)

Thirdly, as Tullock and Mishan[60] have argued producer's surplus may not exist at all for a competitive industry whose supply curve is an average cost curve which at the equilibrium output yields a zero profit. To the extent this is true the area that is generally earmarked as a transfer from consumer's surplus to producer's surplus in the triangle measurement should be added to the total amount of loss and the usual measures of welfare loss would therefore have a distinct downward bias.

Fourthly, the inclusion of a positive terms of trade effect from the imposition of a tariff, which perhaps appropriate for a large country, is inappropriate for a small country which is unable to influence world prices.

Finally, the height of the triangle (i.e., the change in price) in the familiar triangle-type measurement is usually taken to be the uniform average tariff level or the weighted average of the different tariff rates. This homogeneity assumption of tariffs obviously ignores the distortions resulting from various cross effects that occur when uneven tariffs are present. Even the studies which emphasize the lack of homogeneity among tariff rates, such as those which calculate effective rates of protection,

---

[60]E. J. Mishan, "A Note on the Case of Tariffs, Monopolies and Thefts," Western Economic Journal, VII (September, 1969), 230-233; G. Tullock, "The Welfare Costs of Tariffs, Monopolies and Thefts ," Western Economic Journal, V (June, 1967), 224-232.

have made no attempt to relax the homogeneity assumption when it comes to estimating the welfare cost of tariffs. Some authors, such as Johnson and Harberger, have on occasion mentioned the existence of this source of downward bias in their estimates, but no study has yet been done to see how important this bias might be. It is for this reason that we have chosen to devote this study to the investigating the effects on welfare cost calculations of the relaxation of this assumption.

Data availability. The value given to tariff-distorted imports or import-equivalents may differ depending on whether the tariff is calculated from import levels when tariffs are imposed or after they are removed. If measured in terms of the import level with tariffs, the loss may be understated while their valuation in terms of the import level after tariffs are abolished may lead to overestimation of the welfare loss of tariffs. Sometimes it becomes difficult to classify the goods taxed. It also may become difficult to find the ad valorem equivalent of import taxes which are often of the specific variety or of the equivalent of quotas and other non-tariff restrictions. Sometimes it is difficult to get data for total import taxes as the tariff schedules give only import duties and do not include other taxes that may be imposed on imports.

Furthermore, the schedules in tariff rates differ in terminology, description, and organization for the information sources (usually census, survey or national accounting data) for the production and consumption data with which the tariff data needs to be related. The broader the categories and less appropriate the comparisons, the more likely it is that welfare loss will be underestimated unless some allowance is consciously made for such factors. In many less developed countries smuggling of certain goods may conceal some information particularly in cases where the tariffs are high. Sometimes domestic production is so heterogeneous that it is extremely difficult to determine which of two or more tariffs apply to a given product class. Thus additional information or additional empirical assumptions become necessary.

Empirical assumptions. It is apparent that in order to obviate some of these difficulties in the classification of goods, the inclusion of non-tariff import taxes and other restriction, the treatment of uneven tariff rates, the choice of the kind and nature of weights to be used and of the kind of price and the quantity to be used, etc., additional information or particular assumptions are necessary. Without going into the individual assumptions the authors have made in considerable detail, it is impossible to say whether the bias is downward or upward. Since data are particularly deficient and classification particularly haphazard in less developed countries, the few empirical attempts to estimate welfare costs in these countries are undoubtedly subject to large margins of error. It is often necessary to fall back on guesses as to income and price elasticities where estimation is subject to well known pitfalls.[61]

Despite the rather enormous difficulties in making quantitative estimates of the welfare effects of trade distortions, they do seem possible. The methods that have been developed by Johnson, Hicks and others make it possible to obtain quantitative measurements of welfare loss in terms of any commodity or money, even when it is not possible to attach cardinal numbers to utility preferences. By and large the concept of a community indifference curve, upon which all such studies are based, seems viable. The size and pattern of tariffs, particularly in less developed countries would seem to make the further advancement in such calculations an important activity.

Of all the studies surveyed, the most promising would appear to be the simulation study of Johnson.[62] Johnson's study has the advantage of being a general equi-

---

[61]See G. Orcutt, "Measurement of Price Elasticities in International Trade." Review of Economics and Statistics, XXXII (May, 1950), 117-132.

[62]Johnson, "The Cost of Protection and Scientific Tariff," op. cit., pp. 357-370.

librium approach without the disadvantage of being dependent on a specific set of assumptions about the values of individual paramters. Its approach is general enough that one can easily substitute alternative parameters or even different types of utility or production functions. However, in view of its attempt to provide a general equilibrium analysis and realistic results, its assumption of a uniform tariff rate seems regrettable. The relaxation of this assumption in Johnson's study shall be the subject of succeeding chapters. The empirical results confirm our intuitive feeling that this is an important generalization in the measurement of the welfare costs of trade restrictions.

CHAPTER III

JUSTIFICATION FOR THE USE OF A THREE-COMMODITY MODEL

Our main purpose in using a three-commodity model is to demonstrate the effect of variable tariff rates on welfare. Our hypothesis is that unequal tariff rates result in welfare losses that are larger than those resulting from uniform tariff rates. The purpose of this brief chapter will be to demonstrate that the considerations of differential tariff rates is realistic and empirically relevant.

The study of the tariff structure of the Latin American countries as reported in Macario's[1] study of "Protectionism and Industrialization in Latin America" shows glaring evidence of the existence of unequal tariff rates. Table 1 is indicative of this, showing that the tariff rates in some broad classes of goods may be four or five times the rates applying to other classes. If individual items are chosen, as has been done in Table 2, this variability in tariff rates is further accentuated.

---

[1]S. Macario, "Protectionism and Industrialization in Latin America," Economic Bulletin for Latin America, United Nations, IX (March, 1964), 61-101.

Empirical evidence

TABLE 1

WEIGHTED AVERAGES OF THEORETICAL INCIDENCE OF CUSTOMS DUTIES AND CHARGES OF EQUIVALENT EFFECT ON THE C.I.F. VALUE OF IMPORTS IN SELECTED LATIN AMERICAN COUNTRIES
(Percentages)

Country	Duties and Charges in Force at:	Primary Commodities	Capital, Intermediate and Durable Consumer Goods	Non-Durable Consumer Manufactures
Argentina (1959)	30.IV.60	18.5	64.7	66.5
Bolivia (1957-58)	31.XII.59	9.9	13.4	34.2
Brazil (1957-59)	31.VIII.60	2.9	36.9	40.4
Chile (1957-58)	15.III.60	20.2	39.6	56.8
Colombia (1956-58)	30.IX.59	28.3	28.3	48.2
Ecuador (1957-58)	1.IX.59	24.7	40.7	62.3
Mexico (1957-58)	31.XII.59	4.7	14.1	30.8
Paraguay (1957 58)	30.IX.60	26.8	61.9	59.9
Peru (1957-58)	15.IX.59	14.5	78.6	35.9
Uruguay (1957)	15.VII.60	9.4	19.3	19.2
Venezuela (1959)	23.II.60	35.6	12.6	66.3

Source: S. Macario, "Protectionism and Industrialization in Latin America," Economic Bulletin for Latin America, IX (March, 1964), 69.

TABLE 2

WEIGHTED AVERAGES OF THEORETICAL INCIDENCE OF CUSTOMS DUTIES AND OTHER DUTIES OR CHARGES OF EQUIVALENT EFFECT ON THE C.I.F. VALUE OF IMPORTS IN SELECTED LATIN AMERICAN COUNTRIES
(Percentages)

Country	Duties and Charges in Force at:	Raw Materials	Durable Consumer Goods	Processed Food Stuffs and Tobacco
Argentina (1959)	30.IV.60	42.7	699.7	142.4
Bolivia (1957-58)	31.XII.59	16.6	29.4	19.1
Brazil (1957-58)	31.VIII.60	22.0	79.1	50.5
Chile (1957-58)	15.III.60	16.1	83.7	62.8
Colombia (1956-58)	30.IX.59	19.3	113.7	160.5
Ecuador (1957-58)	1.IX.59	36.2	75.2	114.0
Mexico (1957-58)	31.XII.59	6.5	56.2	132.8
Paraguay (1957-58)	30.IX.60	50.0	72.6	55.4
Peru (1957-58)	15.IX.59	22.7	33.3	26.2
Uruguay (1957)	15.VII.60	12.4	20.3	23.3
Venezuela (1959)	23.II.60	68.1	14.3	87.3

Source: Macario, Ibid., p. 69.

Secondly, the empirical literature on the effective tariff structure reveals both that effective rates of protection are subject to even greater variability than are nominal tariff rates and that the experience of the Latin American countries reviewed by Macario is also common in other parts of the world. For example, Balassa's[2] study of the U.S. Tariff structure shows that the effective tariff on ingots and steel forms is ten times as high as the nominal rate. Also his study shows that the effective rate on agricultural machinery is negative. That effective rates of protection tend to be higher and more variable is confirmed by the Grubel and Johnson[3] study of the Common Market countries. Similarly Nugent's[4] study for Central America confirms this variability of tariff rates between primary inputs or capital goods and consumer items, as do Islam's[5] study for Pakistan, Humphrey's[6] study for Argen-

---

[2]B. Balassa, "Tariff Protection in Industrial Countries: An Evaluation," Journal of Political Economy, VII (December, 1965), 573-594.

[3]H. G. Grubel and H. G. Johnson, "Nominal Tariff, Indirect Taxes and Effective Rates of Protection: The Common Market Countries, 1959," The Economic Journal, LXVII (December, 1967), 761-776.

[4]J. B. Nugent, "La Estructura avancelaria y el costo de Proteccion en America Central," El Trimestre Economico, XXXV (October-December, 1968), 751-766.

[5]N. Islam, "Comparative Costs, Factor Proportions and Industrial Efficiency in Pakistan," The Pakistan Development Review, VII (Summer, 1967).

[6]D. B. Humphrey, "Measuring the Effective Rate of

tina, the Bergsman and Malah[7] study for Brazil, the Balassa[8] study for Japan, Sweden, United Kingdom and the EEC, the Korean study,[9] Power's[10] study for the Phillipines and Sun's[11] study for Taiwan. This shows that the effective tariff rates in these countries are much larger than the nominal rates. This rapidly expanding literature on effective protection confirms the following propositions:

a. That nominal tariff rates are unequal;

b. That effective protection rates are even more unequal commodity to commodity than are nominal tariff rates;

c. Final goods have higher nominal tariffs than capital goods, raw materials or intermediate goods--thus raising the effective protection on final goods.

Protection: Direct and Indirect Effects," Journal of Political Economy, LXXVII (September-October, 1969), 834-844.

[7]J. Bergsman and P. Malah, "The Structure of Protection in Brazil," February, 1968. (Mimeographed.)

[8]B. Balassa, "Trade Creation and Trade Division in the European Common Market," The Economic Journal, LXXIII (March, 1967), 1-21.

[9]Korean Development Association, "Effective Protective Rates for Korean Industries," Seoul, 1967.

[10]J. H. Power, "Import Substitution as an Industrialization Strategy," The Philippine Economic Journal, V (January, February, 1966), 167-204.

[11]I-S Sun, "Trade Policies and Economic Development in Taiwan," October, 1966. (Mimeographed.)

Furthermore, a survey of the current theoretical literature[12] would show that there is a trend toward the use of three commodity models permitting one to distinguish between imports, exports and home goods or between consumer goods, intermediate goods and capital goods. Also the theory of import substitution for economic devlopment is being analyzed in some such framework where at least three interrelated commodities are considered.

---

[12] I. F. Pearce, "The Problem of the Balance of Payments," International Economic Review, XII (January, 1961), 1-28; S. W. Arndt, "Customs Union and the Theory of Tariffs," American Economic Review, LIX (March, 1969), 108-118; R. I. McKinnon, "Intermediate Products and Differential Tariffs: A Generalization of Lerner's Symmetry Theorem," The Quarterly Journal of Economics, LXXX (November, 1966), 584-615.

CHAPTER IV

AN EXTENSION OF JOHNSON'S GENERAL EQUILIBRIUM MODEL OF WELFARE COST MEASUREMENT

In this chapter we shall develop a model for estimating consumption cost, production cost and the cost of protection as a whole when tariff rates are not homogeneous. Johnson's rather successful general equilibrium model (reviewed in Chapter II) has pioneered the way by utilizing a two-good CES utility function (except in the special case in which the elasticity of substitution is unity, a two-good Cobb-Douglas has been utilized) for simulating consumption costs and a simple two-good production transformation function for simulating production costs. We propose to extend Johnson's model to three goods so as to enable us to simulate the production and consumption costs of tariffs when tariff rates differ from sector to sector.

4.1 Consumption Cost in the Single Tariff Case

The three good social utility function of the CES type is as follows:

$$U = (AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta} \qquad (1)$$

where A, a and c are distribution paramters, β is a substitution parameter, and σ is the elasticity of substitution from the relationship $\frac{1}{1+\beta} = \sigma$.

The marginal utilities of the three goods are:

$$\begin{aligned} U_X &= A(U/X)^{\beta+1} \\ U_Y &= a(U/Y)^{\beta+1} \\ U_Z &= c(U/Z)^{\beta+1} \end{aligned} \tag{2}$$

If a tax t is imposed on conusmption of Y, utility will be maximized when

$$U_Y/U_X = a/A\ (X/Y)^{\beta+1} = 1+t;$$

$$U_Y/U_Z = a/c\ (Z/Y)^{\beta+1} = 1+t;$$

and

$$U_Z/U_X = c/a\ (X/Z)^{\beta+1} = 1$$

Total income is fixed in terms of world prices,

$$M = X + Y + Z \tag{3}$$

Let us assume that the tax revenue collected is returned to the economy in the form of subsidies. Consumption of the three goods under protection at equilibrium will be:

$$X = \frac{\{A/a\ (1+t)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1} \cdot M \tag{4}$$

$$Z = \frac{\{A/a\ (1+t)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}\ (1+t)\}^{\frac{1}{\beta+1}} (\frac{A}{c})^{\frac{1}{\beta+1}} + (\frac{A}{c})^{\frac{1}{\beta+1}} + \frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot M$$

$$Y = \frac{1}{(\frac{A}{a})^{\frac{1}{\beta+1}}[\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}] + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

Total utility can now be expressed as a function of real income and the tariff rate as follows:

$$U_t = \frac{M_a^{\frac{-1}{\beta}} \left\{ (\frac{A}{a})^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} \right\}^{-\frac{1}{\beta}}}{[\frac{A}{a}(1+t)]^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1}$$

The ratio of total utility under a tariff to that without a tariff is now given by:

$$\frac{U_t}{U_o} = \frac{\left\{ (\frac{A}{a})^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} \right\}^{-\frac{1}{\beta}} (\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1}{(\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}}{}^{-\frac{1}{\beta}} [\frac{A}{a}(1+t)]^{\frac{1}{\beta+1}} + \frac{c}{a}(1+t)^{\frac{1}{\beta+1}} + 1} \qquad (5)$$

Let R_0 be the ratio of the consumption of X to that of Y in the free trade situation. As shown in Appendix A, $R_0 = (\frac{A}{a})^{1/\beta+1}$. Similarly R_1 may be obtained as the ratio of the consumption of Z to that of Y; $R_1 = (\frac{c}{a})^{1/\beta+1}$ and again under free trade. It may also be shown that $R_0 = \frac{r_1}{1-(r_1+r_3)}$ and $R_1 = \frac{r_3}{1-(r_3+r_1)}$ where $r_1 = X/M$ and $r_3 = Z/M$ represent the proportions of income spent on consumption of X and on consumption of Z without a tariff. Hence different values of r_1 and r_3 would represent different preference possibilites. Substituting the expressions for R_0 and R_1 and for $1/1+\beta$ in (5) we obtain:

$$\frac{U_t}{U_o} = \frac{[R_0(1+t)^{\sigma-1} + 1 + R_1(1+t)^{\sigma-1}]^{\sigma/\sigma-1} \quad R_0 + R_1 + 1}{[R_0+R_1+1]^{\sigma/\sigma-1}[R_0(1+t)^{\sigma} + R_1(1+t)^{\sigma} + 1]}$$

$$= \frac{[R_0(1+t)^{\sigma-1} + 1 + R_1(1+t)^{\sigma-1}]^{\sigma/\sigma-1}[R_0+R_1+1]^{1/1-\sigma}}{[R_0(1+t)^{\sigma} + R_1(1+t)^{\sigma} + 1]} \tag{6}$$

Since it is difficult to manipulate the case of a unit elasticity of substitution from a CES function, the Cobb-Douglas of the following form is utilized for that special case:

$$U = X^{\alpha} Y^{\beta} Z^{1-(\alpha+\beta)} \tag{7}$$

Again as shown in Appendix A we obtain the following expression for the ratio of the total utility with a tariff to that under free trade when the elasticity of substitution in consumption is unity:

$$\frac{U_t}{U_o} = \frac{(1+t)^{1-r_2}}{1+t(1-r_2} \qquad \text{where } \alpha = r_1;\ \beta = r_2 \tag{8}$$

$(1 - U_t/U_o)$, where U_t/U_o is given by equations (6) or (8) above expresses the consumption cost of protections as a percentage of the free trade position U_o.

4.2 Consumption Cost in the Differential Tariff Case

We wish now to extend this formulation of consumption cost to a case where two commodities are taxed. This may be treated as a general case with the previous case as the special case in which one of the two goods taxed happens to have a tax rate of zero.

Let Y and Z be the two goods that are taxed. By setting the ratios of marginal utilities equal to the tariff-inclusive prices in Appendix B we have derived the following solutions:

$$\frac{U_Y}{U_X} = \frac{a}{A}(X/Y)^{\beta+1} = 1 + t_1 \quad \text{or} \quad X/Y = \{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}$$

$$\frac{U_Y}{U_Z} = \frac{a}{c}(Z/Y)^{\beta+1} = 1 + t_2 \quad \text{or} \quad Z/Y = \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} \qquad (9)$$

$$\frac{U_Z}{U_X} = \frac{c}{a}(X/Z)^{\beta+1} = 1 + t_3 \quad \text{or} \quad X/Z = \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}$$

Again let total gross income be fixed in terms of world prices as M = X + Y + Z. Consumption of X, Y and Z when Y and Z are taxed can be shown to be:

$$X = \frac{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$Y = \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}\{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

(10)

$$Z = \frac{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}} \cdot M$$

Again expressing total utility as a function of real income and the tariff rates and assuming for simplicity that $1+t_1/1+t_3 = (1+t_2)$ we obtain:

$$U_t = \frac{M_a^{-1/\beta}}{[\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1]} \times \left[(\frac{A}{a})^{\frac{1}{\beta+1}}(1+t_1)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}}(1+t_2)^{\frac{-\beta}{\beta+1}}\right]^{-1/\beta} \qquad (11)$$

The ratio of the utility of consumption when Y and Z are subject to tariffs to that without tariffs is now given by:

$$\frac{U_t}{U_o} = \frac{\left[(\frac{A}{a})^{\frac{1}{\beta+1}}(1+t_1)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}}(1+t_2)^{\frac{-\beta}{\beta+1}}\right]^{-1/\beta} \times (\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1}{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1\left[(\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}}\right]^{-1/\beta}}$$

(12)

Defining R_0 and R_1 as before for the free trade situation and as well as in equation (12) and substituting them we obtain the following simpler relationship for U_t/U_o

$$\frac{U_t}{U_o} = \frac{R_0(1+t_1)^{\sigma-1} + 1 + R_1(1+t_2)^{\sigma-1}{}^{\frac{1}{\sigma-1}} \quad [R_0+R_1+1]^{\frac{1}{1-\sigma}}}{[R_0(1+t_1)^{\sigma} + R_1(1+t_2)^{\sigma} + 1]} \tag{13}$$

Again for the unitary elasticity of substitution case, the Cobb-Douglas function is utilized to obtain the following ratio of utilities:

$$\frac{U_t}{U_o} = \frac{(1+t_2)^{1-r_1-r_2} \; (1+t_1)^{r_1}}{r_1 t_1 + 1 + t_2(1-r_1-r_2)} \tag{14}$$

Once again the difference between U_t/U_o and unity gives us loss in utility of consumption as a percentage of free trade utility derived from the distortion of prices due to the imposition of tariffs.

4.3 Production Costs in the Single Tariff Case

A simple case of the normal type of transformation function is

$$MX^2 + LY^2 + NZ^2 = K \tag{15}$$

where M, L, N and K are positive constants, and X, Y, Z denote quantities produced.

The marginal rate of transformation of one commodity into another is given by:

$$\frac{dK}{dX} = 2MX$$

$$\frac{dK}{dY} = 2LY \qquad (16)$$

$$\frac{dK}{dZ} = 2NZ$$

In order to achieve the maximum value of output at domestic prices with a tariff (t) on Y but not on X or Z, we set the marginal rates of transformation equal to their respective prices:

$$\frac{LY}{MX} = P = 1+t \quad \text{or} \quad X = \frac{LY}{M(1+t)} ,$$

$$\frac{LY}{NZ} = P = 1+t \quad \text{or} \quad Y = \frac{NZ(1+t)}{L} ,$$

$$\frac{MX}{NZ} = P = 1 \quad \text{or} \quad Z = \frac{MX}{N} . \qquad (17)$$

Since M, L and N are parameters without any direct economic interpretation, it would be useful to relate them to their equivalent elasticities of supply of Y at free trade point. Substituting the values of X and Z from (17) in (15) we obtain:

$$\frac{Y^2(NL^2 + MNLP^2 + ML^2P^4)}{MNP^2} = K \qquad (18)$$

Defining the expression within parentheses as A, we obtain

$$Y^2(A) = K \qquad (19)$$

or $$Y = K/A)^{1/2} \tag{20}$$

In terms of logarithms (20) can be written as:

$$\text{Log } Y = 1/2 \text{ Log } K - 1/2 \text{ Log } A \tag{21}$$

Differentiating (21) with respect to A

$$\frac{d \text{ Log } Y}{dA} = \frac{1}{2}\frac{\text{Log } K}{dA} - \frac{1}{2}\frac{d \text{ Log } A}{Da}$$

$$= \frac{1}{Y}\frac{dY}{dA} = -\frac{1}{2A} \tag{22}$$

Manipulating (22) in order to obtain an expression for the elasticity of Y with respect to P, we get

$$\frac{1}{Y}\frac{dY}{dP} = -\frac{1}{2A}\frac{dA}{dP} \tag{23}$$

Substituting the expression for A from (18) in (23) simplifying, one obtains for the elasticity of supply of Y in terms of P at free trade,

$$\varepsilon = -\frac{1}{2}\frac{MNP^2}{NL^2 + MNLP^2 + ML^2P^4} \times \left[\frac{MNP^2(2MNLP + 4P^3ML^2) - (NL^2+MNLP^2+ML^2P^4)2MNP}{(MNP^2)^2}\right] \tag{24}$$

After algebraic manipulation, this expression reduces to

$$\varepsilon = -\frac{L(N-MP^4)}{(NL+MNP^2+MLP^4)P} \tag{25}$$

Since P=1 at free trade, in the absence of tariffs (25) can then be simplified to:

$$\varepsilon = -\frac{L(N-M)}{NL+MN+ML} \tag{26}$$

Thus the elasticity of supply of Y with respect to P in free trade is:

$$\varepsilon = -\frac{L(N-M)}{NL+MN+ML} \tag{27}$$

By substituting the values of X. Y, Z from (17) into the transformation function (15), we obtain

$$X = \frac{L}{M}\frac{\sqrt{MNK}}{\sqrt{ML^2+NL^2+MLN(1+t)^2}}$$

$$Y = \frac{\sqrt{MNK}(1+t)}{\sqrt{ML^2+NL^2+MLN(1+t)^2}} \tag{28}$$

$$Z = \frac{\sqrt{MNK}}{\sqrt{ML\ +NL\ +MLN(1+t)^2}}$$

Gross income may be expressed as:

$$M = X + Y + Z$$

The ratio of national income under protection to that under free trade (M_t/M_o) may be shown to be:

$$\frac{M_t}{M_o} = \frac{[LN+MN(1+t)+ML]}{\sqrt{ML+NL+MN(1+t)^2}}\ \frac{}{LN+MN+ML} \tag{30}$$

$(1-M_t/M_o)$ is the reduction in real output derived from protection expressed as a percentage of free trade output.

4.4 Production Cost in the Differential Tariff Case

To the extent that the calculation of production cost to the differential tariff case in which tariffs are imposed on Y and Z but not on X, we set the marginal transformation rates equal to the tariff-inclusive price ratios.

$$
\begin{aligned}
LY/MX &= 1+t \quad , \quad X = LY/M(1+t_1) \\
LY/NZ &= 1+t \quad , \quad Y = \frac{NZ(1+t_2)}{L} \\
NZ/MX &= 1+t \quad , \quad Z = \frac{MX(1+t_3)}{N} \quad \frac{LY}{N} \quad \frac{1+t_3}{1+t_1}
\end{aligned}
\tag{31}
$$

Substituting these expressions for X, Y and Z in (15) above, we can obtain the following expression for X. Y and Z

$$
\begin{aligned}
X &= \frac{L}{M} \cdot \frac{\sqrt{MNK}}{\sqrt{NL^2+MNL(1+t_1)^2+ML^2(1+t_3)^2}} \\
Y &= \frac{\sqrt{MNK}\ (1+t_1)}{\sqrt{NL^2+MNL(1+t_1)^2+ML^2(1+t_3)^2}} \\
Z &= \frac{L(1+t_3)}{N} \quad \frac{\sqrt{MNK}}{NL^2+MNL(1+t_1)^2+ML\ (1+t_3)^2}
\end{aligned}
\tag{32}
$$

Now $\quad M = X + Y + Z$

$$
= \frac{\sqrt{K}\ [NL+MN(1+t_1)+ML(1+t_3)]}{\sqrt{MN}\ \sqrt{NL^2+MNL(1+t_1)^2+ML\ (1+t_3)^2}}
$$

Also it may be shown that:

$$\frac{M_t}{M_o} = \frac{LN+MN(1+t_1)+ML(1+t_3)}{\sqrt{NL+MN(1+t_1)^2+ML(1+t_3)^2}} \times \frac{1}{\sqrt{LN+MN+ML}}$$

4.5 The Total Cost of Protection (with the CES Utility Function)

A. The Single Tariff Case

Substituting the value of M_t from the production cost calculation from equation (30) in equation (4a), we obtain (35) for the tariff case and (36) for the non-tariff case, respectively.

$$U_t = \left[\left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} + 1 + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} \right]^{-1/\beta}$$

$$\times \frac{a^{-1/\beta}}{\left[\frac{A}{a}(1+t)\right]^{\frac{1}{\beta+1}} + \left[\frac{c}{a}(1+t)\right]^{\frac{1}{\beta+1}} + 1} \times \frac{\sqrt{K}\ [LN+MN(1+t)+ML]}{\sqrt{MN}\ \sqrt{ML^2+NL^2+MLN(1+t)^2}} \qquad (35)$$

$$U_o = \frac{\left[\left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} + 1 + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} \right]^{-1/\beta}}{\left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} + 1} \cdot a^{-1/\beta} \frac{\sqrt{K}\ (LN+MN+ML)}{\sqrt{MN}\ \sqrt{L^2N+MLN+L^2M}} \qquad (36)$$

Hence, the ratio of the total utility enjoyed with a tariff to that enjoyed under free trade may be expressed as:

$$\frac{U_t}{U_o} = \left[(\frac{A}{a})^{\frac{1}{\beta+1}}(1+t)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}}(1+t)^{\frac{-\beta}{\beta+1}}\right]^{-1/\beta}$$

$$\times \frac{1}{\frac{A}{a}(1+t)^{\frac{1}{\beta+1}} + \frac{c}{a}(1+t)^{\frac{1}{\beta+1}}} \times \frac{[LN+MN(1+t)+ML]}{\sqrt{MN}\ \sqrt{ML^2+NL^2+MLN(1+t)^2}}$$

$$\frac{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1}{\left[(\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}}\right]^{1/\beta}}, \qquad \frac{\sqrt{MN}\ \sqrt{L^2N+MLN+L^2M}}{\sqrt{LN+MN+ML}} \tag{37}$$

Using the substitutions for R_0 and R_1 as in (6) above, this expression can be simplified to:

$$\frac{U_t}{U_o} = \frac{[R_0(1+t)^{\sigma-1} + R_1(1+t)^{\sigma-1} + 1]^{\frac{\sigma}{\sigma-1}}\ [R_0+R_1+1]^{\frac{1}{1-\sigma}}}{[R_0(1+t)^{\sigma} + R_1(1+t)^{\sigma} + 1]}$$

$$\times \frac{[LN+MN(1+t)ML]}{\sqrt{LN+MN+ML}} \times \frac{1}{\sqrt{ML+NL+MN(1+t)^2}} \tag{38}$$

B. The Differential Tariff Case

Substituting the values of M tariff and free trade for the differential tariff case from (29) in the corresponding expressions for U_t in (11) we obtain (39) for ratio of utility enjoyed under differential tariffs to that enjoyed under free trade:

$$\frac{U_t}{U_o} = \frac{\left[(\frac{A}{a})^{\frac{1}{\beta+1}} (1+t_1)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} (1+t_2)^{\frac{-\beta}{\beta+1}} \right]^{-1/\beta}}{[\{(\frac{A}{a})(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1]}$$

$$\times \frac{[LN+MN(1+t_1)+ML(1+t_3)]}{\sqrt{MN}\ \sqrt{NL^2+MNL(1+t_1)^2+ML^2(1+t_3)^2}}$$

$$\times \frac{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1}{\left[(\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} \right]^{-1/\beta}} \cdot \frac{\sqrt{MN}\ \sqrt{L^2N+MLN+L^2M}}{(LN+MN+ML)} \tag{39}$$

Once again making the substitutions for R_0 and R_1 (39) simplifies to

$$\frac{U_t}{U_o} = \frac{[R_0(1+t_1)^{\sigma-1} + R_1(1+t_2)^{\sigma-1} + 1]^{\frac{\sigma}{\sigma-1}}}{R_0(1+t_1)^{\sigma} + R_1(1+t_2)^{\sigma} + 1} \times (R_0+R_1+1)^{\frac{1}{1-\sigma}}$$

$$\times \frac{[LN+MN(1+t_1)+ML(1+t_3)}{\sqrt{LN+MN+ML}} \times \frac{1}{\sqrt{LN+MN(1+t_1)^2+ML(1+t_3)^2}} \tag{40}$$

4.6 Total Cost of Protection in the Unitary Elasticity of Substitution Case

Since a different utility function, the Cobb-Douglas function given by (7) was used for consumption cost in the special case in which the elasticity of substitution in consumption is unity, the values for M with and without a single tariff from (29) are substituted in the alternative equation for consumption (8) to obtain the following solution for the single tariff case:

$$\frac{U_t}{U_o} = \frac{(1+t)^{1-\beta} \dfrac{\sqrt{K}\ (bc+ca(1+t)+ab)}{\sqrt{ac}\ \sqrt{ab^2+cb^2+abc(1+t)^2}}}{1+t-t\beta \dfrac{\sqrt{K}\ (bc+ca+ab)}{\sqrt{ac}\ \sqrt{b^2c+abc+b^2a}}} \tag{41}$$

Accordingly for the differential tariff case, the differential tariff and free trade values for M from (33) are substituted in (14) to obtain:

$$\frac{U_t}{U_o} = \frac{(1+t_1)\ (1+t_2)^{1-(\alpha+\beta)}}{\alpha t_1 + 1 + t_2^{(1-\alpha-\beta)}} \cdot \frac{bc(1+t_3) + ac(1+t_1)(1+t_3) + ab}{\sqrt{bc(1+t\ )^2 + ac(1+t\ )^2(1+t\ )^2}}$$

$$\times \frac{\sqrt{bc+ca+ab}}{bc+ca+ab} \tag{42}$$

CHAPTER V

SIMULATION OF THE MODEL: EMPIRICAL ESTIMATES

As has been mentioned above, the Johnson model has been chosen as the vehicle for the present work because it improved on the ordinary triangle measurements of welfare cost by including simultaneously both income and substitution effects. The extension of Johnson's two-commodity general equilibrium model to three commodities that has been formulated in the previous chapter has been used for simulating the sensitivity of the welfare loss calculations to changes in the values of the parameters in the formulas and in particular to estimate the importance of the explicit consideration of differential tariff rates which we have previously shown to be a most realistic consideration.

The specific formulas utilized (directly taken from Chapter III) are as follows:

A. Consumption cost in the single tariff case

For the general case:

$$\frac{U_t}{U_o} = \frac{[R_0(1+t)^{\sigma-1}+1+R_1(1+t)^{\sigma-1}]^{\frac{\sigma}{\sigma-1}}[R_0+R_1+1]^{\frac{1}{1-\sigma}}}{[R_0(1+t)^{\sigma} + R_1(1+t)^{\sigma} + 1]}$$

and for the special case in which the elasticity of substitution in consumption is unity:

$$\frac{U_t}{U_o} = \frac{(1+t)^{1-r_2}}{1+t(1-r_2)}$$

B. Consumption Cost in the Differential Tariff Case

For the general case:

$$\frac{U_t}{U_o} = \frac{\left[R_0(1+t)^{\sigma-1} + 1 + R_1(1+t_2)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}} \left[R_0+R_1+1\right]^{\frac{1}{1-\sigma}}}{\left[R_0(1+t_1)^{\sigma} + R_1(1+t_2)^{\sigma} + 1\right]}$$

and for the special case in which the elasticity of substitution in consumption is unity:

$$\frac{U_t}{U_o} = \frac{(1+t)^{1-r_1-r_2}(1+t)^{r_1}}{r_1 t_1 + 1 + t_2(1-r_1-r_2)}$$

C. Production Cost in the Single Tariff Case

$$\frac{M_t}{M_o} = \frac{[LN+MN(1+t)+ML]}{\sqrt{ML+NL+MN(1+t)^2}\ \sqrt{LN+MN+ML}}$$

D. Production Cost in the Differential Tariff Case

$$\frac{M_t}{M_o} = \frac{LN+MN(1+t_1)+ML(1+t_3)}{\sqrt{NL+MN(1+t_1)^2+ML(1+t_3)^2}}\ \frac{1}{\sqrt{LN+MN+ML}}$$

As can easily be seen, these formulas reduce the welfare cost calculations to the interaction of a small number of parameters. In the consumption cost case in place of

Johnson's single parameter representing the proportion of income spent on the untaxed commodity in the absence of tariff, now we have two r_1 and r_3 instead of one tariff rate in Johnson's model. Similarly in the production cost in place of Johnson's single coefficient, we have the three coefficients L, M, and N. The other parameter, i.e., elasticity of substitution in consumption, is the same in the extended model as in Johnson's model.

Thus the procedure and the parameters involved are similar to those used by Johnson. In fact, the calculations that have been carried out on the basis of these formulas have been designed so as to make the welfare loss calculations of the differential tariff case as comparable as possible with those derived from the single tariff case employed by Johnson.

As in Johnson's study, the welfare cost calculations have been separated into those of production cost and of consumption cost. In the consumption cost calculations the following parameters have been varied:

1. Tariff rates (t)
2. The elasticity of substitution (σ)
3. The proportion of real income spent on the untaxed commodities (r)

In the production cost calculations the following parameters have been changed:

1. Tariff rates (t)
2. The curvature coefficients (L,M,N) (which can be related to elasticity of supply)

As will be shown in the subsequent tables, the results indicate that both the consumption and production costs of the tariff are higher when different tariffs are imposed on different products than when a single tariff is imposed even though the differential tariff rates were chosen so as to have the same weighted average as in the single tariff case.

As in Johnson's study the welfare cost of the tariff is shown to increase with the tariff rate, the elasticity of substitution in consumption, and with the elasticity of supply.[1] In all cases the welfare cost computations are arrived at by comparing the free trade situation with that in which one or more tariffs are imposed.

The alternative values of the parameters considered in the subsequent calculations are as follows:

Tariff rates: 20 percent, 60 percent, 100 percent or combinations thereof (In the free trade case, of course, the tariff rate is zero.)

---

[1]Actually Johnson ("The Costs of Production and Self-Sufficiency," op. cit., p. 364) seems to conclude that the production cost of the tariff is inversely related to the elasticity of supply, but this conclusion is obviously inconsistent with the results of his simulation (i.e., his Table II, p. 365) which show that at any tariff rate, the production cost of the tariff increases with the elasticity of supply.

Elasticity of substitution in consumption parameter: .25 to 3.00

Proportion of income spent on the taxed and untaxed commodities: 0 percent to 100 percent

Curvature coefficients (in production):

L from 1.05 to 1.5
M from 1.05 to 1.5
N from .05 to 1.5

The range of tariff rates, elasticity of substitution, and proportion of income spent on taxed and untaxed goods are exactly the same as those considered by Johnson. The range of values for L,M,N implies a range of elasticities of supply ranging from 0.0 to about 1 which is approximately the same range utilized by Johnson.

5.1 Consumption Cost Calculations

As a check on the validity of the formulae derived for the consumption cost in the three goods model, we have assumed parameters identical to those assumed in Johnson's two goods model. When the tariff rates on two of the three goods are equal and also identical to the rate used by Johnson for one of his two goods, we obtain the results shown in Table 3B which may be compared with Johnson's results which are reproduced in Table 3A.

The first row in each part of the table shows the elasticity of substitution (ranging from .25 through 3.00) and the first two columns show the percentages of national

TABLE 3

CONSUMPTION COST $(1-U_t/U_o)$ OF 20 PERCENT, 60 PERCENT, AND 100 PERCENT TARIFF ON A SINGLE COMMODITY FOR VARIOUS VALUES OF r AND σ UNDER TWO COMMODITY AND r_1, r_3 AND σ UNDER THREE COMMODITY MODELS
(In Percentages of U_o)

Section A: Johnson's Two-Commodity Model

r/σ	.25	.50	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
				20 Percent Tariff						
0.10	0.04	0.07	0.11	0.16	0.20	0.25	0.29	0.34	0.45	0.57
0.20	0.07	0.13	0.20	0.27	0.35	0.43	0.51	0.59	0.76	0.94
0.30	0.09	0.17	0.26	0.36	0.45	0.55	0.64	0.74	0.94	1.15
0.40	0.10	0.20	0.30	0.40	0.51	0.61	0.71	0.82	1.02	1.23
0.50	0.10	0.21	0.31	0.41	0.52	0.62	0.72	0.82	1.02	1.21
0.60	0.10	0.20	0.30	0.39	0.49	0.58	0.67	0.76	0.93	1.09
0.70	0.09	0.17	0.26	0.34	0.42	0.50	0.57	0.64	0.78	0.91
0.80	0.07	0.13	0.20	0.26	0.31	0.37	0.42	0.47	0.57	0.65
0.90	0.04	0.07	0.11	0.14	0.17	0.20	0.23	0.26	0.31	0.35
				60 Percent Tariff						
0.10	0.23	0.50	0.79	1.12	1.48	1.89	2.32	2.80	3.87	5.09
0.20	0.42	0.88	1.38	1.91	2.49	3.09	3.73	4.39	5.77	7.19
0.30	0.56	1.15	1.77	2.43	3.09	3.77	4.46	5.15	6.51	7.78
0.40	0.65	1.31	1.99	2.67	3.36	4.03	4.68	5.32	6.50	7.54

TABLE 3, Section A--Continued

r/σ	.25	.50	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.50	0.69	1.37	2.04	2.70	3.33	3.94	4.62	5.06	6.01	6.80
0.60	0.67	1.31	1.93	2.52	3.06	3.57	4.04	4.46	5.18	5.73
0.70	0.59	1.15	1.67	2.14	2.57	2.96	3.31	3.61	4.11	4.46
0.80	0.46	0.88	1.26	1.59	1.89	2.15	2.37	2.56	2.86	3.06
0.90	0.26	0.50	0.70	0.87	1.02	1.15	1.26	1.35	1.48	1.56
					100 Percent Tariff					
0.10	0.49	1.08	1.77	2.57	3.48	4.51	5.66	6.92	9.74	12.81
0.20	0.89	1.90	3.03	4.28	5.61	7.03	8.50	10.00	12.96	15.67
0.30	1.19	2.48	3.86	5.30	6.76	8.23	9.67	11.05	13.53	15.52
0.40	1.39	2.83	4.29	5.75	7.17	8.51	9.77	10.91	12.80	14.13
0.50	1.48	2.94	4.37	5.72	6.97	8.11	9.13	10.00	11.34	12.16
0.60	1.45	2.83	4.11	5.27	6.30	7.19	7.95	8.57	9.45	9.90
0.70	1.30	2.48	3.53	4.44	5.21	5.85	6.37	6.77	7.29	7.49
0.80	1.20	1.90	2.65	3.27	3.78	4.18	4.48	4.71	4.96	5.01
0.90	0.59	1.08	1.47	1.79	2.03	2.21	2.34	2.43	2.51	2.54

TABLE 3--Continued

Section B: Three Commodity Model

r_1	r_3/σ	.25	.60	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
					20 Percent Tariff						
0.05	0.05	0.04	0.07	0.11	0.16	0.20	0.25	0.29	0.34	0.45	0.57
0.10	0.10	0.07	0.13	0.20	0.28	0.35	0.43	0.51	0.59	0.76	0.94
0.15	0.15	0.09	0.17	0.26	0.36	0.45	0.55	0.64	0.74	0.94	1.15
0.20	0.20	0.10	0.20	[illegible]	0.40	0.51	0.61	0.71	0.82	1.02	1.23
0.25	0.25	0.10	0.21	0.31	0.41	0.52	0.62	0.72	0.82	1.02	1.21
0.30	0.30	0.10	0.20	0.30	0.39	0.49	0.58	0.67	0.76	0.93	1.09
0.35	0.35	0.09	0.17	0.26	0.34	0.42	0.50	0.57	0.64	0.78	0.91
0.40	0.40	0.07	0.13	0.20	0.26	[illegible]	0.37	0.42	0.47	0.57	0.65
0.45	0.45	0.04	0.07	0.11	0.14	0.17	0.20	0.23	0.26	0.31	0.35
					60 Percent Tariff						
0.05	0.05	0.23	0.50	0.79	1.12	1.48	1.89	2.33	2.80	3.87	5.09
0.10	0.10	0.42	0.88	1.38	1.91	2.49	3.09	3.73	4.39	5.77	7.19
0.15	0.15	0.56	1.15	1.77	2.43	3.09	3.77	4.46	5.15	6.51	7.78
0.20	0.20	0.65	1.31	1.99	2.67	3.36	4.03	4.68	5.32	6.50	7.54
0.25	0.25	0.69	1.37	2.04	2.70	3.33	3.94	4.62	5.06	6.01	6.80
0.30	0.30	0.67	1.31	1.93	2.52	3.06	3.57	4.04	4.46	5.18	5.73
0.35	0.35	0.59	1.15	1.67	2.14	2.57	2.96	3.31	3.61	4.11	4.46
0.40	0.40	0.46	0.88	1.26	1.59	1.89	2.15	2.37	2.56	2.86	3.06
0.45	0.45	0.26	0.50	0.70	0.87	1.02	1.15	1.26	1.35	1.48	1.56

TABLE 3, Section B--Continued

r_1	r_3/σ	.25	.60	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
					100 Percent Tariff						
0.05	0.05	0.49	1.08	1.77	2.57	3.48	4.51	5.66	6.92	9.74	12.81
0.10	0.10	0.89	1.90	3.03	4.28	5.61	7.03	8.50	10.00	12.96	15.67
0.15	0.15	1.19	2.48	3.86	5.30	6.76	8.23	9.67	11.05	13.53	15.52
1.20	0.20	1.39	2.83	4.29	5.75	7.17	8.51	9.77	10.91	12.80	14.13
0.25	0.25	1.43	2.94	4.37	5.72	6.97	8.11	9.13	10.00	11.34	12.16
0.30	0.30	1.45	2.83	4.11	5.27	6.30	7.19	7.95	8.57	9.45	9.90
0.35	0.35	1.30	2.48	3.53	4.44	5.21	5.85	6.37	6.77	7.29	7.49
0.40	0.40	1.02	1.90	2.65	3.27	3.78	4.18	4.48	4.71	4.96	5.01
0.45	0.45	0.59	1.08	1.47	1.79	2.03	2.21	2.34	2.43	2.51	2.54

Source: H. G. Johnson, "Costs of Protection and Self-Sufficiency," Quarterly Journal of Economics, LXXIX (August, 1965), 361-362, Tables 1A, 1B and 1C.

income that are not taxed. For example, the second row of Table 3B indicates that if the sum of the two proportions untaxed at free trade prices were .05 and .15, the welfare loss of a 20 percent tariff is exactly the same as in the corresponding calculation by Johnson represented here in Table 3A where the untaxed proportion amounted to .10. Subsequently, similar comparisons are shown for tariff rates of 60 percent and 100 percent. Again identical results are obtained thereby confirming the formulae used and the computational routine derived for the three goods model.

In Table 4, Johnson's consumption cost calculations for a single 60 percent tariff rate (Table 4A) is compared with those of Table 4B obtained from the three goods model with differential tariff rates whose weighted average is 60 percent. The results indicate that when tariff rates are uneven but the weighted average of tariff is the same as that in the two-commodity single tariff case, the welfare loss in the uneven tariff case is generally considerably higher than when a single commodity is taxed. For example, when r=.05, and σ=3.00, the welfare cost in the two-commodity model is 3.08 percent whereas in the three-commodity model, the corresponding welfare loss is 11.71 percent (shown in Table 4B). The only cases in which the consumption costs are higher in the single tariff case of Table 4A than in the differential tariff case of Table 4B are those in which both the elasticity of substitution and

TABLE 4

CONSUMPTION COST $(1-U_t/U_c)$ OF A 60 PERCENT SINGLE TARIFF AND DIFFERENTIAL TARIFF OF 20 PERCENT AND 100 PERCENT ON Y AND Z COMMODITY WITH THE WEIGHTED AVERAGE OF 60 PERCENT FOR VARIOUS VALUES OF r AND σ

Section A: Johnson's 60 Percent Single Tariff Case in His Two Commodity Model

r/σ	.25	.50	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
.10	.23	.50	.79	1.12	1.49	1.89	2.33	2.80	3.87	5.09
.20	.42	.88	1.38	1.91	2.49	3.09	3.73	4.39	5.77	7.19
.30	.56	1.15	1.77	2.42	3.09	3.77	4.46	5.15	6.51	7.78
.40	.65	1.31	1.99	2.67	3.36	4.03	4.68	6.32	6.50	7.54
.50	.69	1.37	2.04	2.70	3.33	3.94	4.52	5.06	6.01	6.80
.60	.67	1.31	1.93	2.52	3.06	3.57	4.04	4.45	5.18	5.73
.70	.59	1.15	1.67	2.14	2.57	2.96	3.31	3.61	4.11	4.46
.80	.46	.88	1.26	1.59	1.89	2.15	2.37	2.56	2.86	3.06
.90	.26	.50	.70	.87	1.02	1.15	1.26	1.35	1.48	1.56

TABLE 4--Continued

Section B: Three Commodity Model with Differential Tariffs of 20 Percent and 100 Percent on Two Commodities with Their Weighted Average of 60 Percent

r_1	r_3/σ	.25	.50	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
0.10	0.45	1.35	2.71	4.06	5.36	6.60	7.75	8.80	9.74	11.25	12.23
0.20	0.40	1.22	2.47	3.72	4.96	6.17	7.31	8.39	9.37	11.04	12.29
0.30	0.35	1.09	2.21	3.36	4.52	5.67	6.79	7.87	8.89	10.69	12.14
0.40	0.30	0.95	1.94	2.98	4.04	5.11	6.19	7.24	8.27	10.17	11.80
0.50	0.25	0.80	1.65	2.56	3.50	4.43	5.48	6.43	7.49	9.42	11.21
0.60	0.20	0.65	1.35	2.11	2.92	3.77	4.66	5.58	6.52	8.43	10.29
0.70	0.15	0.49	1.04	1.63	2.28	2.97	3.72	4.51	5.34	7.10	8.93
0.80	0.10	0.33	0.71	1.12	1.53	2.09	2.64	3.25	3.90	5.35	6.97
0.90	0.05	0.17	0.36	0.58	0.82	1.10	1.41	1.75	2.14	3.04	4.14

Source: H. G. Johnson, Ibid., p. 361.

the percentage taxed are very low.

Tables 5A, 5B and 5C show weighted average tariffs of 80 percent, 40 percent and 60 percent respectively. Note that the variance of the tariff rates in Table 5C is greater than in the other two cases, in which the variance is identical (in absolute terms). The comparisons of the 80 percent and 40 percent cases show that when the variance is held constant, but the tariff rate is increased, the consumption cost is larger. However, the comparisons of the results of Tables 5A and 5C show that when the tariff rates are more unequal, even though their average may be lower, the welfare cost is greater. Thus the calculations of Table 5C are frequently higher than those of Table 5A even though the average tariff rates used in 5C are higher than those of 5A.

5.2 Production Cost Calculations

In calculating the production cost of tariffs, two different programs were run. The first program was designed for the single tariff case, i.e., when one commodity is taxed at a single rate, with less estimated for tariffs ranging from 0.05 to 1.50 percent. In the other program there are three commodities, and three curvature coefficients (M,L,N). Since these curvature coefficients (parameters in the transformation function) have no direct interpretation in economic terms they have to be converted into

TABLE 5

CONSUMPTION COST $(1-U_t/U_o)$ OF A WEIGHTED AVERAGE OF 80 PERCENT, 40 PERCENT, AND 60 PERCENT TARIFF FOR VARIOUS VALUES OF r AND σ

Section A: Weighted Average of 80 Percent Tariffs of Individual Rates of 60 Percent and 100 Percent

r_1	r_2/σ	.25	.50	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
.05	.35	1.30	2.66	4.05	5.44	6.81	8.13	9.36	10.51	12.45	13.80
.10	.35	1.29	2.61	3.94	5.26	6.53	7.73	8.85	9.88	11.60	12.85
.15	.35	1.26	2.54	3.80	5.02	6.19	7.29	8.30	9.21	10.73	11.82
.20	.35	1.22	2.44	3.62	4.75	5.82	6.81	7.71	8.51	9.84	10.79
.25	.30	1.15	2.30	3.42	4.51	5.54	6.50	7.38	8.19	9.54	10.55
.30	.30	1.10	2.18	3.23	4.22	5.15	6.01	6.79	7.50	8.68	9.55
.35	.30	1.04	2.04	2.99	3.89	4.72	5.48	6.17	6.78	7.80	8.56
.40	.30	.96	1.87	2.73	3.52	4.25	4.91	5.51	6.04	6.92	7.57
.45	.30	.86	1.67	2.42	3.11	3.74	4.31	4.82	5.27	6.02	6.59
.50	.30	.75	1.45	2.09	2.67	3.20	3.67	4.10	4.48	5.11	5.61
.60	.25	.59	1.13	1.62	2.07	2.[illegible]3	2.85	3.19	3.50	4.02	4.46
.70	.20	.42	.80	1.15	1.47	1.76	2.03	2.28	2.51	2.93	3.29
.80	.15	.24	.46	.67	.86	1.04	1.21	1.37	1.52	1.82	2.10
.90	.05	.18	.34	.48	.61	.72	.83	.92	1.01	1.18	1.32

TABLE 5--Continued

Section B: Weighted Average of 40 Percent Tariffs of Individual Rates of 20 Percent and 60 Percent

r_1	r_2/σ	.25	.50	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
.05	.35	.59	1.21	1.85	2.49	3.14	3.80	4.44	5.08	6.29	7.38
.10	.40	.50	1.22	1.84	2.46	3.07	3.67	4.26	4.82	5.87	6.79
.15	.35	.56	1.13	1.71	2.30	2.89	3.47	4.05	4.62	5.69	6.66
.20	.35	.54	1.08	1.64	2.19	2.75	3.30	3.85	4.38	5.38	6.30
.25	.30	.49	.99	1.50	2.02	2.55	3.08	3.60	4.13	5.14	6.09
.30	.30	.47	.94	1.43	1.92	2.42	2.92	3.41	3.90	4.85	5.74
.35	.30	.44	.89	1.35	1.82	2.28	2.75	3.21	3.67	4.56	5.40
.40	.30	.42	.84	1.27	1.70	2.14	2.57	3.00	3.43	4.26	5.05
.45	.30	.39	.78	1.18	1.58	1.99	2.39	2.79	3.19	3.96	4.70
.50	.30	.36	.72	1.09	1.46	1.83	2.20	2.57	2.94	3.66	4.35
.60	.25	.29	.59	.89	1.21	1.52	1.84	2.16	2.48	3.14	3.78
.70	.20	.23	.46	.70	.94	1.20	1.46	1.73	2.00	2.56	3.14
.80	.15	.16	.32	.50	.68	.87	1.06	1.27	1.48	1.93	2.42
.90	.05	.07	.14	.22	.30	.39	.48	.57	.67	.89	1.13

TABLE 5--Continued

Section C: Weighted Average of 60 Percent Tariffs of Individual Rates of 20 Percent and 100 Percent

r_1	r_2/σ	.25	.50	.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00
.05	.35	1.27	2.62	4.01	5.42	6.83	8.20	9.51	10.73	12.85	14.45
.10	.40	1.31	2.62	4.04	5.37	6.68	7.92	9.08	10.14	11.92	13.20
.15	.35	1.21	2.47	3.77	5.08	6.38	7.65	8.86	10.00	11.98	13.52
.20	.35	1.17	2.39	3.64	4.90	6.15	7.37	8.54	9.63	11.55	13.06
.25	.30	1.05	2.17	3.35	4.55	5.77	6.99	8.18	9.33	11.43	13.17
.30	.30	1.02	2.10	3.23	4.39	5.56	6.73	7.87	8.98	11.01	12.71
.35	.30	.98	2.02	3.10	4.21	5.18	6.10	6.95	7.73	9.05	10.05
.40	.30	.95	1.94	2.98	4.04	5.11	6.19	7.24	8.27	10.17	11.80
.45	.30	.90	1.85	2.84	3.86	4.88	5.91	6.92	7.91	9.75	11.34
.50	.30	.86	1.26	2.70	3.67	4.65	5.63	6.60	7.55	9.33	10.89
.60	.25	.72	1.49	2.31	3.16	4.05	4.96	5.88	6.81	8.62	10.31
.70	.20	.58	1.20	1.88	2.61	3.38	4.18	5.02	5.89	7.66	9.42
.80	.15	.43	.90	1.43	2.00	2.62	3.29	4.01	4.77	6.39	8.11
.90	.05	.17	.36	.58	.82	1.10	1.41	1.75	2.14	3.04	4.14

the elasticities of supply of the taxed commodities Y and Z.

As noted above, in the three-goods model we have utilized a transformation function suggested by Allen[2] that is not strictly comparable with Johnson's. Indeed the formulation of the transformation function we have utilized permits a parametric simulation for production cost that is much more comparable to the parametric simulation Johnson has undertaken for consumption cost than that used by Johnson for production cost. However, as the curvature coefficients L, M, and N approach equality at 1.0, the results of the three goods transformation function are quite comparable to Johnson's two-goods transformation model in the special case in which his cross product coefficient (M) is equal to zero. However, in this case the elasticity of supply in the three goods model is zero, whereas in Johnson's special case the elasticity of supply is 1/2. It is interesting to see (in Table 6) that his calculations of production loss from the two commodity model in which the elasticity of supply is 1/2 are similar in magnitude to our calculations using the three commodity model in which the elasticity of supply is zero. However, with the same elasticity the differential tariff case of the three commodity

---

[2]R. G. D. Allen, Mathematical Analysis for Economists (London: Macmillan, 1956), p. 284.

TABLE 8

PRODUCTION COST $(1-M_t/M_0)$ FOR VARIOUS VALUES OF M, L, M AND t (% OF M_0)

Elasticity Y	Z	L	M	M/t	.05	.10	.15	.20	.25	.30	.40	.50	.60	.70	.80	.90	1.00	
		.0*	-	-	.03	.11	.24	.41	.61	.86	1.26	1.54	2.56	[illegible]	3.85	4.49	5.13	Johnson Single Tariff
1/2	-	.05	.05	.05	.03	.10	.23	.29	.59	.82	1.36	1.53	2.67	3.43	4.16	4.93	5.72	Three Commodity Models: Single Tariff
0	-	.05	.05	.05	0	.20	.63	1.05	1.54	2.41	3.24	4.53	5.58	7.37	9.75	10.59	11.94	Three Commodity Models: Differential Tariff
0	0	.05	.50	.05	.03	.10	.23	[illegible]	.59	.82	[illegible]	1.58	2.67	[illegible]	4.16	4.93	5.72	Three Commodity Models: Single Tariff
.43	.43	.05	.50	.05	.03	.11	.24	.41	.62	.85	1.26	1.57	2.62	3.27	3.94	4.61	5.27	
.46	.46	.05	1.00	.05	.03	.11	.24	.41	.61	.84	1.27	1.55	2.55	3.24	3.90	4.56	5.21	
.48	.48	.05	1.50	.05	.03	.11	.21	.41	.61	[illegible]	1.27	1.53	[illegible]	3.23	3.88	[illegible]	[illegible]	
0	0	.05	.25	.05		.20	.63	1.05	1.54	[illegible]	3.24	4.58	[illegible]	[illegible]	9.75	10.59	11.94	Three Commodity Models: Differential Tariff
.43	.43	.05	.50	.05		[illegible]	[illegible]	1.50	[illegible]	3.36	[illegible]	6.33	[illegible]	9.15	11.10	12.57	14.47	
.46	.46	.05	1.00	.05		[illegible]	.92	1.53	2.25	3.41	4.73	6.45	8.13	9.33	11.36	12.77	[illegible]	
.48	.48	.05	1.50	.05		[illegible]	.92	1.55	2.27	3.47	[illegible]	6.51	8.25	[illegible]	11.42	12.82	14.92	
0	.75	.05	.50	.50	.02	.06	.12	.20	.20	.49	.62	.85	1.09	1.33	1.56	1.79	2.00	Three Commodity Models: Single Tariff
0	.86	.05	1.00	1.00	.01	.03	.07	.12	.17	.23	.35	.48	.62	.75	.87	1.00	1.11	
0	.91	.05	1.50	1.50	.01	.02	.05	.08	.12	.16	.25	.34	.43	.52	.61	.69	.77	
0	.75	.05	.50	.50		.12	.25	[illegible]	.60	.87	1.15	1.55	1.94	2.33	2.64	2.96	3.30	Three Commodity Models: Differential Tariff
0	.86	.05	1.00	1.00		.07	.14	[illegible]	.34	.49	.64	.86	1.07	1.27	1.46	1.63	[illegible]	
0	.91	.05	1.50	1.50		.05	.10	[illegible]	.24	.34	.45	.60	.74	.86	1.00	1.12	1.27	

Notes: In differential tariff cases listed under .10, two tariff rates were .05 and .15; under .15 two tariff rates utilized were .10 and .20, etc.

yields production losses that are much higher than those obtained in the single commodity case.

An attempt has been made to test the sensitivity of the production cost calculation comparisons to changes in curvature coefficients by showing different sets of such comparisons when one parameter at a time is changed. In every case the results demonstrate that the existence of differential tariff rates yield much larger production losses due to tariffs than those obtained for the single tariff case. The findings also confirm Johnson's hypothesis that production costs are likely to be larger than consumption costs. But it invariably supports our contention that differential tariffs produce systematically higher welfare costs than obtained from uniform tariffs. The maximum production loss in the single tariff case with a 100 percent tariff is about 6 percent while with differential tariffs averaging 100 percent this production cost ranged from 14 to 17 percent of national income.

CHAPTER VI

POLICY IMPLICATIONS AND CONCLUSIONS

Our review in Chapter II of the rather numerous studies on the measurement of the welfare cost of trade restrictions revealed a number of weaknesses in these studies mainly due to the simplifying assumptions which have apparently been necessary in order to make the approaches operational. Most of these weaknesses tend to give the estimates a distinct downward bias. It is our hypothesis that the welfare costs of trade restrictions, particularly in less developed countries, are much higher than those that have been indicated in previous studies.

Among the more restrictive assumptions that have been incorporated in all existing studies has been the assumption of homogeneity of tariff rates. When occasionally the authors (e.g., Harberger and Johnson) have admitted to the reality of unequal tariff rates, they have promptly avoided complications by using a single weighted average of the different tariff rates in their calculation. In this study we have adopted the most promising approach to welfare cost measurement, namely the general equilibrium simulation model developed by Harry Johnson. This model provides for both substitution and income effects in welfare cost calculations and reduces the welfare cost calculations to a few measurable parameters. However, we have extended the model to the case of three goods so as to simulate the consumption and production costs of tariffs when two unequal tariff rates are imposed. The results indicate that the homogeneity of tariff rates assumption in existing studies has led to a substantial downward bias in the estimates of welfare cost and confirms our hypothesis that the true welfare cost of such trade restrictions may be much larger than previous empirical studies have indicated.

Like other such studies, the present study uses an entirely static approach. However, it should be recognized that the dynamic effects may not be independent of the static ones. For example, McKinnon[1] and Briton,[2] have argued tha high and varied tariff rates may be one of the more important explanations for the disappointing rates of technological change achieved by many Latin American countries.

We do not wish to display ignorance of a sometimes justifiable argument in favor of differential tariffs.

---

[1]R. I. McKinnon, "Import Substitution and Economic Development," Paper presented at the World Conference of the Society for International Development, New York, March 16-18, 1966. (Mimeographed.)

[2]H. J. Briton, "Productivity Growth in Latin America," American Economic Review, LVII, No. 5 (December, 1967), 1099-1116.

Indeed, if there are distortions already existing in the economy due to the presence of monopoly power, government interference in the market, resource immobility, etc., then differential tariffs can sometimes be justified in order to compensate for these other distortions.[3] This is what has sometimes been referred to as "the theory of second best."[4] However, the ad hoc origin to most tariffs and their haphazard scatter make it most unlikely that actual tariffs display any such function.[5]

The policy implications of this study are rather obvious. The welfare costs of the high tariff rates in less developed countries may be greatly underestimated. Aside from suggesting that tariff rates should be lowered, the results generated in this study suggest that the height of tariff rates may not be as detrimental to welfare as the variance in tariff rates from sector to sector. That conclusion would seem to support the policy proposal of Kaldor[6]

---

[3]S. I. Hagen, "An Economic Justification of Protectionism," Quarterly Journal of Economics, LXII (November, 1958), 496-514.

[4]See R. G. Lipsey, "The Theory of Customs Unions: A General Survey," Economic Journal, LXX (September, 1960), 496-513.

[5]See J. Macario, "Protectionism and Industrialization in Latin America," Economic Bulletin for Latin America, IX (March, 1964), 61-101.

[6]N. Kaldor, "Dual Exchange Rates and Economic Development," Economic Bulletin for Latin America, IX, No. 2 pp. 215-223.

and McKinnon[7] that if protection is to be provided for domestic manufacturing in less developed countries, the tariff or dual exchange rate should be the same for all subsectors.

[7]R. I. McKinnon, op. cit.

B I B L I O G R A P H Y

BIBLIOGRAPHY

Books

Allen, R. G. D. Mathematical Analysis for Economists. London: Macmillan, 1956.

Bergson, A. Essays in Normative Economics. Cambridge, Mass.: Harvard University Press, 1966.

Janssen, L. H. Free Trade, Protection and Customs Union. Leiden: H. E. Stenfertkroese N.V. , 1961.

Johnson, H. G. "Optimal Trade Intervention in the Presence of Domestic Distortions," Trade, Growth and the Balance of Payments, Essays in Honor of Gottfried Haberler. Edited by R. E. Baldwin et al. Chicago: Rand McNally, 1965.

Lawrence, R. "Primary Product Preferences and Economic Welfare: The EEC and Africa," The Open Economy. Edited by P. Kenen and R. Lawrence. New York: Columbia University Press, 1968.

Machlup, F. "Professor Hicks' Revision of Demand Theory," Readings in Microeconomics. Edited by W. Breit and H. M. Hochman. New York: Holt, Rinehart and Winston, Inc., 1967.

Meade, J. E. Trade and Welfare. London: Oxford University Press, 1955.

Minhas, B. S. An International Comparison of Factor Cost and Factor Use. Amsterdam: North Holland Publishing Company, 1963.

Nerlove, M. "Recent Emprical Studies of the CES and Related Production Functions," The Theory and Empirical Analysis of Production, Studies in Income and Wealth. Vol. 31. New York: National Bureau of Economic Research, 1967, 55-122.

Samuelson, P. ed. International Economic Relations. New York: St. Martin's Press, 1969.

Scitovsky, T. Economic Theory and Western European Integration. Stanford, Calif.: Stanford University Press, 1958.

Scitovsky, T. Papers on Welfare and Growth. Stanford, Calif.: Stanford University Press, 1964.

Taussing, F. W. Selected Readings in International Trade and Tariff Problems. Boston: Ginn and Company, 1921.

Travis, W. P. The Theory of Trade and Protection. Cambridge, Mass.: Harvard University Press, 1964.

Vaccara, B. N. Employment and Output in Protected Manufacturing Industries. Washington, D.C.: The Brookings Institution, 1960.

Young, J. H. Canadian Commercial Policy. Ottawa: Royal Commission on Canada's Economic Prospects, 1957.

Articles and Periodicals

Anderson, J., and Naya, S. "Substitution and Two Concepts of Effective Rate of Protection," The American Economic Review, LIX (September, 1969), 607-611.

Arrow, K. J. "Import Substitution in Leontief Models," Econometrica, XXII (October, 1954), 481-492.

Arrow, K. J., Chenery, H. B., Minhas, B. S., and Solow, R. M. "Capital-Labor Substitution and Economic Efficiency," Review of Economics and Statistics, XLIII (August, 1961), 225-250.

Balassa, B. "Trade Creation and Trade Diversion in the European Common Market," Economic Journal, LXXIII (March, 1967), 1-21.

Balassa, B., and Kreinen, M. E. "Trade Liberalization under the Kennedy Round: The Static Effects," Review of Economics and Statistics, XLIX (May, 1967), 125-137.

Balassa, B., and Schydlowsky, D. M. "Effective Tariffs, Domestic Cost of Foreign Exchange, and the Equilibrium Exchange Rate," Journal of Political Economy, LXXVI (May/June, 1968), 348-360.

Baldwin, R. E. "The Case against Infant-Industry Tariff Protection," Journal of Political Economy, LXXVII (May/June, 1969), 295-305.

Baseir, G. "The Restrictive Effect of the U.S. Tariff," American Economic Review, LVII (September, 1968), 840-852.

Basevi, G. "The United States Tariff Structure: Estimates of Effective Rates of Protection of United States Industries and Industrial Labor," Review of Economics and Statistics, XLVIII (May, 1966), 147-160.

Bergstrom, R. "Linear Programming, Import Controls and the Exchange Rates Where the Supply of Exports is Inelastic," The Economic Record, XXXVI (August, 1960), 385-392.

Bhagwati, J., and Johnson, H. G. "Notes on Some Controversies in the Theory of International Trade," The Economic Journal, LXX (March, 1960), 74-93.

Brigden, J. B., et al. The Australian Tariff: An Economic Enquiry. Melbourne: Melbourne University Press, 1929.

Bruton, H. J. "Productivity Growth in Latin America," American Economic Review, LVII, No. 5 (December, 1967), 1099-1116.

Chipman, J. S. "A Survey of the Theory of International Trade," Parts 2 and 3, Econometrica, XXXV and XXXIV (October, 1965; January, 1966) 685-757; 19-76.

Corden, W. M. "The Calculation of the Cost of Protection," Economic Record, XXXIII (April, 1957), 28-51.

Corden, W. M. "Effective Protective Rates in the General Equilibrium Model: Geometrical Note," Oxford Economic Papers, XXI (July, 1969), 135-141.

Corden, W. M. "The Structure of a Tariff System and the Effective Protective Rates," Journal of Political Economy, LXXIV (June, 1966), 221-237.

Dardis, R. "Intermediate Goods and the Gains from Trade," The Review of Economics and Statistics, XLIX (November, 1967), 502-509.

Grubel, H. G., and Johnson, H. G. "Nominal Tariffs, Indirect Taxes and Effective Rates of Protection: The Common Market Countries 1959," The Economic Journal, LXVII (December, 1967), 761-776.

Guisinger, S. E. "Negative Value Added and the Theory of Effective Protection," Quarterly Journal of Economics, LXXXIII (August, 1969), 415-433.

Hagen, E. "An Economic Justification of Protectionism," Quarterly Journal of Economics, LXII (November, 1958), 496-514.

Harberger, A. C. "The Measurement of Waste," American Economic Review, Papers and Proceedings, LIV (May, 1964), 58-76.

Harberger, A. C. "Using the Resources at Hand More Effectively," American Economic Review, Papers and Proceedings, XLIX (May, 1959), 134-146.

Hause, J. C. "The Welfare Cost of Disequilibrium Exchange Rates," Journal of Political Economy, LXXIV (August, 1966), 333-352.

Hicks, J. R. "The Four Consumer's Surplus," The Review of Economic Studies, XI (Winter, 1943), 31-44.

Holzman, F. D. "Comparison of Different Forms of Trade Barriers," The Review of Economics and Statistics, LI (May, 1969), 159-165.

Humphrey, D. B. "Measuring the Effective Rate of Protection: Direct and Indirect Effects," Journal of Political Economy, LXXVII (September-October, 1969), 834-844.

Hutcheson, T. L. "Factor Intensity Reversals and the CES Production Function," Review of Economics and Statistics, LI (November, 1969), 468-469.

Islam, N. "Comparative Costs, Factor Proportions and Industrial Efficiency in Pakistan," The Pakistan Development Review, VII (Summer, 1967).

Johnson, H. G. "The Cost of Protection and Scientific Tariff," Journal of Political Economy, LXVII (August, 1965), 327-345.

Johnson, H. G. "The Costs of Protection and Self-Sufficiency," The Quarterly Journal of Economics, LXXIX (August, 1965), 356-372.

Johnson, H. G. "The Gains from Trade with Europe: An Estimate," Manchester School of Economic and Social Studies, XXVI (September, 1958).

Johnson, H. G. "A Model of Protection and the Exchange Rate," Review of Economic Studies, XXXIX, No. 2 (April, 1966), 159-163.

Johnson, H. G. "The Welfare Costs of Exchange Rate Stabilization," Journal of Political Economy, LXXIX (October, 1966), 512-518.

Kaldor, N. "Dual Exchange Rates and Economic Development," Economic Bulletin for Latin America, IX, No. 2, pp. 215-223.

Krueger, A. O. "Some Economic Costs of Exchange Control: The Turkish Case," Journal of Political Economy, LXXIV (October, 1966), 466-480.

Leibenstein, H. "Allocative Efficiency vs. 'X-Efficiency'," American Economic Review, LVI (June, 1966), 392-415.

Leith, J. C. "Substitution and Supply Elasticities in Calculating the Effective Protective Rate," The Quarterly Journal of Economics, LXXXII (November, 1968), 588-601.

Leontief, W. "An International Comparison of Factor Costs and Factor Use." American Economic Review, LIV (June, 1964), 335-345.

Lewis, S. R., Jr., and Guisinger, S. E. "Measuring Protection in a Developing Country: The Case of Pakistan," Journal of Political Economy, LXXVI (December, 1968), 1170-1198.

Lipsey, R. G. "The Theory of Customs Unions: A General Survey," The Economic Journal, LXX (September, 1960), 496-513.

Loveday, A. "The Australian Tariff: A Criticism," The Economic Record, VI (November, 1930).

Marcario, S. "Protectionism and Industrialization in Latin America," Economic Bulletin for Latin America, IX (March, 1964), 61-101.

McKinnon, R. I. "Intermediate Products and Differential Tariffs: A Generalization of Lerner's Symmetry Theorem," The Quarterly Journal of Economics, LXXX (November, 1966), 584-615.

Melvin, J. R. "Intermediate Goods, the Production Possibility Curve, and Gains from Trade," The Quarterly Journal of Economics, LXXXIII (February, 1969), 141-151.

Mishan, E. J. "A Survey of Welfare Economics, 1939-1959," The Economic Journal, LXX (June, 1960), 197-265.

Mishan, E. J. "Welfare and Trade: A Reply to Kemp," Notes and Memoranda, The Economic Journal, LXXVIII (June, 1968), 481-482.

Nugent, J. B. "La estructura arancelaria y el costo de Proteccion en America Central," El Trimestre Economico, XXXV (October-December, 1968), 751-766.

Ophir, T. "The Interaction of Tariffs and Quotas," American Economic Review, LIX (December, 1969), 1002-1005.

Orcutt, G. H. "Measurement of Price Elasticities in International Trade," Review of Economics and Statistics, XXXII (May, 1950), 117-132.

Otto, A. D., and Winston, A. B. "Welfare Economics and the Theory of Second Best," Review of Economic Studies, XXXII, No. 89 (January, 1965), 1-14.

Power, J. H. "Import Substitution as an Industrialization Strategy," The Philippine Economic Journal, LII (1966), 167-204.

Reddaway, W. B. "Some Effects of the Australian Tariff," The Economic Record, XIII (June, 1937), 9.

Reitsma, A. J. "The Excess Costs of a Tariff and Their Measurement," The Economic Record, XXXVII (December, 1961), 442-455.

Samuelson, P. "Prices of Factors and Goods in General Equilibrium," The Review of Economic Studies, XXI (October, 1953), 1-20.

Snape, R. H. "Sugar: Costs of Protection and Taxation," Economica, XXXVI (February, 1969), 29-41.

Soligo, R., and Stern, J. J. "Tariff Protection, Import Substitution, and Investment Efficiency," Pakistan Development Review, V (Summer, 1965), 249-270.

Stern, R. M. "The U.S. Tariff and the Efficiency of the U S Economy," American Economic Review, LIV (May, 1964), 459-470.

Tan, A. H. H. "Differential Tariffs, Negative Value Added and the Theory of Effective Protection," American Economic Review, LX (March, 1970), 107-116.

Thomas, R. P. "A Quantitative Approach to the Study of the Effects of British Imperial Policy upon Colonial Welfare: Some Preliminary Findings," Journal of Economic History, XXV (December, 1965), 615-638.

Verdoorn, P. J. "A Customs Union for Western Europe--Advantages and Feasibility," World Politics, VII (July, 1954), 482-500.

Vinelle, L. D. de la. "La creation du commerce attributable ou marche common et son incidence sur le volume du produit national de la communoute," Information Statistiques, IV (1965), 61-70; V (1966), 5-31.

Viner, J. "The Australian Tariff--An Economic Enquiry," The Economic Record, V (November, 1929).

Viner, J. Memoranda on Commercial Policy, Vol. XI, International Economics. Glencoe, Ill.: The Free Press, 1936.

Waelbroeck, J. "Le commerce de la Communoute Europeene avec les pays tiers," Integration Europeene et Realite Economique, Bruges, 1964, pp. 139-164.

Walker, F. V. "The Restrictive Effect of the U.S. Tariff," American Economic Review, LIX (December, 1965), 963-966.

Wemelsfelder, W. "The Short-Term Effect of Lowering Import Duties in Germany," The Economic Journal, LXX (March, 1960), 94-104.

Wilford, W. T. "Trade Creation in the Central American Common Market," Western Economic Journal, VIII (March, 1970), 61-69.

Zarembka, P. "On the Empirical Relevance of the CES Production Function," Review of Economics and Statistics, LII (February, 1970), 47-54.

A P P E N D I X E S

APPENDIX A

THE DERIVATION OF THE CONSUMPTION COST MEASURE IN THE SINGLE TARIFF CASE

THE DERIVATION OF THE CONSUMPTION COST MEASURE
IN THE SINGLE TARIFF CASE

$$U = (AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta}$$

Let $AX^{-\beta} + aY^{-\beta} + cZ^{-\beta} = V$

Then $U = (V)^{-1/\beta}$

By using the chain rule of partial differentiation

$$\frac{dU}{dX} = \frac{dU}{dV}\frac{dV}{dX} = \text{marginal utility of X}$$

$$= -\frac{1}{\beta} V^{-1/\beta-1}(- AX^{-\beta-1})$$

$$= -\frac{1}{\beta}(AX^{-\beta} + aY^{-\beta})^{-1/\beta-1}(- AX^{-\beta-1})$$

$$= (AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta-1}(AX^{-\beta-1})$$

$$= \frac{(AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta}}{AX^{-\beta} + aY^{-\beta} + cZ^{-\beta}} \cdot AX^{-\beta-1}$$

$$= \frac{(AX^{-\beta} + aY^{-\beta} + cZ^{-\ })^{-1/\beta}}{AX^{-\beta} + aY^{-\beta} + cZ^{-\beta}} \cdot \frac{AX^{-\beta}}{X}$$

$$= \frac{AU}{X} \quad \frac{X^{-\beta}}{AX^{-\beta} + aY^{-\beta} + cZ^{-\beta}}$$

$$= A\,\frac{U}{X}\;\frac{(AX^{-\beta} + AY^{-\beta} + cZ^{-\beta})^{-1/\beta}}{(AX^{-\beta} + aY^{-\beta} \_ cZ^{-\beta})^{-1/\beta}} \cdot \frac{X^{-\beta}}{AX^{-\beta} + aY^{-\beta} + cZ^{-\beta}}$$

$$= A\,\frac{U}{X}\;\frac{(AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta}}{Ax^{-\beta} + aY^{-\beta} + cz^{-\beta}} \cdot \frac{X^{-\beta}}{(AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta}}$$

$$= A\,\frac{U}{X}\;\frac{(AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta-1}}{(AX^{-\beta} + aY^{-\beta} + cz^{-\beta})^{-1/\beta}} \cdot X^{-\beta}$$

$$= A\,\frac{U}{X} \cdot (AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1} \cdot X^{-\beta}$$

$$= A\,\frac{U}{X}\;(AX^{-\beta} + aY^{-\beta} + cZ^{-\beta})^{-1/\beta\;\beta} \cdot X^{-\beta}$$

$$= A\,\frac{U}{X} \cdot \frac{U^{\beta}}{X^{\beta}}$$

$$= A\left(\frac{U}{X}\right)^{\beta+1} = U_x$$

Similarly $\quad \dfrac{dU}{dY} = a\left(\dfrac{U}{Y}\right)^{\beta+1}$

$$\frac{dU}{dZ} = c\left(\frac{U}{Z}\right)^{\beta+1} = U_z$$

So with tariff:

$$\frac{U_y}{U_x} = a\left(\frac{U}{Y}\right)^{\beta+1} \times \frac{1}{A}\left(\frac{X}{U}\right)^{\beta+1} = \frac{a}{A}\left(\frac{X}{Y}\right)^{\beta+1} = 1 + t$$

$$\frac{U_Y}{U_Z} = a\left(\frac{U}{Y}\right)^{\beta+1} \times \frac{1}{c}\left(\frac{Z}{U}\right)^{\beta+1} = \frac{a}{c}\left(\frac{Z}{Y}\right)^{\beta+1} = 1 + t$$

$$\frac{U_Z}{U_X} = c\left(\frac{U}{Z}\right)^{\beta+1} \times \frac{1}{A}\left(\frac{X}{U}\right)^{\beta+1} = \frac{c}{a}\left(\frac{X}{Z}\right)^{\beta+1} = 1$$

(1) $\left[\frac{A}{a}(1+t)\right]^{\frac{1}{\beta+1}} = \frac{X}{Y}$

(2) $\left[\frac{c}{a}(1+t)\right]^{\frac{1}{\beta+1}} = \frac{Z}{Y}$

(3) $\left(\frac{A}{c}\right)^{\frac{1}{\beta+1}} = \frac{X}{Z}$

$$\frac{U_Y}{U_X} = \frac{a}{A}\left(\frac{X}{Y}\right)^{\beta+1} = 1 + t$$

$$\left(\frac{X}{Y}\right)^{\beta+1} = \left\{\frac{A}{a}(1+t)\right\}^{\beta+1}$$

$$\frac{X}{Y} = \frac{A}{a}(1+t)^{\beta+1}$$

$$\frac{U_Y}{U_Z} = \frac{a}{c}\left(\frac{Z}{Y}\right)^{\beta+1} = 1 + t$$

$$= \left(\frac{Z}{Y}\right)^{\beta+1} = \frac{c}{a}(1+t)$$

$$= \frac{Z}{Y} = \left\{\frac{c}{a}(1+t)\right\}^{\frac{1}{\beta+1}}$$

$$\frac{U_z}{U_x} = \frac{c}{a}\left(\frac{X}{Z}\right)^{\beta+1} = 1$$

$$= \left(\frac{X}{Z}\right)^{\beta+1} = \left(\frac{A}{c}\right)$$

$$= \frac{X}{Z} = \left(\frac{A}{c}\right)^{\frac{1}{\beta+1}}$$

Given a total gross income fixed in terms of world prices

$$M = X + Y + Z$$

implying that tax revenue collected on consumption of Y are returned to the community as income subsidies, consumption of the three goods, determined by equations 5, 6, 7 and 8

$$X = \left\{\frac{A}{a}(1+t)\right\}^{\frac{1}{\beta+1}} \cdot Y$$

$$Y = \frac{1}{\left\{\frac{c}{a}(1+t)\right\}^{\frac{1}{\beta+1}}} \cdot Z$$

$$Z = \frac{1}{\left(\frac{A}{c}\right)^{\frac{1}{\beta+1}}} \cdot X$$

$$X = \left\{\frac{A}{a}(1+t)\right\}^{\frac{1}{\beta+1}} \cdot \frac{Y}{X+Y+Z} \cdot X + Y + Z$$

$$= \left\{\frac{A}{a}(1+t)\right\}^{\frac{1}{\beta+1}} \cdot \frac{1}{\frac{X+Y+Z}{Y}} \cdot X + Y + Z$$

$$= \left\{\frac{A}{a}(1+t)\right\}^{\frac{1}{\beta+1}} \cdot \frac{1}{\frac{X}{Y} + 1 + \frac{Z}{Y}} \cdot M$$

$$= \{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} \frac{1}{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$= \frac{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$Y = \frac{1}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot \frac{Z}{X+Y+Z} \cdot x + Y + Z$$

$$= \frac{1}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot \frac{1}{\frac{X+Y+Z}{Z}} \cdot M$$

$$= \frac{1}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot \frac{1}{\frac{X}{Z}+\frac{Y}{Z} + 1} \cdot M$$

$$= \frac{1}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}} + \frac{1}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} + 1} \cdot M$$

$$= \frac{1}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot \frac{1}{\frac{(\frac{A}{c})^{\frac{1}{\beta+1}}[\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}] + 1 + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}}} \cdot M$$

$$= \frac{1}{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot \frac{\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}}{(\frac{A}{c})^{\frac{1}{\beta+1}}[\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}] + 1 + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot M$$

$$= \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}}[\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$Z = \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}}} \cdot \frac{X}{X+Y+Z} \cdot X + Y + Z$$

$$= \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}}} \cdot \frac{1}{\frac{X+Y+Z}{X}} \cdot X + Y + Z$$

$$= \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}}} \cdot \frac{1}{1 + \frac{Y}{X} + \frac{Z}{X}} \cdot M$$

$$= \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}}} \cdot \frac{1}{1 + \frac{1}{\{\frac{A}{c}(1+t)\}^{\frac{1}{\beta+1}}} + \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}}}} \cdot M$$

$$= \frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}}} \cdot \frac{1}{\frac{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}\{(\frac{A}{c})^{\frac{1}{\beta+1}}\} + (\frac{A}{c})^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}\}\{(\frac{A}{c})^{\frac{1}{\beta+1}}\}}} \cdot M$$

$$= \frac{1}{\left(\frac{A}{c}\right)^{\frac{1}{\beta+1}}} \cdot \frac{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}\{(\frac{A}{c})^{\frac{1}{\beta+1}}\}}{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}\{(\frac{A}{c})^{\frac{1}{\beta+1}}\} + (\frac{A}{c})^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}} \cdot M$$

$$= \frac{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}\{(\frac{A}{c})^{\frac{1}{\beta+1}}\} + (\frac{A}{c})^{\frac{1}{\beta+1}} + \{\frac{A}{c}(1+t)\}^{\frac{1}{\beta+1}}} \cdot M$$

Substituting the values of X, Y, Z, in the original utility function to express it as a function of real income and the tariff rate:

$$U = \left[\frac{A \cdot \{\frac{A}{a}(1+t)\}^{\frac{-\beta}{\beta+1}}}{[\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}} \cdot M^{-\beta} \right.$$

$$+ a \frac{M^{-\beta}}{[\{(\frac{A}{c})^{\frac{1}{\beta+1}}\}[\{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}] + \{\frac{c}{a}(1+t)^{\frac{1}{\beta+1}} + 1]^{-\beta}}$$

$$\left. + \frac{c \cdot [\{\frac{A}{a}(1+t)\}^{\frac{-\beta}{\beta+1}}]}{[\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}\{(\frac{A}{c})^{\frac{1}{\beta+1}}\} + (\frac{A}{c})^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}}]^{-\beta}} \cdot M^{-\beta} \right]^{-1/\beta}$$

or

$$U = \left[\frac{A\cdot\{[\frac{A}{a}(1+t)]^{\frac{-\beta}{\beta+1}}\}}{[\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}} \cdot M^{-\beta} \right.$$

$$+ a \frac{M^{-\beta}}{[\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}}$$

$$\left. + \frac{c \cdot [[\frac{A}{a}(1+t)]^{\frac{-\beta}{\beta+1}}]}{[(\frac{A}{c})^{\frac{1}{\beta+1}}\{\frac{A}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1 + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}}]^{-\beta}} \cdot M^{-\beta} \right]^{-1/\beta}$$

or

$$U = \left[\frac{M^{-\beta}}{[\frac{A}{a}(1+t)]^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}} \left(A\cdot[\frac{A}{a}(1+t)^{\frac{-\beta}{\beta+1}} + a + c\cdot[\frac{A}{a}(1+t)]^{\frac{-\beta}{\beta+1}} \right) \right]^{-1/\beta}$$

$$= \left[\frac{M\, a^{-1/\beta}}{[\frac{A}{a}(1+t)]^{\frac{1}{\beta+1}} + \frac{c}{a}(1+t)^{\frac{1}{\beta+1}} + 1]} \right] \left[(\frac{A}{a}\; [\frac{A}{a}(1+t)]^{\frac{-\beta}{\beta+1}} + 1 + \frac{\frac{c}{a}\; [\frac{A}{a}(1+t)]^{\frac{-\beta}{\beta+1}}}{\{(\frac{A}{c})\}^{\frac{-\beta}{\beta+1}}} \right]^{-1/\beta}$$

Right hand fraction:

$$= \left[\left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} + 1 + \frac{\frac{c}{a}\left[\frac{\Lambda}{a}(1+t)\right]^{\frac{-\beta}{\beta+1}}}{\left(\frac{\Lambda}{c}\right)^{\frac{-\beta}{\beta+1}}} \right]^{-1/\beta}$$

$$= \left[\left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} \left(\frac{\Lambda}{c}\right)^{\frac{-\beta}{\beta+1}} + \frac{\left(\frac{\Lambda}{c}\right)^{\frac{-\beta}{\beta+1}} + \frac{c}{a}\left[\frac{\Lambda}{c}(1+t)\right]^{\frac{-\beta}{\beta+1}}}{\left(\frac{\Lambda}{c}\right)^{\frac{-\beta}{\beta+1}}} \right]^{-1/\beta}$$

$$= \left\{ \frac{\left(\frac{\Lambda}{c}\right)^{\frac{-\beta}{\beta+1}} \left[\left\{\left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} + 1 + \frac{c}{a}\left(\frac{c}{a}\right)^{\frac{-\beta}{\beta+1}} I1+t)^{\frac{-\beta}{\beta+1}}\right]}{\left(\frac{\Lambda}{c}\right)^{\frac{-\beta}{\beta+1}}} \right\}^{-1/\beta}$$

$$= \left[\left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} + 1 + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} \right]^{-1/\beta}$$

$$\frac{U_t}{U_o} = \frac{M\ a^{-1/\beta}}{\left[\frac{\Lambda}{a}(1+t)\right]^{\frac{1}{\beta+1}} + \left\{\frac{c}{a}(1+t)\right\}^{\frac{1}{\beta+1}} + 1} \cdot \left\{ \left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} \{(1+t)^{\frac{-\beta}{\beta+1}} + 1 \right.$$

$$\left. + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} \right\}^{-1/\beta} \times \frac{\left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} + 1}{M\ a^{-1/\beta}\left[\left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} + 1 + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}}\right]^{-1/\beta}}$$

$$\left[\left(\frac{\Lambda}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} + 1 + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} (1+t)^{\frac{-\beta}{\beta+1}} \right]^{-1/\beta}$$

$$= \frac{X\ [(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1}{[(\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}}]^{-1/\beta}\ X}$$

$$[\frac{A}{a}(1+t)]^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t)\}^{\frac{1}{\beta+1}} + 1$$

Denoting (in the absence of tariff) ratio of X to Y as

$R_0 = (A/a)^{1/(\beta+1)}$

$$R_0 = \frac{r_1}{1-r_1-r_3} = \frac{X}{M\ X-Z} \quad \text{since } X+Y+Z = M$$

$$r_1 = X/M$$

$$R_1 = \frac{r_3}{1-r_3-r_1} = \frac{Z}{M-X-Z} = \frac{Z}{Y} \qquad r_2 = Y/M$$

$$r_3 = Z/M$$

$$R_1 = (\frac{c}{a})^{\frac{1}{\beta+1}}$$

$$\frac{Z}{Y} = \frac{Z}{M-X-Z} = R_1 \qquad X+Y+Z = M$$

$$\frac{Z/M}{1-(\frac{Z}{M}\ \frac{X}{M})} = \frac{Z/M}{1-\frac{Z+X}{M}} = \frac{Z/M}{\frac{M-Z-X}{M}} = \frac{Z}{M} \times \frac{M}{M-X-Z} = \frac{Z}{M-X-Z} = \frac{Z}{Y}$$

$$R_1 = \frac{r_3}{1-r_3-r_1} \qquad X/M = r_1$$

$$Y/M = r_2$$

$$Z/M = r_3$$

$$X/Y = \frac{X}{M-X-Z} = \frac{X}{M} \times \frac{M}{M-X-Z} = \frac{X/M}{\frac{M-X-Z}{M}} = \frac{X/M}{1 - \frac{Z+X}{M}} = \frac{X/M}{1 - (\frac{Z}{M}+\frac{X}{M})}$$

$$R_0 = \frac{r_1}{1-r_3-r_1} = \frac{X}{Y}$$

<u>Free Trade Case</u>

$$\frac{U_Y}{U_X} = \frac{a}{A} \left(\frac{X}{Y}\right)^{\beta+1} = 1$$

$$\frac{U_Y}{U_Z} = \frac{a}{c} \left(\frac{Z}{Y}\right)^{\beta+1} = 1$$

$$\frac{U_Z}{U_X} = \frac{c}{a} \left(\frac{X}{Z}\right)^{\beta+1} = 1$$

$$(X/Y)^{\beta+1} = A/a$$

$$X/Y = (A/a)^{\frac{1}{\beta+1}}$$

So $$Z/Y = (c/a)^{\frac{1}{\beta+1}}$$

$$X/Z = (A/c)^{\frac{1}{\beta+1}}$$

Given a total income fixed in terms of world prices,

$$M = X + Y + Z$$

then $$X = \left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} \cdot Y$$

$$Y = \frac{1}{\left(\frac{c}{a}\right)^{\frac{1}{\beta+1}}} \cdot Z$$

$$Z = \frac{1}{\left(\frac{A}{c}\right)^{\frac{1}{\beta+1}}} \cdot X$$

So

$$X = \left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} \cdot \frac{Y}{X+Y+Z} \cdot X + Y + Z$$

$$= \left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} \cdot \frac{1}{\frac{X+Y+Z}{Y}} \cdot X + Y + Z$$

$$= \left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} \cdot \frac{1}{\frac{X}{Y}+1+\frac{Z}{Y}} \cdot X + Y + Z$$

$$= \left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} \frac{1}{\left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$= \frac{\left(\frac{A}{a}\right)^{\frac{1}{\beta+1}}}{\left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$Y = \frac{1}{\left(\frac{A}{c}\right)^{\frac{1}{\beta+1}} \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$= \frac{\quad}{\left(\frac{A}{a}\right)^{\frac{1}{\beta+1}} + \left(\frac{c}{a}\right)^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$Z = \frac{(\frac{A}{a})^{\frac{1}{\beta+1}}}{(\frac{A}{a})^{\frac{1}{\beta+1}} (\frac{A}{c})^{\frac{1}{\beta+1}} + (\frac{A}{c})^{\frac{1}{\beta+1}} + (\frac{A}{c})^{\frac{1}{\beta+1}}} \cdot M$$

Substituting the values of X, Y, Z in the original function

$$U_o = A \left\{ \left[(\frac{A}{a})^{\frac{1}{\beta+1}} \frac{1}{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1} \cdot M \right]^{-\beta} \right.$$

$$+ a \left[\frac{1}{(\frac{A}{c})^{\frac{1}{\beta+1}} (\frac{c}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1} \cdot M \right]^{-\beta}$$

$$\left. + c \left[\frac{(\frac{A}{a})^{\frac{1}{\beta+1}}}{(\frac{A}{a})^{\frac{1}{\beta+1}} (\frac{A}{c})^{\frac{1}{\beta+1}} + (\frac{A}{c})^{\frac{1}{\beta+1}} + (\frac{A}{a})^{\frac{1}{\beta+1}}} \cdot M \right]^{-\beta} \right\}^{-1/\beta}$$

$$= \left\{ A \cdot (\frac{A}{a})^{\frac{-\beta}{\beta+1}} \frac{}{\{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1\}^{-\beta}} \cdot M^{-\beta} \right.$$

$$+ a \frac{M^{-\beta}}{[(\frac{A}{c})^{\frac{1}{\beta+1}} \{(\frac{c}{a})^{\frac{1}{\beta+1}}\} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1]^{-\beta}}$$

$$+ c \frac{(\frac{A}{a})^{\frac{-\beta}{\beta+1}} M^{-\beta}}{[(\frac{A}{c})^{\frac{1}{\beta+1}} (\frac{A}{c})^{\frac{1}{\beta+1}} + (\frac{A}{c})^{\frac{1}{\beta+1}} + (\frac{A}{a})^{\frac{1}{\beta+1}}]^{-\beta}} \Bigg\}^{-1/\beta}$$

$$= \left\{ \frac{M^{-\beta}}{[(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1]^{-\beta}} \left[A \cdot (\frac{A}{a})^{\frac{-\beta}{\beta+1}} + a + \frac{c(\frac{A}{a})^{\frac{-\beta}{\beta+1}}}{[(\frac{A}{c})^{\frac{1}{\beta+1}}]^{-\beta}} \right] \right\}^{-1/\beta}$$

$$= \frac{M \quad a^{-1/\beta}}{[(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1]} \left[\frac{A}{a} \quad (\frac{A}{a})^{\frac{-\beta}{\beta+1}} + 1 + \frac{\frac{c}{a} \, (\frac{A}{a})^{\frac{-\beta}{\beta+1}}}{(\frac{A}{c})^{\frac{-\beta}{\beta+1}}} \right]^{-1/\beta}$$

$$= \frac{M \quad a^{-1/\beta}}{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1} \times \left\{ (\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + \frac{\frac{c}{a}(\frac{A}{a})^{\frac{-\beta}{\beta+1}}}{(\frac{A}{c})^{\frac{1}{\beta+1}}} \right\}^{-1/\beta}$$

$$= \frac{M \quad a^{-1/\beta}}{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1} \left\{ \frac{(\frac{A}{a})^{\frac{1}{\beta+1}} \cdot (\frac{A}{c})^{\frac{-\beta}{\beta+1}} \quad (\frac{A}{c})^{\frac{-\beta}{\beta+1}} \quad \frac{c}{a}(\frac{A}{a})^{\frac{-\beta}{\beta+1}}}{(\frac{A}{c})^{\frac{-\beta}{\beta+1}}} \right\}^{-\frac{1}{\beta}}$$

$$= \frac{M\ a^{-1/\beta}}{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1} \left[(\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} \right]^{-1/\beta}$$

$$\frac{U_t}{U_o} = \frac{[R_0(1+t)^{\sigma-1} + 1 + R_1(1+t)^{\sigma-1}]^{\frac{\sigma}{\sigma-1}} \ [R_0+R_1+1]^{\frac{1}{1-\sigma}}}{[R_0(1+t)^{\sigma} + R_1(1+t)^{\sigma} + 1]}$$

Unit Elasticity Case

Let us formulate a Cobb-Douglas utility function of the following type.

$$U = X^{\alpha} Y^{\beta} Z^{1-(\alpha+\beta)} ,$$

where X = exportable good

Y = intermediate good with zero tariff

Z = importable final good with tariff

Free Trade Case with no tariff

$$\frac{dU}{dX} = \alpha\ X^{\alpha-1}\ Y^{\beta}\ Z^{1-(\alpha+\beta)}$$

$$= \frac{\alpha}{X} X^{\alpha}\ Y^{\beta}\ Z^{1-(\alpha+\beta)} = \frac{U}{X}\ \alpha$$

$$\frac{dU}{dY} = X^{\alpha}\ \beta Y^{\beta-1}\ Z^{1-(\alpha+\beta)} = X^{\alpha}\ \frac{\beta}{Y}\ Y^{\beta}\ Z^{1-(\alpha+\beta)}$$

$$= U/Y^{\beta}$$

$$\frac{dU}{dZ} = X^{\alpha}\ Y^{\beta}\ 1-(\alpha+\beta) Z^{1-(\alpha+\beta)} = X^{\alpha}\ Y^{\beta}\ \frac{1-(\alpha+\beta)}{Z}\ Z^{1-(\alpha+\beta)}$$

$$= \frac{U}{Z}\ 1-(\alpha+\beta)$$

So $U_x = \frac{U}{X}\alpha$

$$U_y = \frac{U}{Y}\beta$$

$$U_z = \frac{U}{Z}\,1-(\alpha+\beta)$$

$$\frac{U_y}{U_x} = \left(\frac{U}{Y}\beta\right)\cdot\frac{X}{U}\frac{1}{\alpha} = \left(\frac{X}{Y}\right)\frac{\beta}{\alpha} = 1$$

$$X/Y = \alpha/\beta \ \ldots$$

$$\frac{U_y}{U_z} = \frac{U}{Y}\cdot\beta\cdot\frac{Z}{U}\frac{1}{1-(\alpha+\beta)}\cdot\frac{Z}{Y}\frac{\beta}{1-(\alpha+\beta)} = 1$$

$$\therefore \quad Z/Y = \frac{1-(\alpha+\beta)}{\beta}$$

$$\frac{U_z}{U_x} = \frac{U}{Z}\,(1-(\alpha+\beta)\cdot\frac{X}{U}\frac{1}{\alpha}\quad\frac{X}{Z}\frac{(1-\alpha+\beta)}{\alpha}$$

$$X/Z = \frac{\alpha}{1-(\alpha+\beta)}$$

Given a total gross income fixed in terms of world prices,

$$M = X + Y + Z$$

$$X = (\alpha/\beta)Y$$

$$Y = \frac{\beta}{1-(\alpha+\beta)}\,Z$$

$$Z = \frac{1-(\alpha+\beta)}{\alpha}\cdot X$$

$$X = \left(\frac{\alpha}{\beta}\right)\frac{Y}{X+Y+Z}\cdot X+Y+Z$$

$$= \left(\frac{\alpha}{\beta}\right)\frac{1}{\frac{X+Y+Z}{Y}}\cdot M = \left(\frac{\alpha}{\beta}\right)\frac{1}{\frac{X}{Y}+1+\frac{Z}{Y}}\cdot M = \left(\frac{\alpha}{\beta}\right)\frac{1}{\frac{\alpha}{\beta}+1+\frac{1-(\alpha+\beta)}{\beta}}\cdot M$$

$$= \frac{(\alpha/M)}{\frac{\alpha+1-\alpha-\beta+\beta}{\beta}} \cdot M = \frac{\alpha}{\beta} \times \frac{\beta}{1} \cdot M = \alpha M$$

Similarly,

$$Y = \frac{\beta}{1-(\alpha+\beta)} \cdot Z$$

$$= \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{Z}{X+Y+Z} \cdot X + Y + Z$$

$$= \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1}{\frac{X+Y+Z}{Z}} \cdot X + Y + Z$$

$$= \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1}{\frac{X}{Z} + \frac{Y}{Z} + 1} \cdot X + Y + Z$$

$$= \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1}{\frac{\alpha}{1-(\alpha+\beta)} + \frac{\beta}{1-(\alpha+\beta)} + 1} \cdot M$$

$$= \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1}{\frac{\alpha+\beta+1-\alpha-\beta}{[1-(\alpha+\beta)]}} \cdot M$$

$$= \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1-(\alpha+\beta)}{1} \cdot M$$

$$= \beta M.$$

$$Z = \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{X}{X+Y+Z} \cdot X + Y + Z$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{\frac{X+Y+Z}{X}} \cdot X + Y + Z$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{1 + \frac{Y}{X} + \frac{Z}{X}} \cdot X + Y + Z$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{1 + \frac{\beta}{\alpha} + \frac{1-(\alpha+\beta)}{\alpha}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{\frac{\alpha+\beta+1-\alpha-\beta}{\alpha}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \alpha \cdot M = 1-(\alpha+\beta)M$$

Substituting these values in the original equation

$$U = X^{\alpha} Y^{\beta} Z^{1-(\alpha+\beta)}$$

$$U = (\alpha M)^{\alpha} (\beta M)^{\beta} [1-(\alpha+\beta)M]^{1-(\alpha+\beta)}$$

<u>Unit Elasticity Case:</u>
<u>Cobb-Douglas with Tariff</u>

$$\frac{U_Y}{U_X} = \frac{X}{Y} \cdot \frac{\beta}{\alpha} + 1 + t$$

$$X/Y = (\alpha/\beta)(1+t)$$

$$\frac{U_Y}{U_Z} = \frac{Z}{Y} \cdot \frac{\beta}{1-(\alpha+\beta)} = 1 + t$$

$$Z/Y = \frac{1-(\alpha+\beta)}{\beta}(1+t)$$

$$\frac{U_z}{U_x} = \frac{X}{Z} \; \frac{1-(\alpha+\beta)}{\alpha} = 1$$

$$X/Z = \frac{\alpha}{1-(\alpha+\beta)}$$

Given a total gross income fixed in terms of world prices,

$$M = X + Y + Z$$

$$X = (\alpha/\beta)(1+t)Y$$

$$Y = \frac{\beta}{1-(\alpha+\beta)} \; \frac{1}{(1+t)} \; Z$$

$$Z = \frac{1-(\alpha+\beta)}{\alpha} \; X$$

$$X = (\alpha/\beta)(1+t) \frac{Y}{X+Y+Z} \cdot X + Y + Z$$

$$= (\alpha/\beta)(1+t) \frac{1}{\frac{X+Y+Z}{Y}} \cdot X + Y + Z$$

$$= (\alpha/\beta)(1=t) \frac{1}{\frac{X}{Y} + 1 + \frac{Z}{Y}} \cdot M$$

$$= (\alpha/\beta)(1+t) \frac{1}{(\alpha/\beta)(1+t) + \frac{1-(\alpha+\beta)}{\beta}(1+t) + 1} \cdot M$$

$$= (\alpha/\beta)(1+t) \frac{1}{(1+t)\left[(\alpha/\beta) + \frac{1-(\alpha+\beta)}{\beta}\right] + 1} \cdot M$$

$$= (\alpha/\beta)(1+t) \frac{1}{(1+t)\frac{1-\beta}{\beta} + 1} \cdot M$$

$$= (\alpha/\beta)(1+t) \frac{1}{(1+t)\cdot\frac{(1-\beta)}{\beta} + 1} \cdot M$$

$$= (\alpha/\beta)(1+t) \frac{1}{(1+t)\frac{(1-\beta)+\beta}{\beta}} \cdot M$$

$$= \alpha(1+t) \frac{1}{(1+t)(1-\beta) + \beta} \cdot M$$

$$= \frac{\alpha(1+t)}{1+t-t\beta} \cdot M = \frac{\alpha(1+t)}{1-t-t\beta} \cdot M$$

$$Y = \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1}{(1+t)} \cdot Z$$

$$= \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1}{(1+t)} \cdot \frac{Z}{X+Y+Z} \cdot X + Y + Z$$

$$= \frac{\beta}{1-(\alpha+\beta)(1+t)} \frac{1}{\frac{X+Y+Z}{Z}} \cdot M$$

$$= \frac{\beta}{(1+t)(1-(\alpha+\beta)} \frac{1}{\frac{X}{Z} + \frac{Y}{Z} + 1} \cdot M$$

$$= \frac{\beta}{(1+t)(1-(\alpha+\beta)} \frac{1}{\frac{\alpha}{1-(\alpha+\beta)} + \frac{\beta}{1-(\alpha+\beta)(1+t)} + 1} \cdot M$$

$$\frac{\beta}{1-(\alpha+\beta)(1+t)} \frac{1}{\frac{\alpha}{1-(\alpha+\beta)} + \frac{\beta}{1-(\alpha+\beta)(1+t)} + 1} \cdot M$$

$$= \frac{\beta}{1-(\alpha+\beta)(1+t)} \quad \frac{1}{\dfrac{\alpha(1+t) + \beta + \{1-(\alpha+\beta)(1+t)\}}{[1-(\alpha+\beta)](1+t)}} \cdot M$$

$$= \frac{\beta}{1-(\alpha+\beta)(1+t)} \quad \frac{[1-(\alpha+\beta)(1+t)]}{\alpha(1+t) + \beta + \{1-(\alpha+\beta)\}(1+t)} \cdot M$$

$$= \frac{\beta}{\alpha(1+t) + \beta + [1-\alpha-\beta)(1+t)]} \cdot M$$

$$= \frac{\beta}{(\alpha + \alpha t) + \beta + (1-\alpha-\beta)(1+t)} \cdot M$$

$$= \frac{\beta}{\beta + 1 - \beta + t - \beta t} \cdot M$$

$$= \frac{\beta}{1 + t - \beta t} \cdot M = \frac{\beta}{1 = t - \beta t} \cdot M$$

$$Z = \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{X}{X+Y+Z} \cdot X + Y + Z$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{\dfrac{X+Y+Z}{X}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{1 + \dfrac{Y}{X} + \dfrac{Z}{X}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{1 + \dfrac{\beta}{(1+t)} + \dfrac{1-(\alpha+\beta)}{\alpha}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{\frac{\alpha(1+t) + \beta + (1-\alpha-\beta)(1+t)}{\alpha(1+t)}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{\alpha(t+1)}{\alpha+\alpha t+\beta+1-\alpha-\beta+t-t\alpha t\beta}$$

$$= \frac{(1-(\alpha+\beta)(t+1)}{\alpha t+1+t-t\alpha-t\beta} \cdot M = \frac{1-(\alpha+\beta)(t+1)}{1+t-t\beta} \cdot M$$

Substituting the values in the original equation

$$U_t = \left\{\frac{\alpha(1+t)}{1+t-t\beta} \cdot M\right\}^{\alpha} \left\{\frac{\beta}{1+t-\beta t} \cdot M\right\}^{\beta} \left\{\frac{1-(\alpha+\beta)(t+1)}{1-t-t\beta} \cdot M\right\}^{1-(\alpha+\beta)}$$

So

$$\frac{U_t}{U_o} = \frac{\left\{\frac{\alpha(1+t)}{1+t-t\beta} \cdot M\right\}^{\alpha} \left\{\frac{\beta}{1+t-\beta t} \cdot M\right\}^{\beta} \left\{\frac{1-\alpha-\beta(t+1)}{1+t-t\beta} \cdot M\right\}^{1-(\alpha+\beta)}}{(\alpha M)^{\alpha} (\beta M)^{\beta} \cdot [1-(\alpha+\beta)M]^{1-(\alpha+\beta)}}$$

$$= \frac{\frac{[\alpha(1+t)]^{\alpha} \cdot M^{\alpha}}{(1+t-t\ )} \frac{\beta^{\beta} \cdot M^{\beta}}{[1+t-\ t]} \frac{\{(1-(\alpha+\beta)(t+1)\}^{(1-\alpha+\beta)} M^{1-(\alpha+\beta)}}{(1+t-t\ )^{1-(\alpha+\beta)}}}{\alpha^{\alpha} M^{\alpha} \beta^{\beta} \cdot M^{\beta}[1-(\alpha+\beta)]^{1-(\alpha+\beta)} M^{1-(\alpha+\beta)}}$$

$$= \frac{[\alpha(1+t)]^{\alpha} \cdot M^{\alpha}}{(1+t-t\beta)^{\alpha}} \cdot \frac{\beta^{\beta} \cdot M^{\beta}}{[1+t-\beta t]^{\beta}} \frac{[(1-(\alpha+\beta)^{1-(\alpha+\beta)} (t+1)^{1-\alpha-\beta} (M)^{1-\alpha-\beta}}{(1+t-t\beta)^{1-(\alpha+\beta)}}$$

$$\times \frac{1}{\alpha^{\alpha} \cdot M^{\alpha} \cdot \beta^{\beta} \cdot M^{\beta}[1-(\alpha+\beta)]^{1-(\alpha+\beta)} M^{1-(\alpha+\beta)}}$$

$$= \frac{(1+t)^{\alpha}\ (1+t)^{1-\alpha-\beta}}{(1+t-t\beta)^{\alpha}\ [1+t-\beta t]^{\beta}\ (1+t-t\beta)^{1-(\alpha+\beta)}} = \frac{(1+t)^{1-\beta}}{1+t-t\beta}$$

Since $\alpha = r_1$

$\beta = r_2$

$1-(\alpha+\beta) = 1 - (r_1+r_2)$

then

$$\frac{(1+t)^{1-r_2}}{(1+t-tr_2)} = \frac{(1+t)^{1-r_2}}{1+t(1-r_2)}$$

APPENDIX B

THE DERIVATION OF THE CONSUMPTION COST MEASURE IN THE DIFFERENTIAL TARIFF CASE

THE DERIVATION OF THE CONSUMPTION COST MEASURE IN THE DIFFERENTIAL TARIFF CASE

$$\frac{U_y}{U_x} = \frac{a}{A}\left(\frac{X}{Y}\right)^{\beta+1} = 1 + t_1 \quad \text{or} \quad \frac{X}{Y} = \left\{\frac{A}{a}(1+t_1)\right\}^{\frac{1}{\beta+1}}$$

$$\frac{U_x}{U_z} = \frac{a}{c}\left(\frac{Z}{Y}\right)^{\beta+1} = 1 + t_2 \quad \text{or} \quad \frac{Z}{Y} = \left\{\frac{c}{a}(1+t_2)\right\}^{\frac{1}{\beta+1}}$$

$$\frac{U_z}{U_x} \quad \frac{c}{A}\left(\frac{X}{Z}\right)^{\beta+1} = 1 + t \quad \text{or} \quad \frac{X}{Z} = \left\{\frac{A}{c}(1+t\ )\right\}^{\frac{1}{\beta+1}}$$

Let total gross income fixed in terms of world prices be

$$X + Y + Z = M$$

$$X = \frac{A}{a}(1+t_1)^{\frac{1}{\beta+1}} \cdot Y$$

$$Y = \frac{1}{\left\{\frac{c}{a}(1+t_2)\right\}^{\frac{1}{\beta+1}}} \cdot Z$$

$$Z = \frac{1}{\left\{\frac{A}{c}(1+t_3)\right\}^{\frac{1}{\beta+1}}} \cdot X$$

From previous calculations

$$X = \frac{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{C}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$Y = \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} \{\frac{C}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + \{\frac{C}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1} \cdot M$$

$$Z = \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}} \cdot \frac{X}{X+Y+Z} \cdot X + Y + Z$$

$$= \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}} \cdot \frac{1}{1 + \frac{Y}{X} + \frac{Z}{X}} \cdot M$$

$$= \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}} \cdot \frac{1}{1 + \frac{1}{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}} + \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}}} \cdot M$$

$$= \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}} \cdot \frac{1}{\{\frac{A}{c}(1+t_1)\}^{\frac{1}{\beta+1}} \quad \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}} \cdot M$$

$$+ \frac{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t\ )\}^{\frac{1}{\beta+1}} \cdot \{\frac{A}{c}(1+t\ )\}^{\frac{1}{\beta+1}}}$$

$$= \frac{1}{\{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}} \cdot \frac{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t\ )\}^{\frac{1}{\beta+1}}} \cdot M$$

$$= \frac{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}}{\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}} \cdot M$$

Substituting these values of X, Y, Z in the original utility function we get

$$U = \left[\frac{\{A \cdot [\frac{A}{a}(1+t_1)]^{\frac{-\beta}{\beta+1}}}{[\{\frac{A}{a}(1+t\ )\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t\ )\}^{\frac{1}{\beta+1}} + 1]^{-\beta}} \cdot M^{-\beta} \right.$$

$$+ a \frac{M^{-\beta}}{[\{\frac{A}{c}(1+t\ )\}^{\frac{1}{\beta+1}} \{\frac{c}{a}(1+t\ )\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t\ )\}^{\frac{1}{\beta+1}} + 1]^{-\beta}}$$

$$\left. \frac{c\ \{\frac{A}{a}(1+t\ )\}^{\frac{1}{\beta+1}} M^{-\beta}}{\left[\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{c}(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}}\right]^{-\beta}} \right]^{\frac{1}{\beta}}$$

$$= \left[\frac{A \cdot [\frac{A}{a}(1+t\ )]^{\frac{-\beta}{\beta+1}} M^{-\beta}}{[\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}} \right.$$

$$+ a \frac{M^{-\beta}}{[\{\frac{A}{a}(1+t_2)(1+t_3)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}}$$

$$\left. + \frac{c \cdot \frac{A}{a}(1+t_1)^{\frac{-\beta}{\beta+1}} M^{-\beta}}{\left[\{\frac{A}{c}(1+t\ )\}^{\frac{1}{\beta+1}} \{\frac{A}{a}(1+t\ )\}^{\frac{1}{\beta+1}} + 1 + \frac{c}{a}\frac{1+t_1}{1+t_3}^{\frac{1}{\beta+1}}\right]^{-\beta}} \right]^{-1/\beta}$$

Assuming $\frac{1+t_1}{L+t_3} = 1+t_2$

$$= \left[\frac{A \cdot [\frac{A}{a}(1+t_1)]^{\frac{-\beta}{\beta+1}} M^{-\beta}}{[\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}} \right.$$

$$+ a \frac{M^{-\beta}}{[\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1]^{-\beta}}$$

$$\left[(\frac{A}{a})^{\frac{1}{\beta+1}} (1+t_1)^{\frac{-\beta}{\beta+1}} + 1 + \frac{c}{a} \left\{ \frac{c}{a} \frac{(1+t_1)}{(1+t_3)} \right\}^{\overline{\beta+1}} \right]^{-1/\beta}$$

$$= \left[(\frac{A}{a})^{\frac{1}{\beta+1}} (1+t_1)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} (1+t_2)^{\frac{-\beta}{\beta+1}} \right]^{-1/\beta}$$

$$\frac{U_t}{U_o} = \frac{M\ a^{-1/\beta}}{[\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1]}$$

$$\times \left[(\frac{A}{a})^{\frac{1}{\beta+1}} (1+t_1)^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} (1+t_2)^{\frac{-\beta}{\beta+1}} \right]^{-1/\beta}$$

$$\times \frac{(\frac{A}{a})^{\frac{1}{\beta+1}} + (\frac{c}{a})^{\frac{1}{\beta+1}} + 1}{M\ a^{-1/\beta} \left[(\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} \right]^{-1/\beta}}$$

$$= \left[(\frac{A}{a})^{\frac{1}{\beta+1}} (1+t\ )^{\frac{-\beta}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} (1+t\ )^{\frac{-\beta}{\beta+1}} \right]^{-1/\beta}$$

$$\frac{\times\ (\frac{A}{a})^{\beta+1} + (\frac{c}{a})^{\beta+1} + 1}{\left[\{\frac{A}{a}(1+t_1)\}^{\frac{1}{\beta+1}} + \{\frac{c}{a}(1+t_2)\}^{\frac{1}{\beta+1}} + 1 \right.}$$

$$\left. (\frac{A}{a})^{\frac{1}{\beta+1}} + 1 + (\frac{c}{a})^{\frac{1}{\beta+1}} \right]^{-1/\beta}$$

Substituting R_0, R_1, we obtain:

$$\frac{[R_0(1+t_1)^{\sigma-1} + 1 + R_1(1+t\ )^{\sigma-1}]^{\frac{\sigma}{\sigma-1}} \, [R_0+R_1+1]^{\frac{1}{1-\sigma}}}{[R_0(1+t_1)^{\sigma} + R_1(1+t_2)^{\sigma} + 1]}$$

<u>Differential Tariff: Unit Elasticity of Substitution Case: Cobb-Douglas</u>

$$\frac{U_Y}{U_X} \cdot \frac{X}{Y} \cdot \frac{\beta}{\alpha} = 1 + t_1$$

$$\frac{X}{Y} = \left(\frac{\alpha}{\beta}\right) (1+t_1)$$

$$\frac{U_Y}{U_Z} = \frac{Z}{Y} \, \frac{\beta}{1-(\alpha+\beta)} = 1 + t_2$$

$$\frac{Z}{Y} = \frac{1-(\alpha+\beta)}{\beta} (1+t_2)$$

$$\frac{U_Z}{U_X} = \frac{X}{Z} \, \frac{1-(\alpha+\beta)}{\alpha} = 1 + t_3$$

$$\frac{X}{Z} = \frac{\alpha}{1-(\alpha+\beta)} (1+t_3)$$

Given a total gross income fixed in terms of world prices:

$$M = X + Y + Z$$

$$X = (\alpha/\beta)(1+t_1) \cdot Y$$

$$Y = \frac{\beta}{1-(\alpha+\beta)} \cdot \frac{1}{(1+t_2)}$$

$$Z = \frac{1-(\alpha+\beta)}{\alpha} \cdot \frac{1}{(1+t_3)} \cdot X$$

From previous calculations

$$X = (\alpha/\beta)(1+t_1) \frac{1}{(\alpha/\beta)(1+t_1) + \frac{1-(\alpha+\beta)}{\beta}(1+t_2) + 1} \cdot M$$

$$= (\alpha/\beta)(1+t_1) \frac{1}{\frac{\alpha t_1+1-\alpha-\beta+t_2-\alpha t_2-t_2\beta+\beta}{\beta}} \cdot M$$

$$= (\alpha/\beta)(1+t_1) \frac{1}{\frac{\alpha t_1+1+t_2-\alpha t_2-t_2\beta}{\beta}} \cdot M$$

$$= \frac{\alpha(1+t_1)}{\beta} X \frac{\beta}{(\alpha t_1+1+t_2-\alpha t_2-t_2\beta)} \cdot M$$

$$= \frac{\alpha + \alpha t_1}{\alpha t_1+1+t_2(1-\alpha-\beta)} \cdot M$$

$$Y = \frac{\beta}{1-(\alpha+\beta)} \quad \frac{1}{(1+t_2)} \cdot Z$$

$$= \frac{\beta}{1-(\alpha+\beta)(1+t_2)} \quad \frac{1}{\frac{\alpha}{1-(\alpha+\beta)}(1+t\ ) + \frac{\beta}{1-(\alpha+\beta)(1+t_2)} + 1} \cdot M$$

$$= \frac{\beta}{1-(\alpha+\beta)(1+t_2)} \quad \frac{1}{\frac{\alpha(1+t_2)(1+t_3)+\beta+1-(\alpha+\beta)(1+t_2)}{1-(\alpha+\beta)\ (1+t_2)}} \cdot M$$

$$= \beta \frac{1}{\alpha(1+t_2)(1+t_3)+\beta+1-(\alpha+\beta)(1+t_2)} \cdot M$$

Substituting $(1+t_2)(1+t_3) = 1 + t_1$

$$= \frac{\beta}{\alpha(1+t_1)+\beta+[1-(\alpha+\beta)(1+t_2)]} \cdot M$$

$$= \frac{\beta}{\alpha t_1+1+t_2-\alpha t_2-\beta t_2} \cdot M$$

$$= \frac{\beta}{\alpha t_1+1+t_2-\alpha t_2-\beta t_2} \cdot M = \frac{\beta}{\alpha t_1+1+t_2(1-\alpha-\beta)} \cdot M$$

$$Z = \frac{1-(\alpha+\beta)}{\alpha} \quad \frac{1}{(1+t\ \ )} \cdot X$$

$$= \frac{1-(\alpha+\beta)}{\alpha(1+t_3)} \cdot \frac{1}{1+\dfrac{\beta}{\alpha(1+t_1)}+\dfrac{1-(\alpha+\beta)}{\alpha(1+t_3)}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{(1+t_3)} \cdot \frac{1}{\dfrac{\alpha(1+t_1)(1+t_3)+\beta(1+t_3)+1-(\alpha+\beta)(1+t_1)}{\alpha(1+t_1)(1+t_3)}} \cdot M$$

$$= \frac{1-(\alpha+\beta)}{\alpha(1+t_3)} \cdot \frac{(1+t_1)\ (1+t_3)}{\alpha(1+t_1)(1+t_3)+\beta(1+t_3)+1-(\alpha+\beta)(1+t_1)} \cdot M$$

$$= \frac{1-(\alpha+\beta)(1+t_1)(1+t_2)}{\alpha(1+t_1)(1+t_3)(1+t_2)+\beta(1+t_3)(1+t_2)+1-(\alpha+\beta)(1+t_1)(1+t_2)} \cdot M$$

Substituting $\frac{1+t_1}{1+t_3} = 1+t_2$

$$= \frac{1-(\alpha+\beta)(1+t_2)}{\alpha(1+t_1)+\beta+1-(\alpha+\beta)(1+t_2)} \cdot M$$

$$= \frac{1-(\alpha+\beta)(1+t_2)}{\alpha+\alpha t_1+\beta+1-\alpha-\beta+t_2-\alpha t_2-\beta t_2} \cdot M$$

$$= \frac{1-(\alpha+\beta)(1+t_2)}{\alpha t_1+1+t_2-\alpha t_2-\beta t_2} \cdot M$$

Substituting the values of X, Y and Z in the original function

$$U_t = \left[\frac{\alpha+\alpha t_1}{\alpha t_1+1+t_2(1-\alpha-\beta)} \cdot M\right]^{\alpha} \left[\frac{\beta}{\alpha t_1+1+t_2(1-\alpha-\beta)} \cdot M\right]^{\beta}$$

$$\left[\frac{1-(\alpha+\beta)(1+t_2)}{\alpha t_1+1+t_2-\alpha t_2-\beta t_2} \cdot M\right]^{1-(\alpha+\beta)}$$

$$= \frac{(\alpha+\alpha t_1)^{\alpha} \cdot M^{\alpha}}{[\alpha t_1+1+t_2(1-\alpha-\beta)]^{\alpha}} \quad \frac{\beta}{[\alpha t_1+1+t_2(1-\alpha-\beta)]^{\beta}} \cdot M^{\beta}$$

$$\frac{[1-(\alpha-\beta)(1+t_2)]^{1-(\alpha+\beta)} \cdot M^{1-(\alpha+\beta)}}{[\alpha t_1+1+t_2(1-\alpha-\beta)]^{1-(\alpha+\beta)}}$$

$$= \frac{(\alpha+\alpha t_1)^{\alpha} \cdot M^{\alpha} \cdot \beta^{\beta} \cdot M^{\beta} \cdot [(1-(\alpha+\beta)(1+t_2)]^{1-\alpha-\beta} \cdot M^{1-(\alpha+\beta)}}{(\alpha t_1+1+t_2(1-\alpha-\beta)}$$

$$= \frac{(\alpha+\alpha t_1)^{\alpha} \cdot M^{\alpha} \cdot \beta^{\beta} \cdot M^{\beta} \cdot [1-(\alpha+\beta)(1+t_2)]^{1-(\alpha+\beta)} \; M^{1-\alpha-\beta}}{\alpha t_1+1+t_2(1-\alpha-\beta)}$$

$$= \frac{M \quad \beta^{\beta}(\alpha+\alpha t_1)^{\alpha}[1-(\alpha+\beta)(1+t_2)]^{1-(\alpha+\beta)}}{\alpha t_1+1+t_2(1-\alpha-\beta)}$$

So

$$\frac{U_t}{U_o} = \frac{M \cdot \beta^{\beta}(\alpha+\alpha t_1)^{\alpha}[1-(\alpha+\beta)(1+t_2)]^{1-\alpha-\beta}}{\alpha t_1+1+t_2(1-\alpha-\beta)}$$

$$\times \frac{1}{(\alpha M)^{\alpha} \cdot (\beta M)^{\beta} \; [1-(\alpha+\beta)M]^{1-(\alpha+\beta)}}$$

$$= \frac{(\alpha+\alpha t_1)^{\alpha}[1-\beta)(1+t_2)]^{1-(\alpha+\beta)}}{\alpha t_1+1+t_2(1-\alpha-\beta)} \quad [1-\beta]^{1-(\alpha+\beta)}$$

$$= \frac{(1+t_2)^{1-(\alpha+\beta)}}{t_1+1+t_2^{1-\alpha-\beta}} \; \frac{(\alpha+\alpha t_1)^{\alpha}}{\alpha^{\alpha}}$$

$$= \frac{(1+t_2)^{1-(\alpha+\beta)}}{t_1+1+t_2(1-\alpha-\beta)} \quad (1+t_1)^{\alpha}$$

$$= \frac{(1+t_2)^{1-\alpha-\beta} \quad (1+t_1)^{\alpha}}{t_1+1+t_2^{(1-\alpha+\beta))}}$$

Substituting

$$X/M = \alpha = r_1$$

$$Y/M = \beta = r_2$$

$$Z/M = 1-\alpha-\beta = 1-r_1-r_2$$

we obtain

$$\frac{(1+t_2)^{1-r_1-r_2}\,(1+t_1)^{r_1}}{r_1 t_1 + 1 + t_2(1-r_1-r_2)}$$